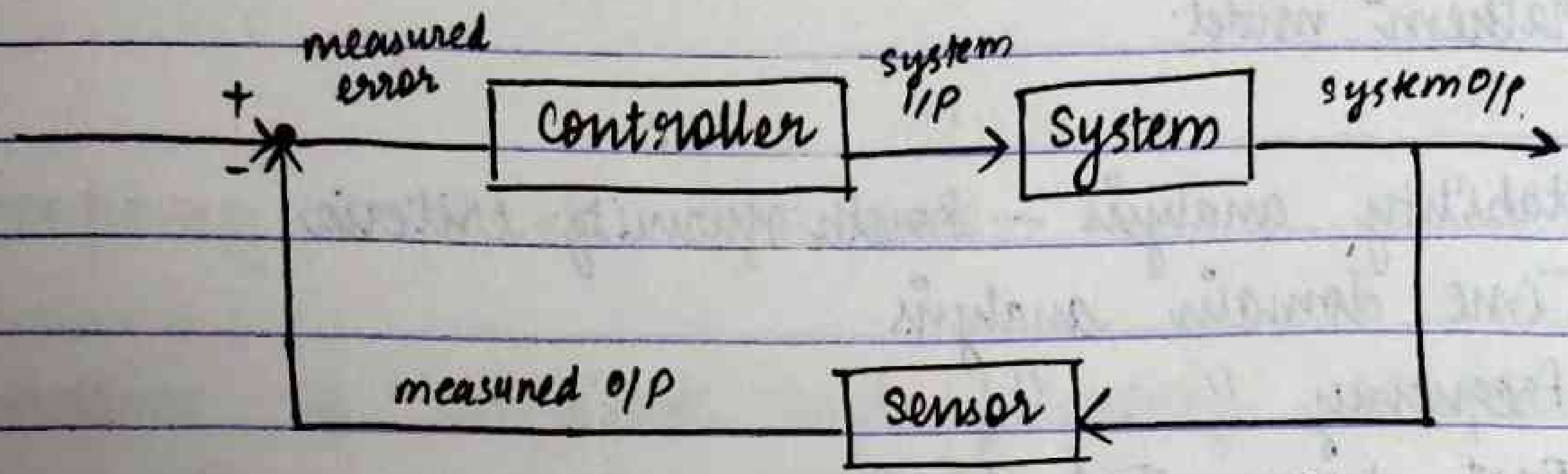
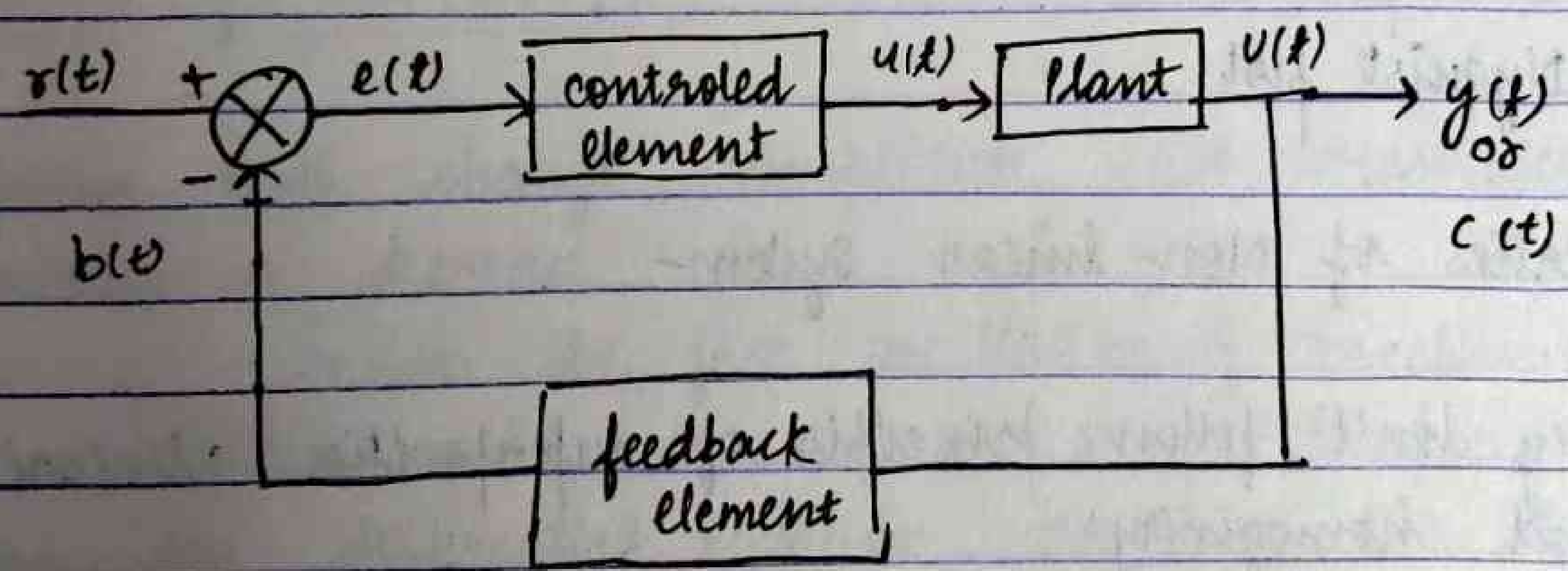
UNIT-I

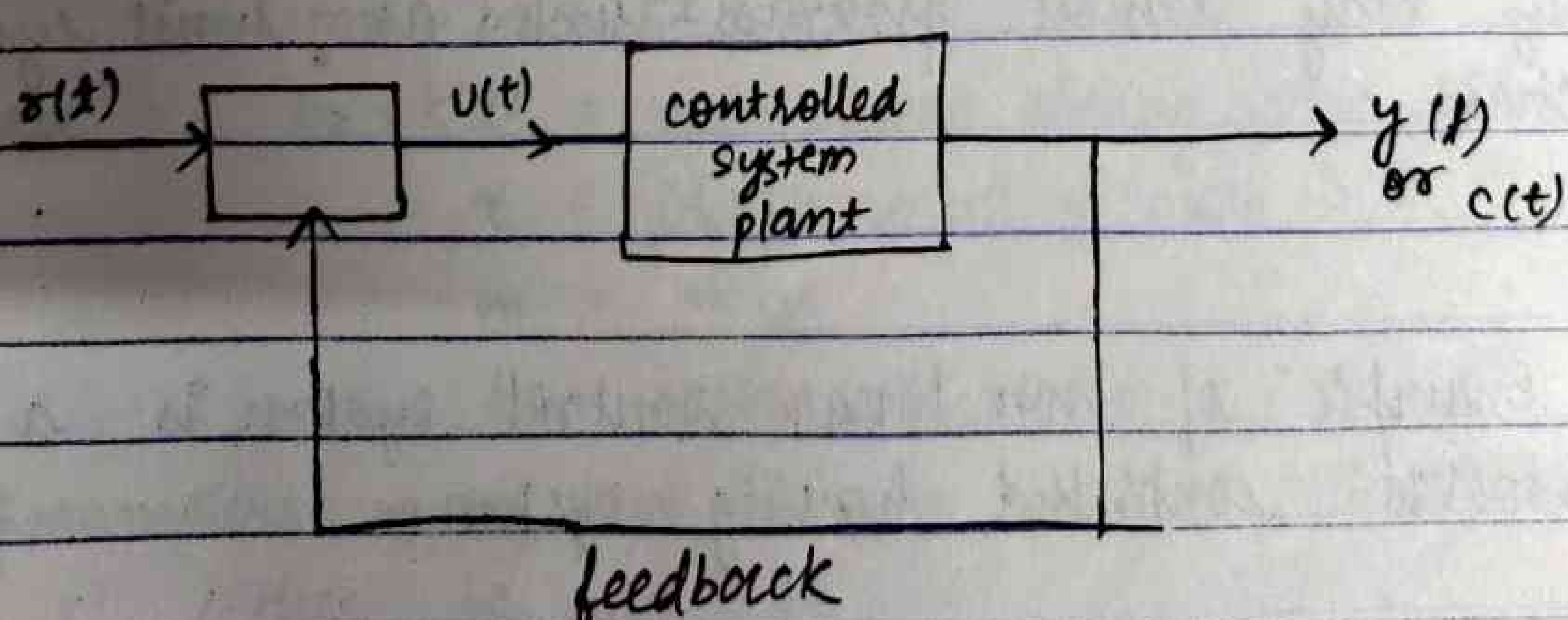
Non - linear control system

- Non-linear control theory is the area of control theory which deals with system that are non-linear, time variant or both

- It concerned with the behaviour of dynamical system with I/P and how to modify O/P by changing in I/P using feedback, feedforward or signal filtering.



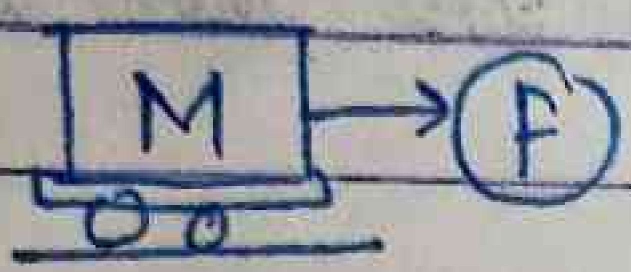
- 



- Two types of system -

(a) Static - it doesn't depend upon time, represented as algebraic eq

(b) Dynamic - it depends upon time, represented as differential

- 

$$M\ddot{x} = F \Rightarrow \dot{x} = f(x, u)$$

- Two types of Models -

(a) Physical model

(b) Mathematical model.

- Stability analysis - Routh Hurwitz criteria.

→ Time domain analysis

→ Frequency " "

→ Root Locus Technique

→ Bode plot

→ Nyquist plot

Properties of Non-linear system-

- They don't follow principle of superposition (linearity and homoginity)
- They may have multiple isolated equilibium points
- They may exhibit properties such as limit cycle, chaos.

Note-

Example of non-linear control system is a thermostat controlled heating system.

* BEHAVIOUR OF NON-LINEAR SYSTEM :- on the basis of frequency

Non-linear system to a particular test signal is no guide to behaviour to other inputs. The non-linear system response may be highly sensitive to I/P amplitude

→ $\dot{X} = AX + BU$

$X = f(x, U)$ $\qquad$ $\dot{X} = AX \quad , U = 0$

$U = u \sin \omega t$ $\qquad$ $U = U_1 + U_2 \neq X_1 + X_2$

$y = f(x, V)$

(a) Frequency-amplitude dependence-

- Dependance of amplitude and frequency characterstic shown by non-linear system

$\boxed{O/P \Rightarrow y = \sin(\omega t + \phi)}$

Response will change, amplitude will dramatically increase or decrease.

Consider the free oscillation of mechanical system which consist of a mass, viscous damper, nonlinear spring. The differential equation describing the dynamic of the system should be written as-

$$m\ddot{x} + b\dot{x} + kx + k'x^3 = 0$$

where $kx + k'x^3$ = non linear spring force

x = displacement of mass

m = mass

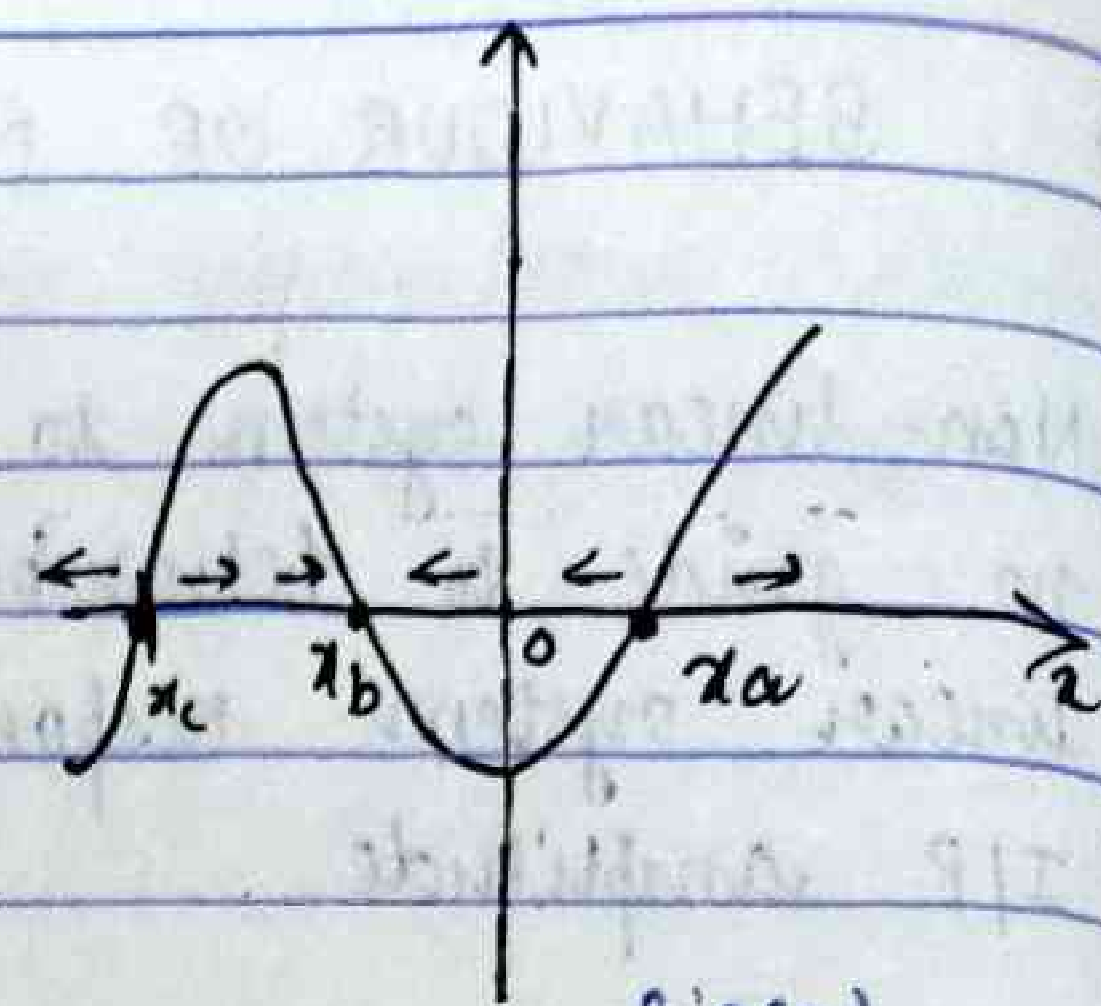
b = viscous friction

The parameters m, b and k are positive constants while k' either be +ve or -ve

(b) Multiple equilibirium-

$\dot{x} = f(x)$

Derivative point should be zero called equilibirium point



fig(a)

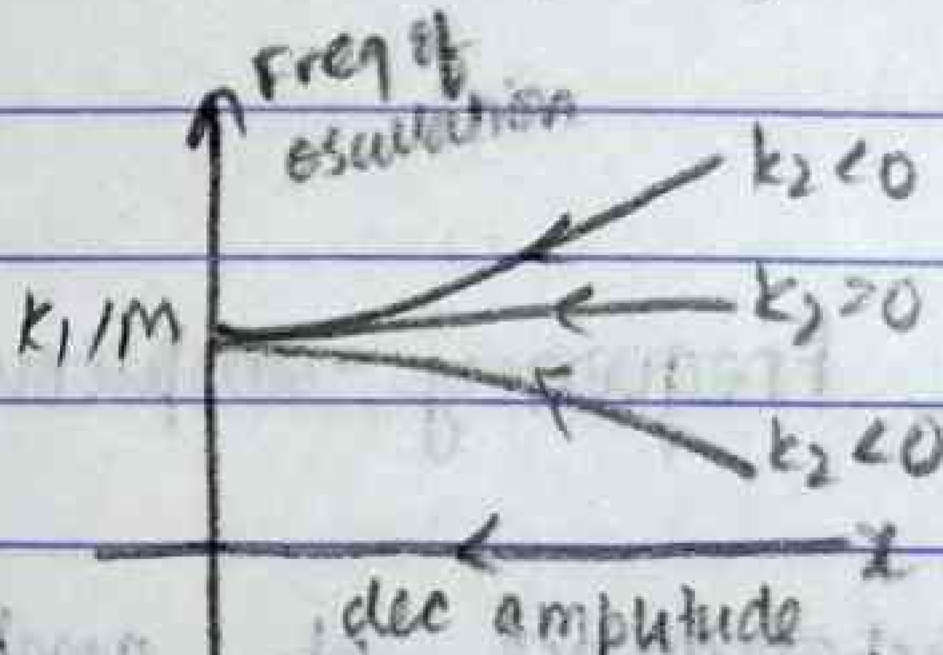
→ From fig (a)-

Case a ⇒ if $x > x_a$, then
$f(x) > 0$
and $x(t)$ is increasing

→← convergent
←→ divergent.

Case b = if $x < x_a$, then
$f(x) < 0$
and $x(t)$ is decreasing.

Hence,
$x_b < x < x_a$
$f(x) < 0 \Rightarrow x(t)$ is decreasing



Freq vs Amp for oscillation of spring mass damper system

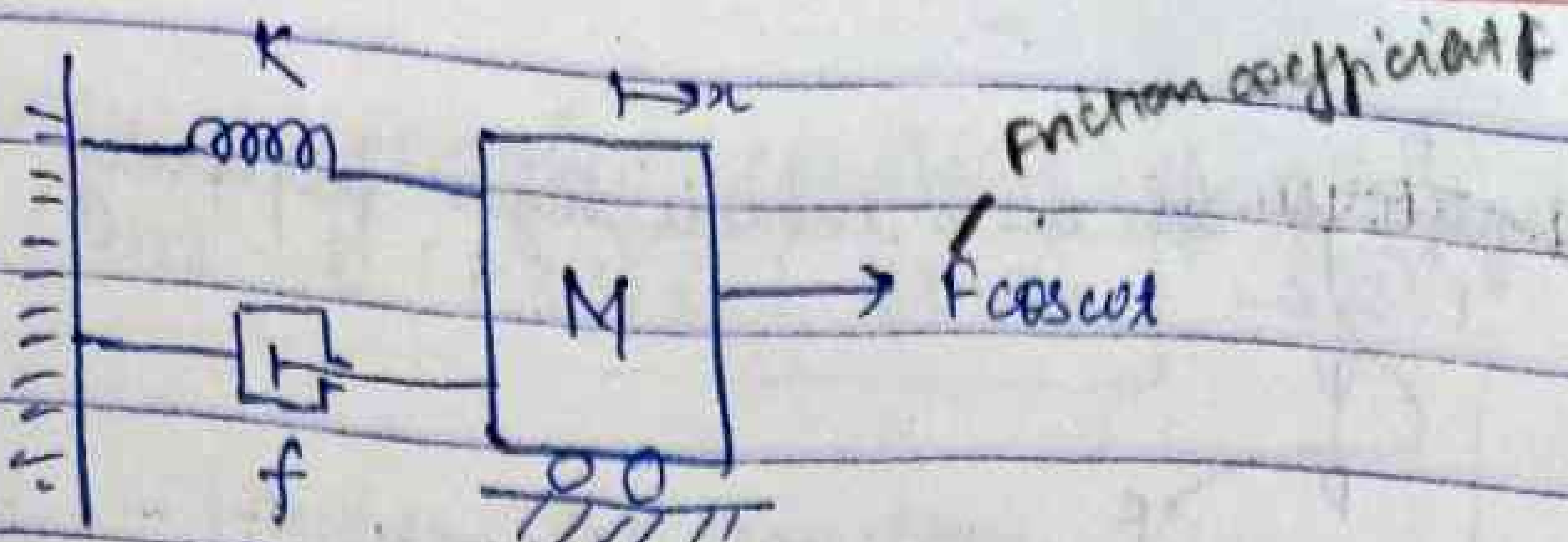
(c) Unstable equilibirium-

Small deviation if occur in any state it will led the change in the state. Deviating point converging in same point and many equilibirium points is there is non linear system, some are stable and some are unstable.

Origin of state point is the equilibirium point for linear system.

(d) Multivalued Responses: (Jump responces)-

Let's take example of one mass spring damper system.



$$M\ddot{x} + f\dot{x} + Kx = F\cos\omega t$$

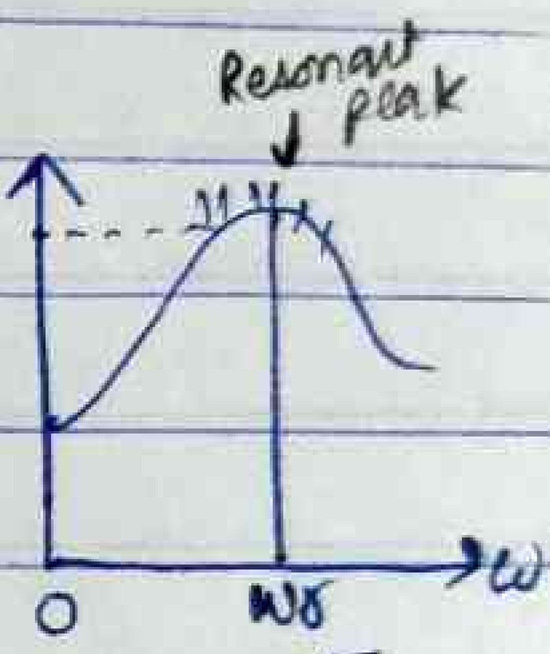
→ Linear system.

For non linear system we got different value at same frequency and for linear system we got different value at different frequency.

→ For non linear system spring eqⁿ is -

$$\boxed{k_1x + k_2x^3 = f_K}$$

restoring force is f_t



$\omega = \sqrt{\frac{K}{m}}$

• Dynamic equation for non-linear system is $\boxed{M\ddot{x} + f\dot{x} + k_1x + Kx^3 = F\cos\omega t}$

$$\boxed{k_1x + k_2x^3 = f_K}$$

If,

$k_2 = 0$ → linear spring (stable limit cycle)

$k_2 > 0$ → non linear spring called hard spring (unstable limit cycle)

$k_2 < 0$ → soft spring

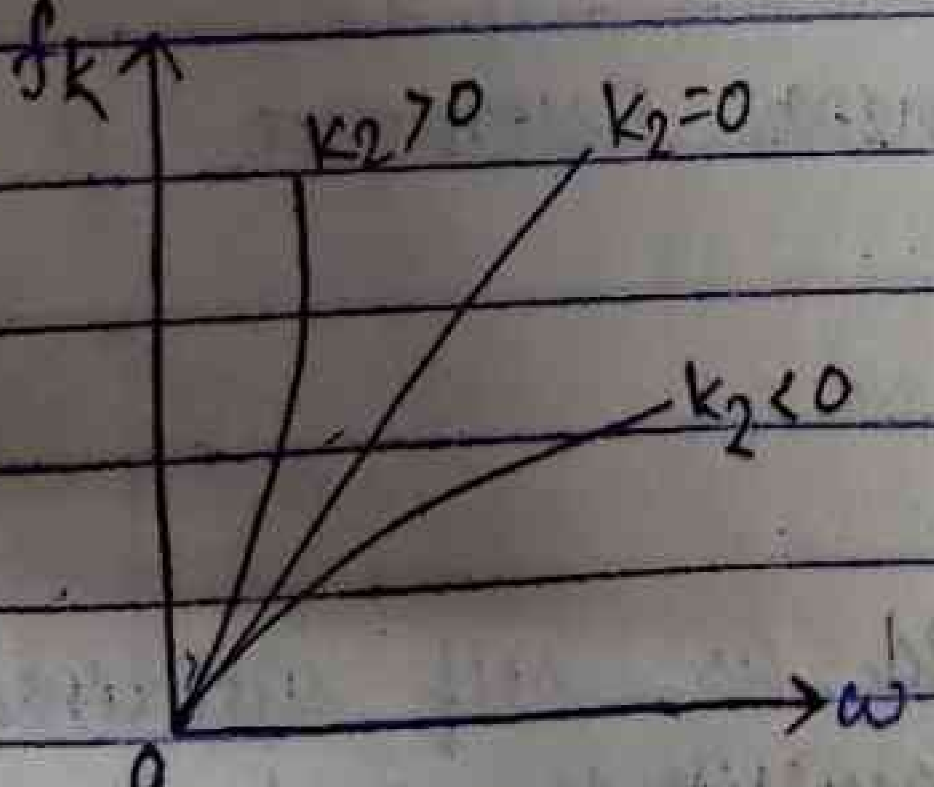


fig (a) (spring characteristic)

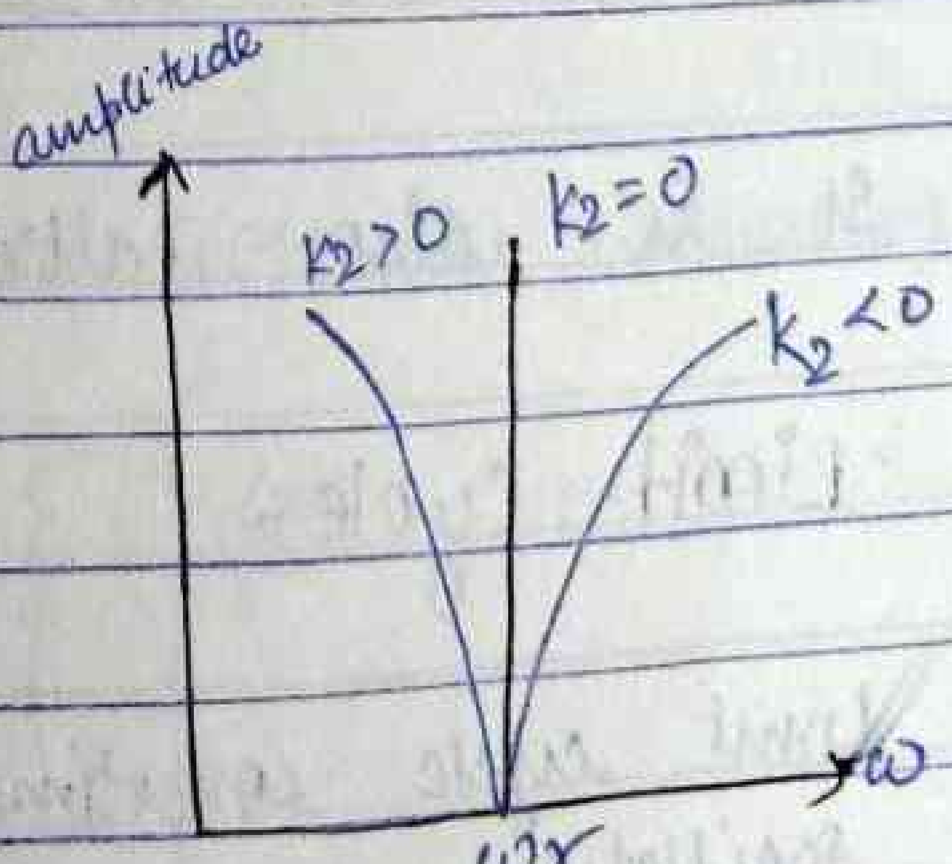
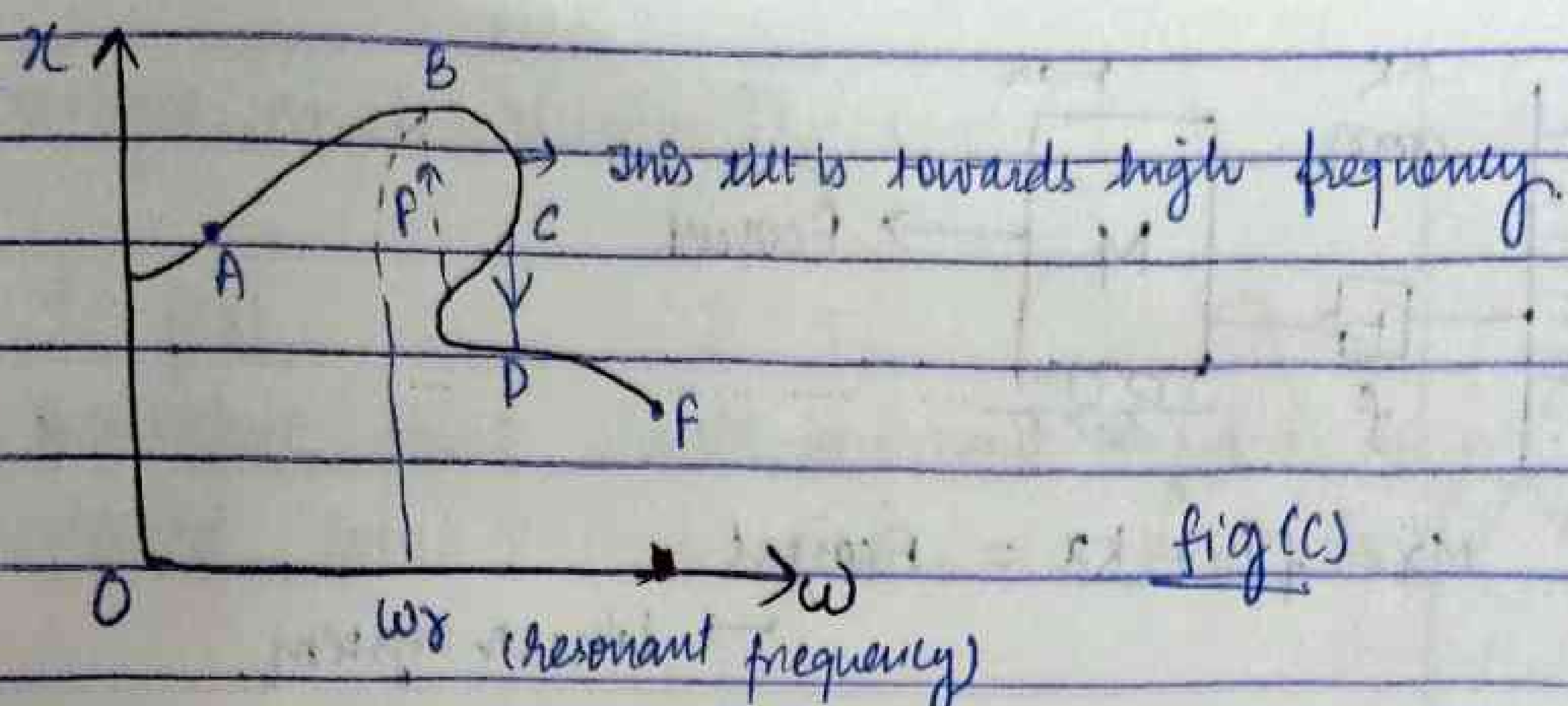


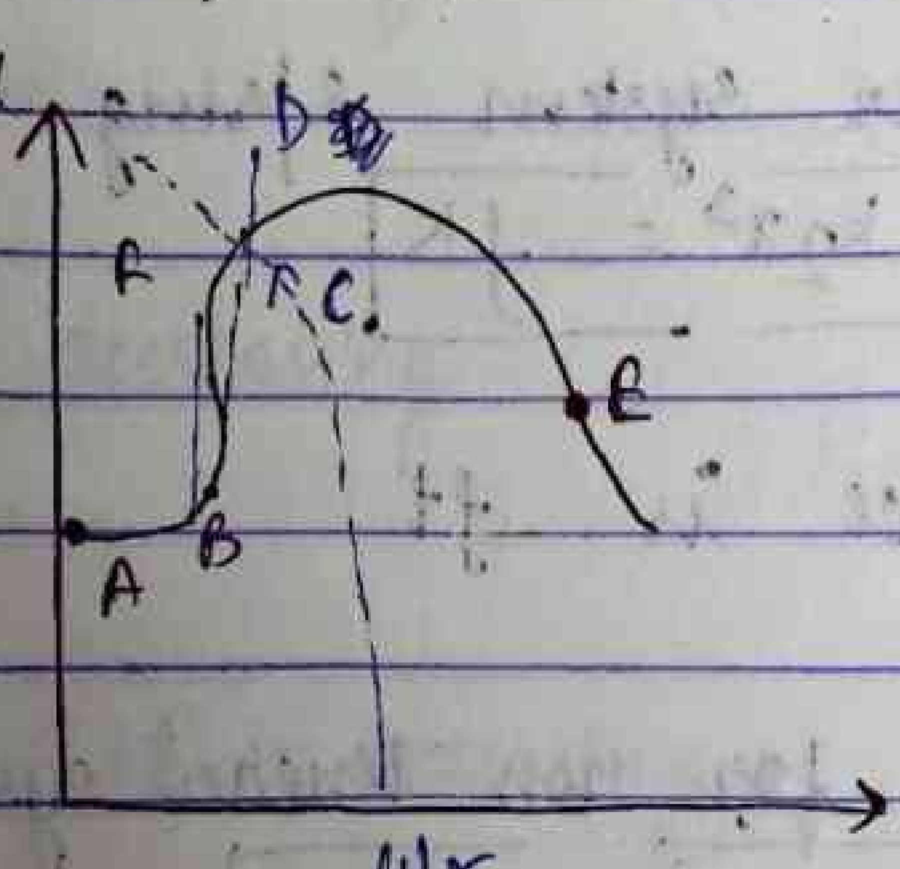
fig (b) (spring



fig(c)

1. For forward direction – ABCDF
2. For Reverse direction – FDFBA.

For soft spring – $k_2 < 0$



fig(d)

- We again observe multivalued response when ω is increasing movement is upward. This phenomenon is called jump response or we can defined as 'The amplitude of variation can increase or decrease abruptly as the excitation frequency is increased or decreased.

- It is also called non-linear resonance.

(e) Limit Cycle –

Limit cycle sometimes denoted as self sustained oscillations i.e., of fixed amplitude and frequency angle.

We know dynamic equation of a system;

$$\boxed{m\ddot{x} - b(1-x^2)\dot{x} + Kx = 0} \quad \text{—} (*)$$

Vander Pol equation ('if $x << 1$)

(mass-damper spring eqn)

- standard equation of damping is–

$$\boxed{s^2 + 2\xi\omega_n s + \omega_n^2 = 0} \quad \text{—} (A)$$

where,

$s^2 =$

$\omega_n =$ undamped natural frequency

$\xi =$ damping factor

Now from eq (A)

$$-\xi\omega_n \pm j\omega_d$$

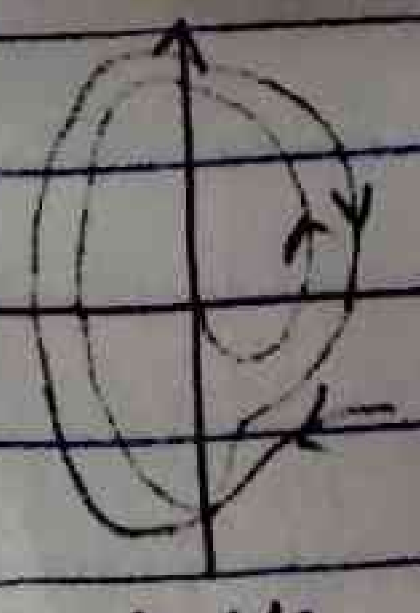
$$\boxed{\omega_d = \omega_n\sqrt{1-\xi^2}}$$

Note – if $(1-x^2)$ is +ve, then b is become +ve

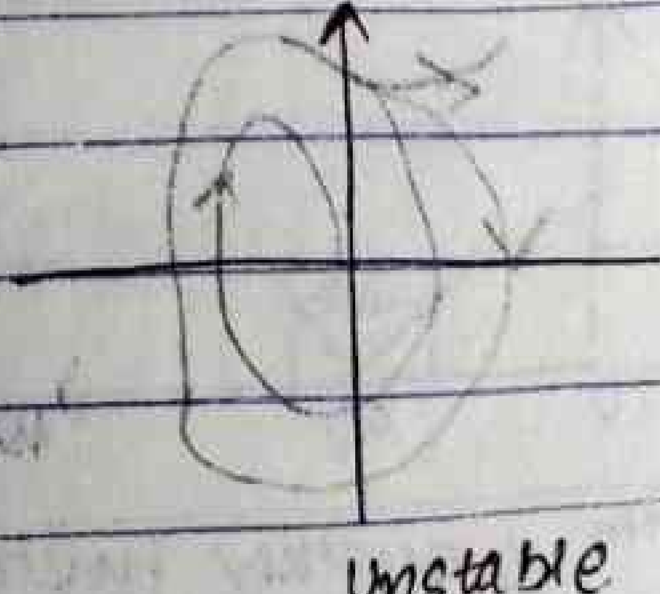
- At steady state time response reaches to it's state system energy is going to decrease.

- Negative damping is obtained when $(1-x^2)$ should be less than 1

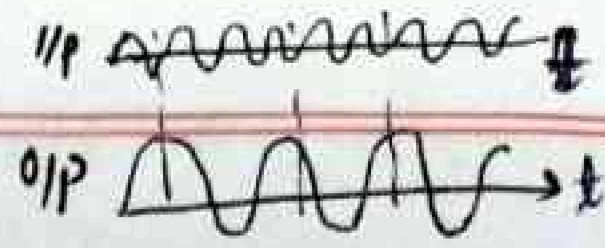
- If system is damped, oscillation is increasing.



Stable



Unstable (diverging limit cycle)



- Consider a system described by following eqn-

$$\boxed{m\ddot{x} - b(1-x^2)\dot{x} + kx = 0}$$

Where, m, b, k are the quantities. This equation is called Vander Pol equation. It is non-linear in the damping term.

- For small value of x damping will be -ve & will actually put energy to the system.

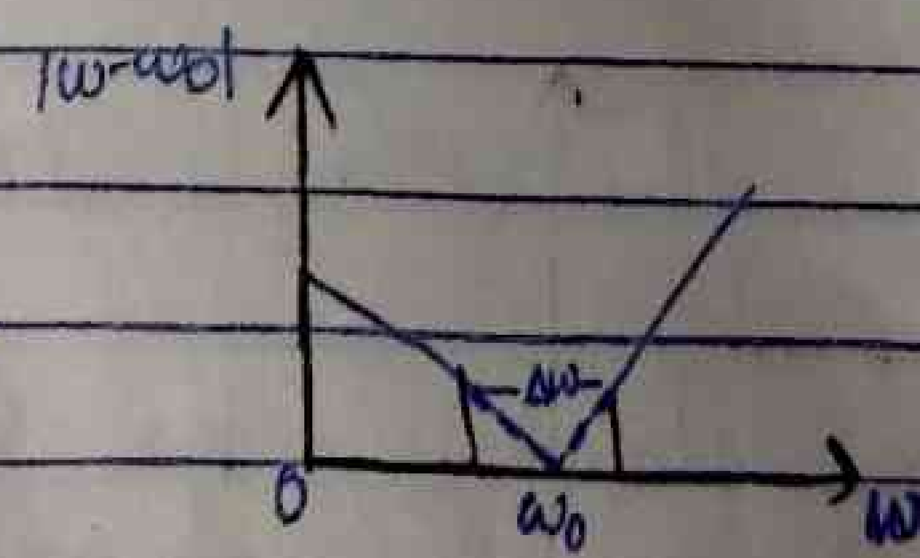
- If a system possess only one limit cycle, the amplitude of this limit cycle doesn't depend upon the initial condition.

(f) Frequency entrainment -

- If a periodic force of frequency ω is applied to a system capable of exhibiting a limit cycle of frequency ω_0, the 'beat phenomenon' is observed.

- As the difference b/w the two frequency decreases the beat frequency also decreased

- The frequency ω_0 of limit cycle falls in synchronicity with, or entrained by forcing freq ω within a certain band of frequencies (this known as freq entrainment).



ω_0 - limit cycle
w - I/P cycle.

→ Rythm of another function
→ Rythm is taken within a particular range of freq in a system

(g) Asynchronous quenching-

Such type of system is called signal stabilization
When ω change, the ω_0 may be quenched (end)

- In a nonlinear system that exhibits a limit cycle of frequency ω_0, it is possible to quench the limit cycle oscillation by forcing the system at frequency ω_1 is called 'asynchronous' quenching or 'signal stabilization'

(h) Subharmonic Oscillations-

- They are non-linear steady state oscillations whose frequencies are an integral submultiple of the forcing frequency.

- Observe in non-linear system

  $u(t) = u \sin \omega t$

  $y(t) = y \sin \omega t + \phi$

- For non-linear, if I/P is sinusoidal then O/P is not same but for linear system it's vice-versa.
  First harmonic = fundamental harmonic

- Submultiple $\rightarrow \frac{\omega}{n} = \frac{\sin \omega t}{n}$

  $\omega \rightarrow$ fundamental freq

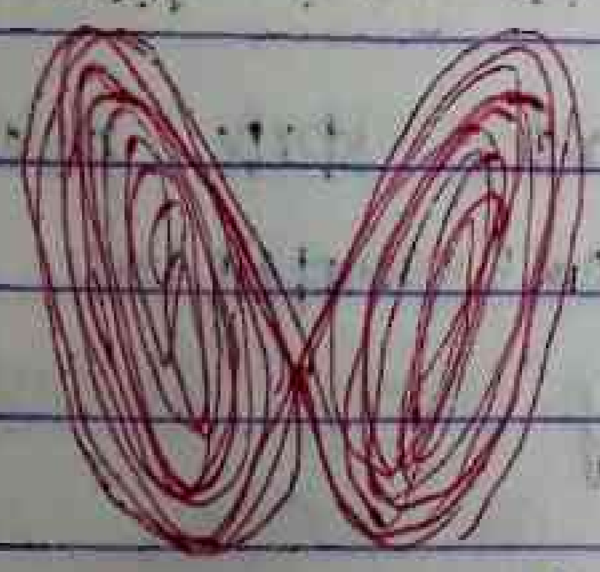
  $n\omega \rightarrow$ harmonic

(iii) Chaotic behaviour -

- It is known as 'butterfly behaviour'
- Highly non-linear system, small change in parameter around the system changes behaviour of system.

- 'Chaos' implies the existence of unpredictable or random behaviour

- Theoretically, chaotic behaviour could not occur in finite state machine because the state vector would eventually be periodic.



'Chaotic behaviour'

- strongly perturbed orbits, like those of planetary ring systems or stellar orbits in a ~~gl~~ galaxy, will display chaotic behaviour.

* Non-Linear System Analysis -

Restoring force, $f_k = k_1x + k_2x^3$

x = distortion

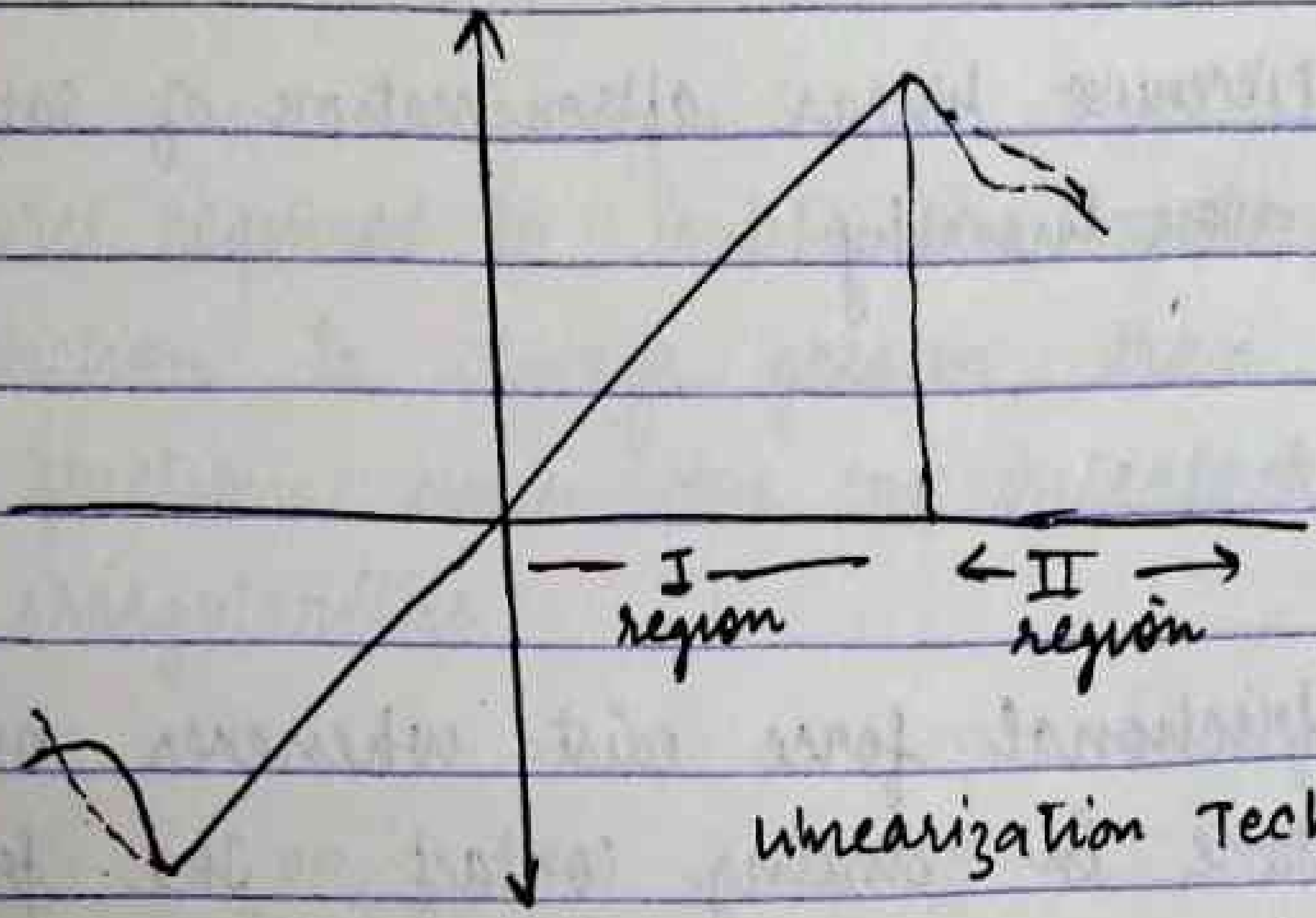
Case(i) - If $\boxed{k_2x^3 << k_1x}$, then k_2x^3 must neglected.

For mass-spring-damper :- $\boxed{M\ddot{x} + f\dot{x} + k_1x + k_2x^3 = 0}$ → non-linear system estimated as

$\boxed{M\ddot{x} + f\dot{x} + k_1x = 0}$ → approximate model

The four analysis of non-linear system are-

(a) Linearization Technique
(b) Describing function Analysis
(c) Phase plane analysis
(d) Lyapunov's stability analysis.



Linearization Techique.

VImp COMMON PHYSICAL NON-LINEARITIES-

(1) Incidental non-linearities } on the basis of magnitude
(2) Intentional non-linearities
(3) Multivariable non-linearities (ex. Torque speed characteristics)

(1) Incidental non-linearities- Saturation, Deadzone, Hysteresis · backlash · static, coulomb friction · Nonlinear spring · compressibility of fluid

They are inherinthy present in the system, also known as inherent non-linearity. Common example of incidental non-linearities are saturation, dead zone, coulomb friction, stiction, backlash, etc.

(a) Saturation-

It is the most common non-linearity due to it's physical limitation.
Ex- amplifiers torque and speed saturation in motors

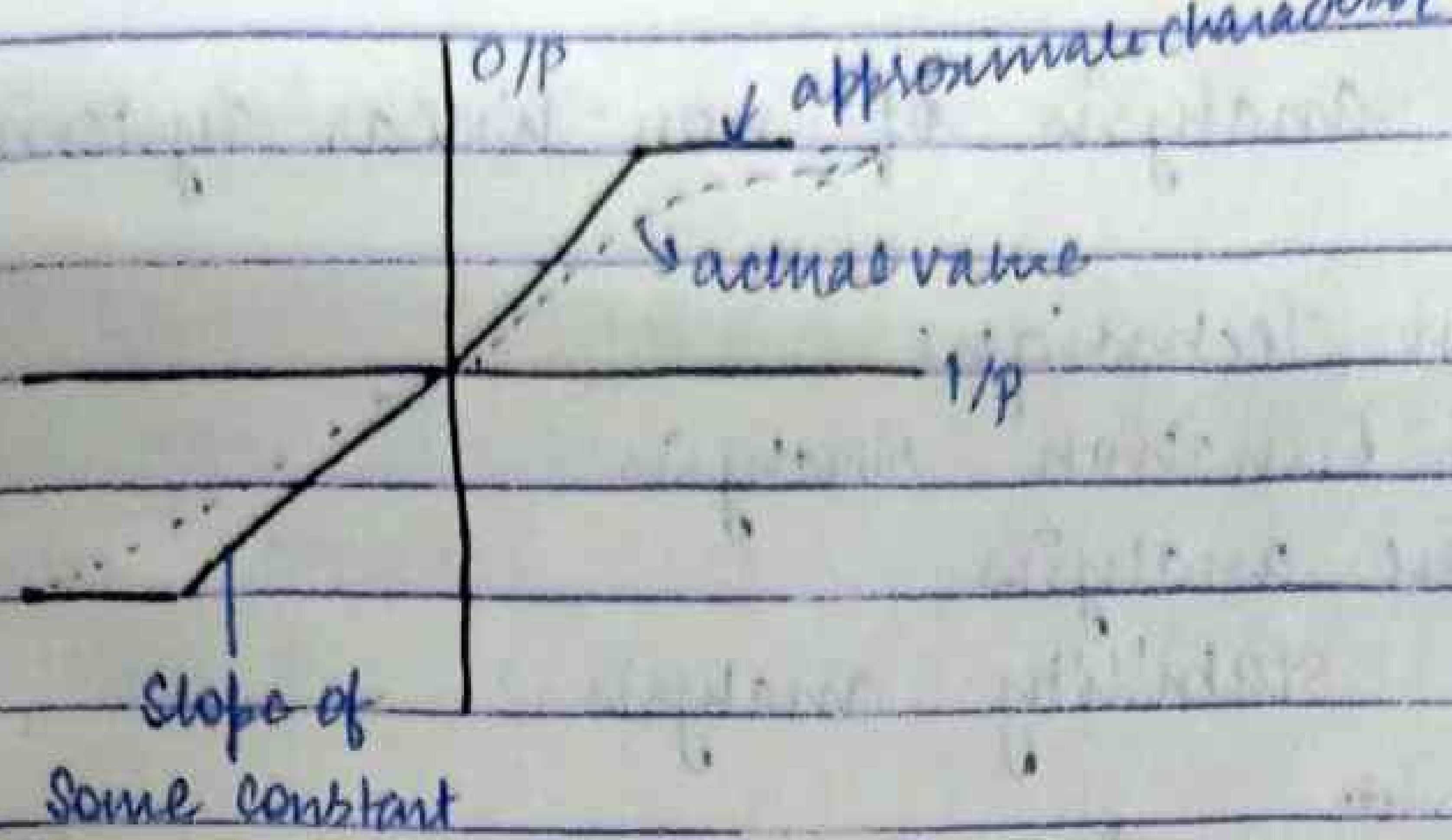


Fig → Piecewise linear approximation of saturation non-linearity

(b) Friction-

Retarding frictional force exist whenever mechanical surfaces come in sliding contact. The predominant frictional force called viscous friction. ($f\dot{x}$)

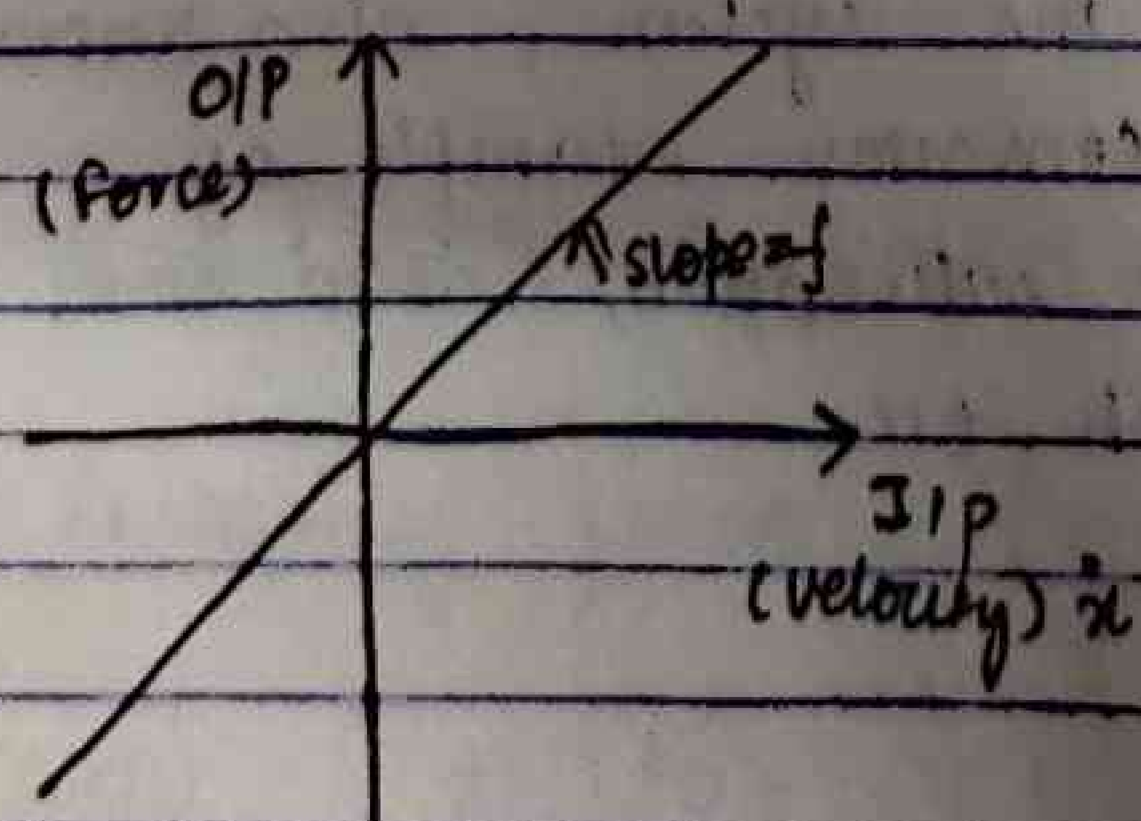
Viscous friction force = $f\dot{x}$

Where f = constant

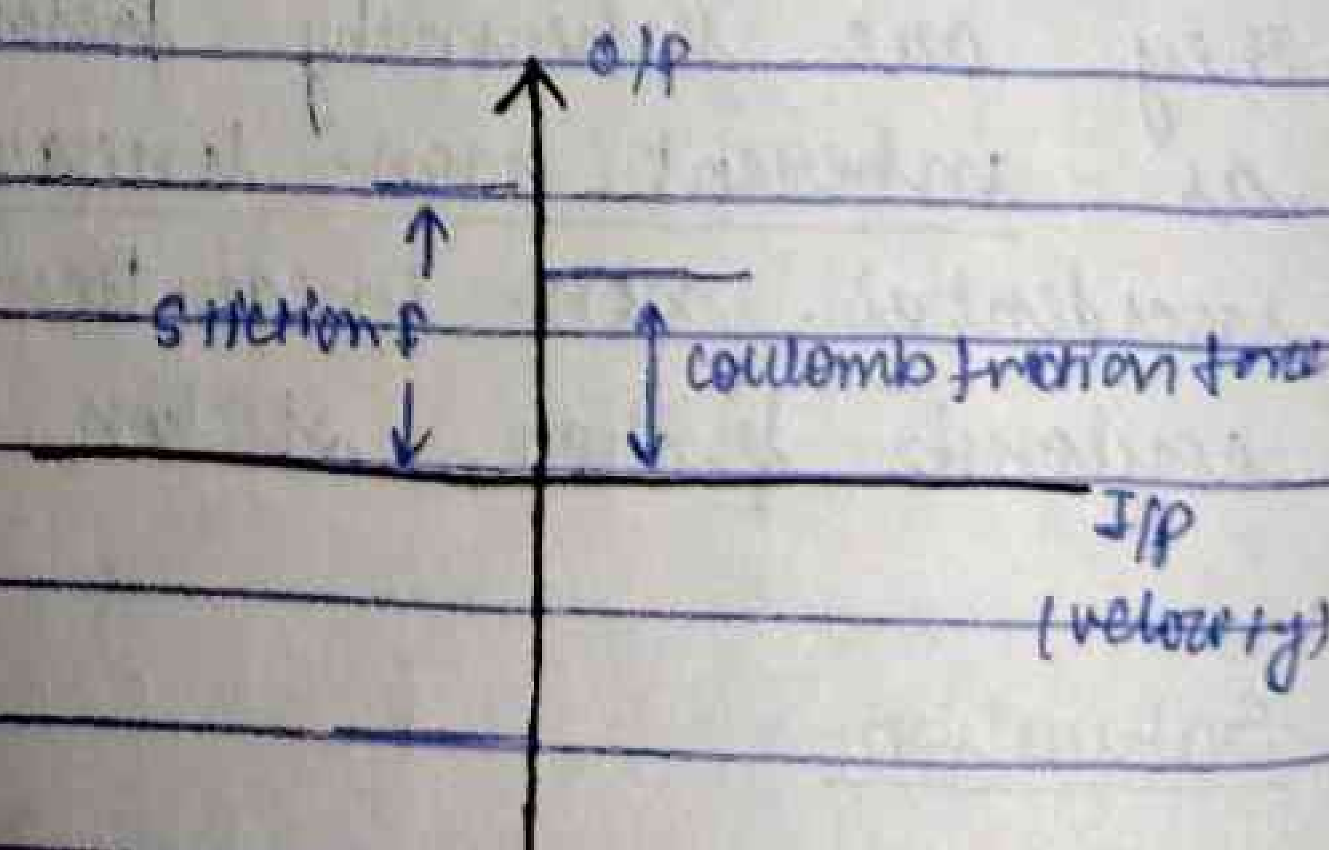
$\dot{x}$ = relative velocity

Friction described in three types-

- Viscous friction
- Coulomb friction
- Stiction friction

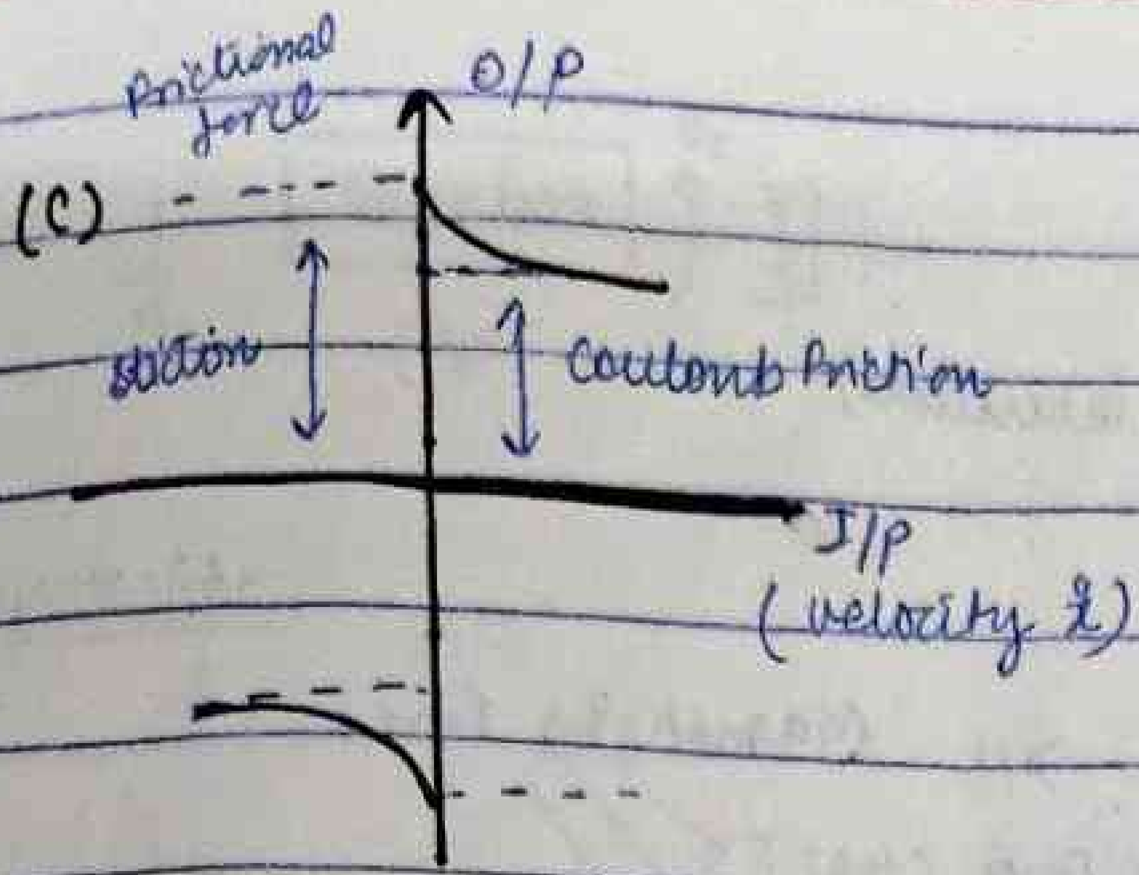


(a) Viscous friction



(b) Colomb friction

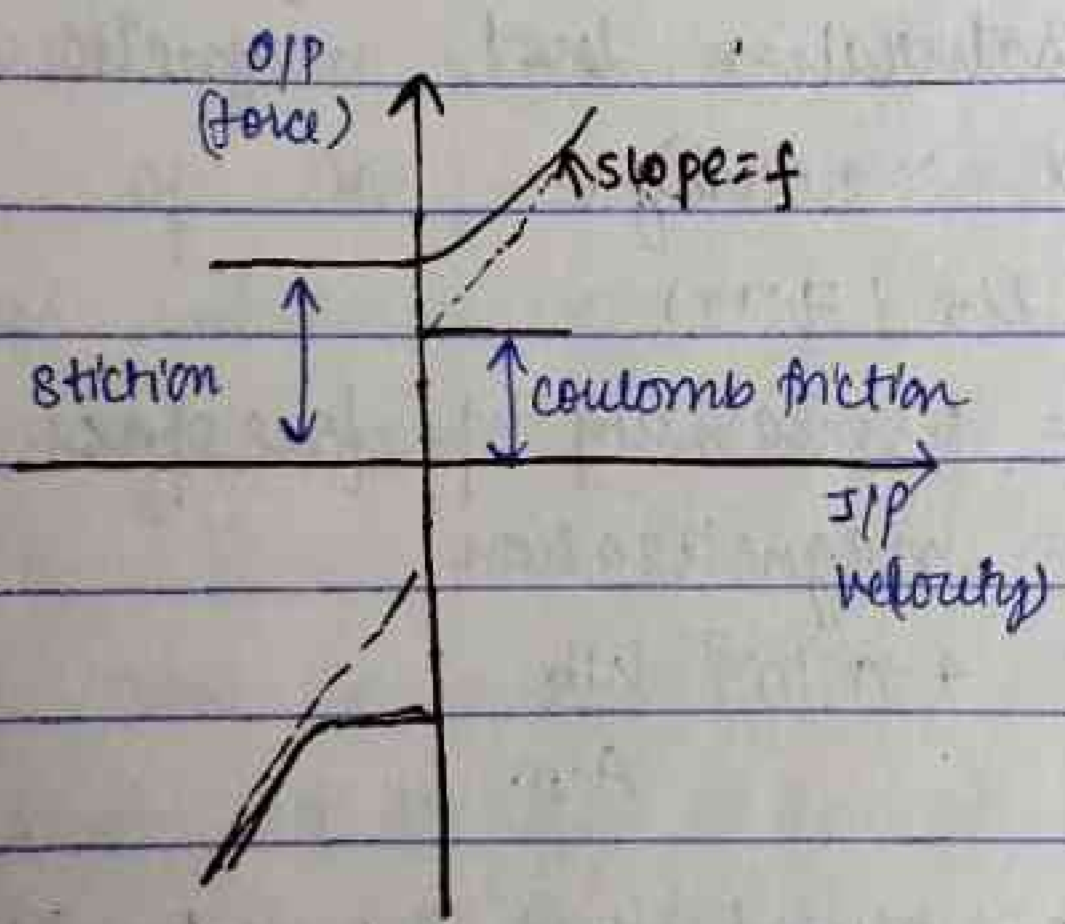
It is constant retarding f (always opposing the relative motion). Const[illegible] friction



fig(c) actual stiction

(need a higher F).

<u>Stiction</u> → force required to initiate the motion. The force ~~of~~ of stiction is always greater than that of coulomb friction since due to interlocking of surface irregularities

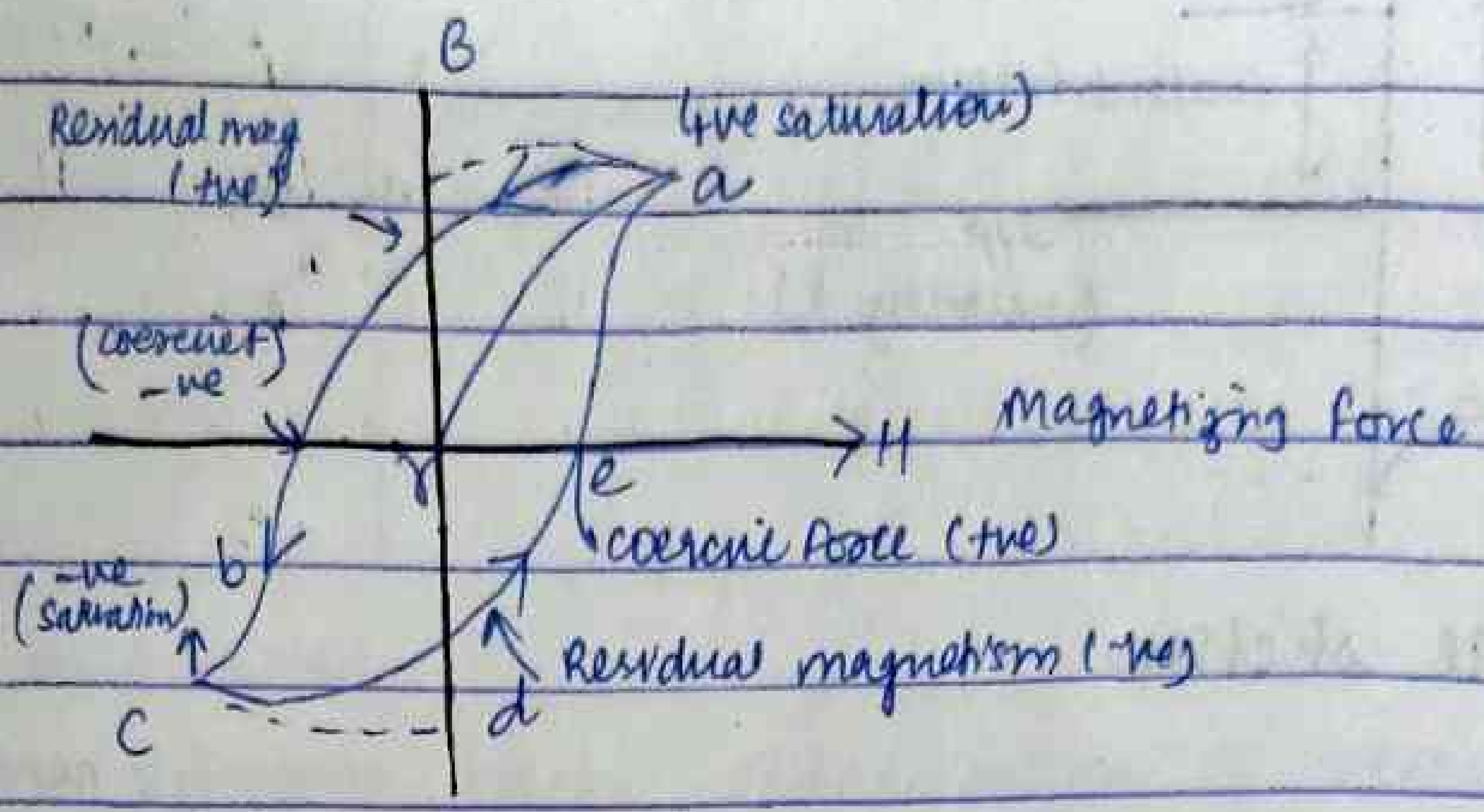


fig(d) - stiction, coulomb and viscous friction.

- If all the friction present or available in the system, then we may express it as fig (d).

Above all fig a, b, c, d are the characteristic of the various types of frictions.

(C) Hysteresis-



They generate multivalued response.

- Hysteresis behaviour shows serious non-linearity b/w I/P and O/P.
- The most imp parameters in a magnetic hysteresis B(H) curve are saturation level, magnetic remanence, or retentivity and coercivity.
- We know, $B = \mu_0 (H+M)$

  where μ_0 = permeability of free space

  M = magnetisation

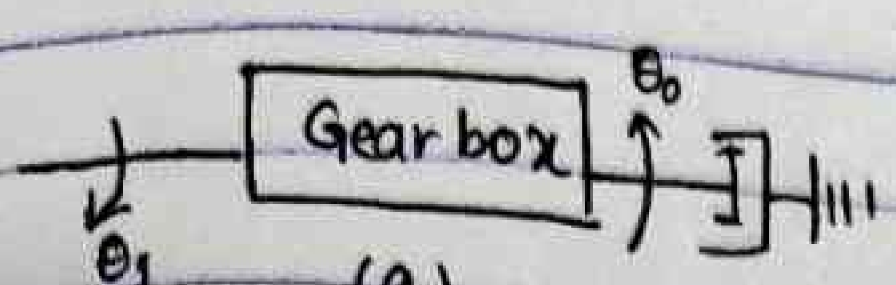
  $\mu_0 = 4\pi 10^{-7} \frac{Wb}{A \cdot m}$

- Hysteresis behaviour reveal that the r/n b/w I/P voltage & O/P displacement is non-linear, multivalued and non-local memoryless.

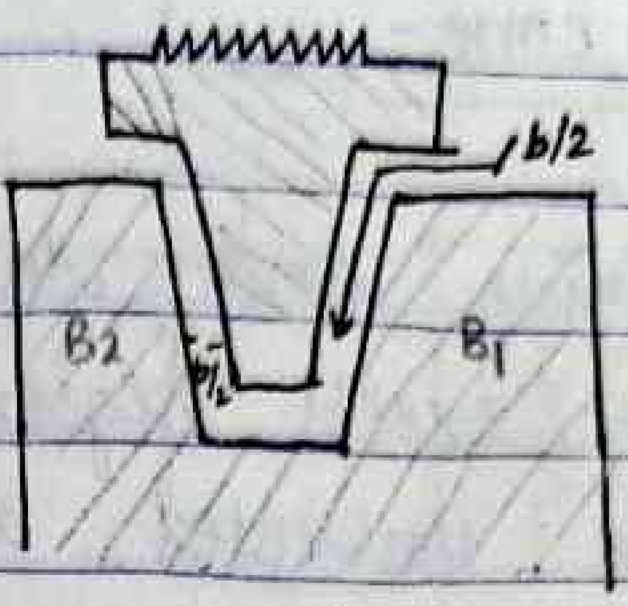
- The shape of B-H curve is non-linear because it indicates that relative permeability of a material isn't constant but it varies.

(d) Backlash-

Backlash differ from magnetic hysteresis, It is the play between the teeth of the driver gear and driven gear

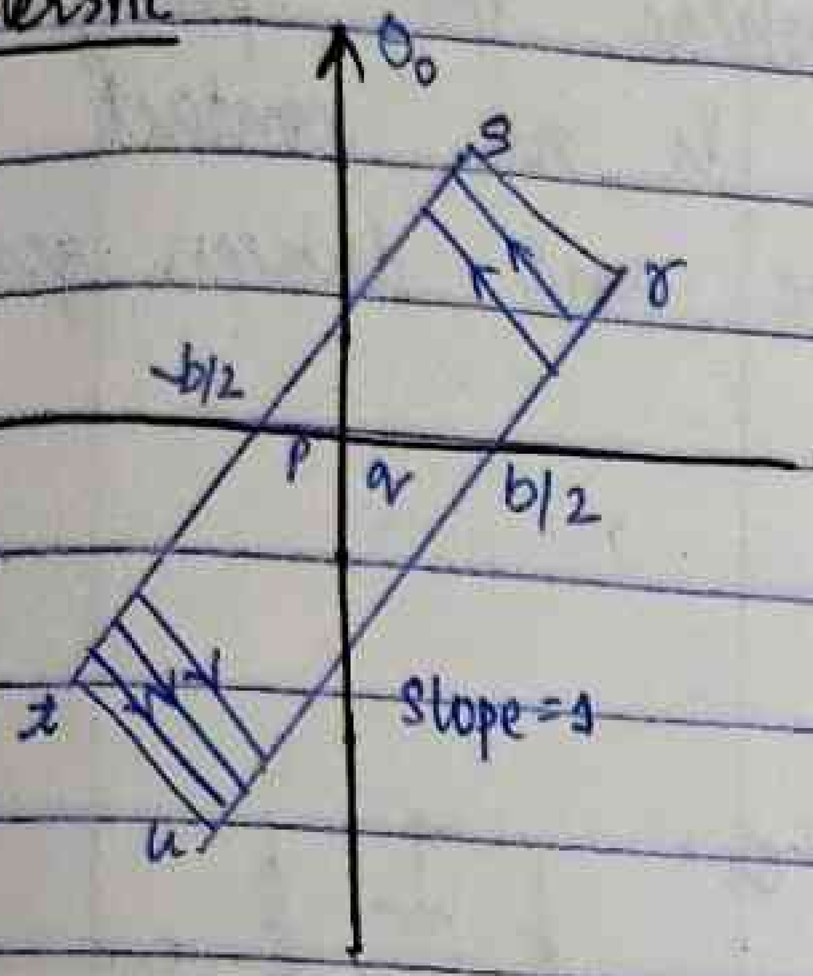


(a)



(b)

Characteristic-

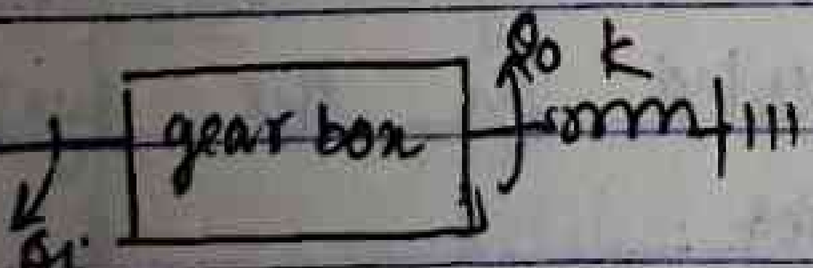


(c)

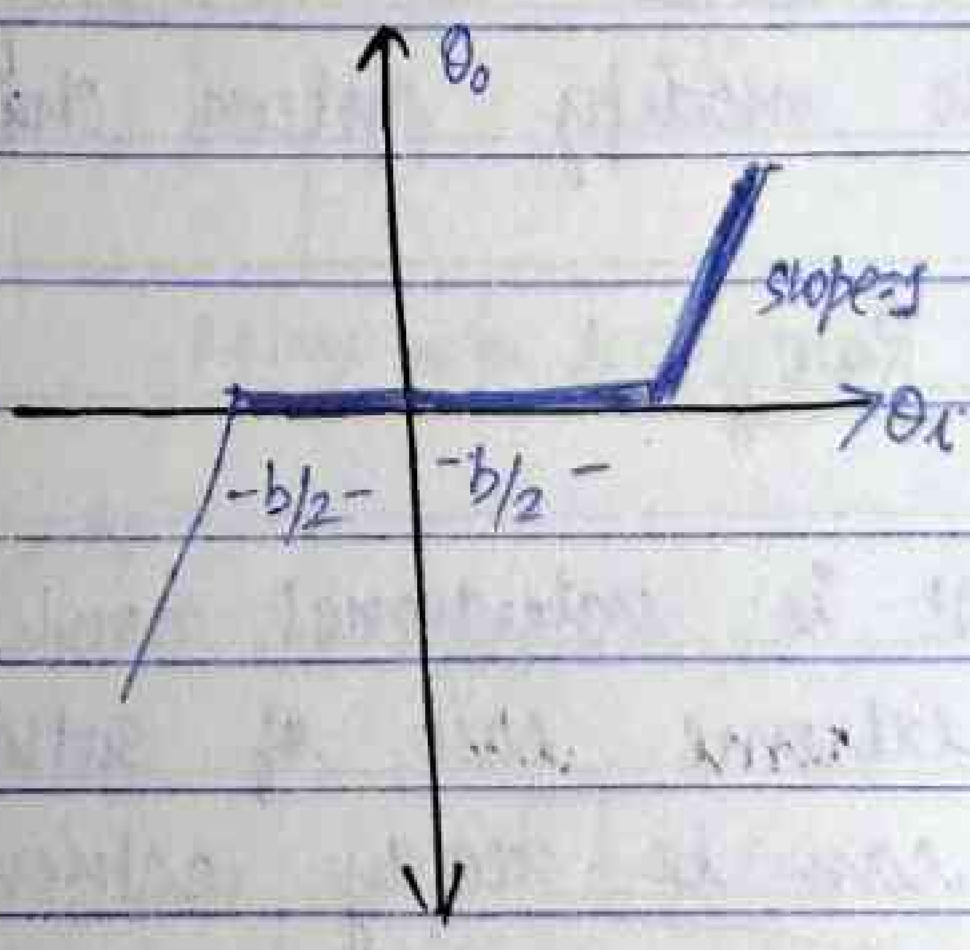
(Backlash non linearity)

- O/P result for a given O/P depends upon the history of the I/P. This type of non-linearity has thus inherent memory and is referred to as memory type non-linearity

- Backlash can be reduced by using high quality gears. and oscillatory behaviour may be improved by using high quality spring arrangement.



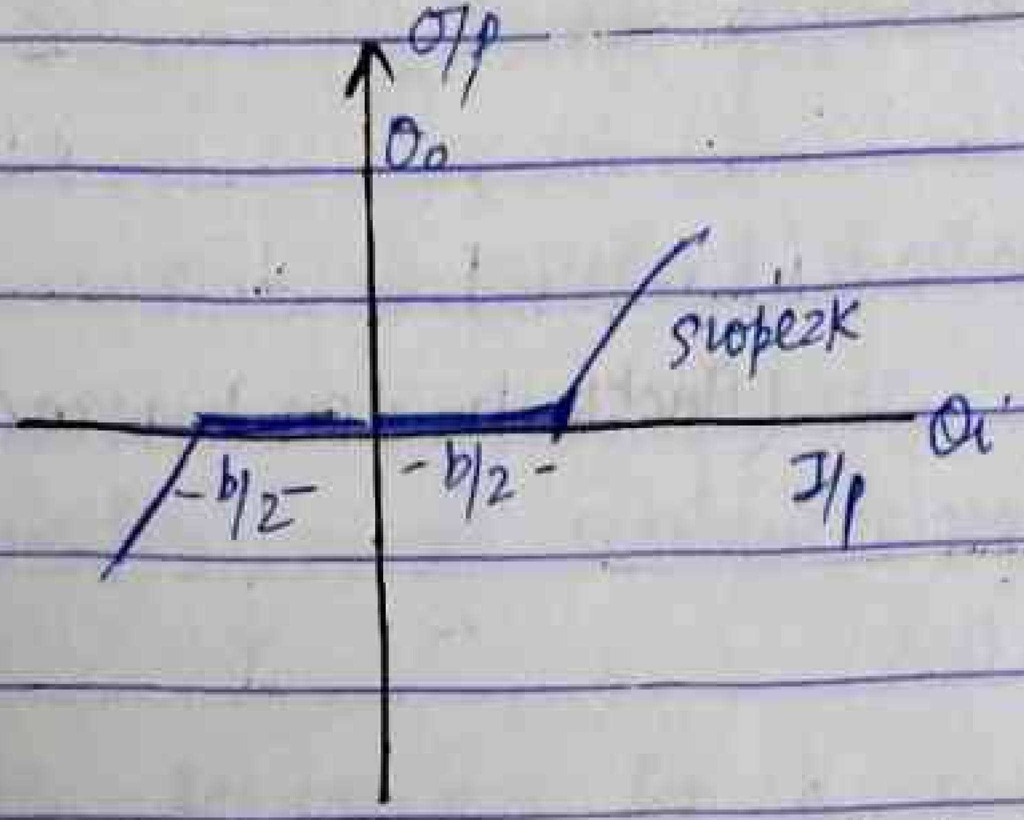
(d)



- Backlash present in most of the mechanical and hydraulic system. "The diff b/w the tootspace & tooth width in mech system, which is essential for rapid working gear transmission is known as backlash."

(c) Dead zone-

The drive and driven gears must now move together except region $\pm b/2$ where the spring remains untwisted and there is no contact b/w the teeth of the drive and driven gears.



- O/P becomes zero when I/P crosses certain limiting value, it may destablize system and reduced positioning accuracy.

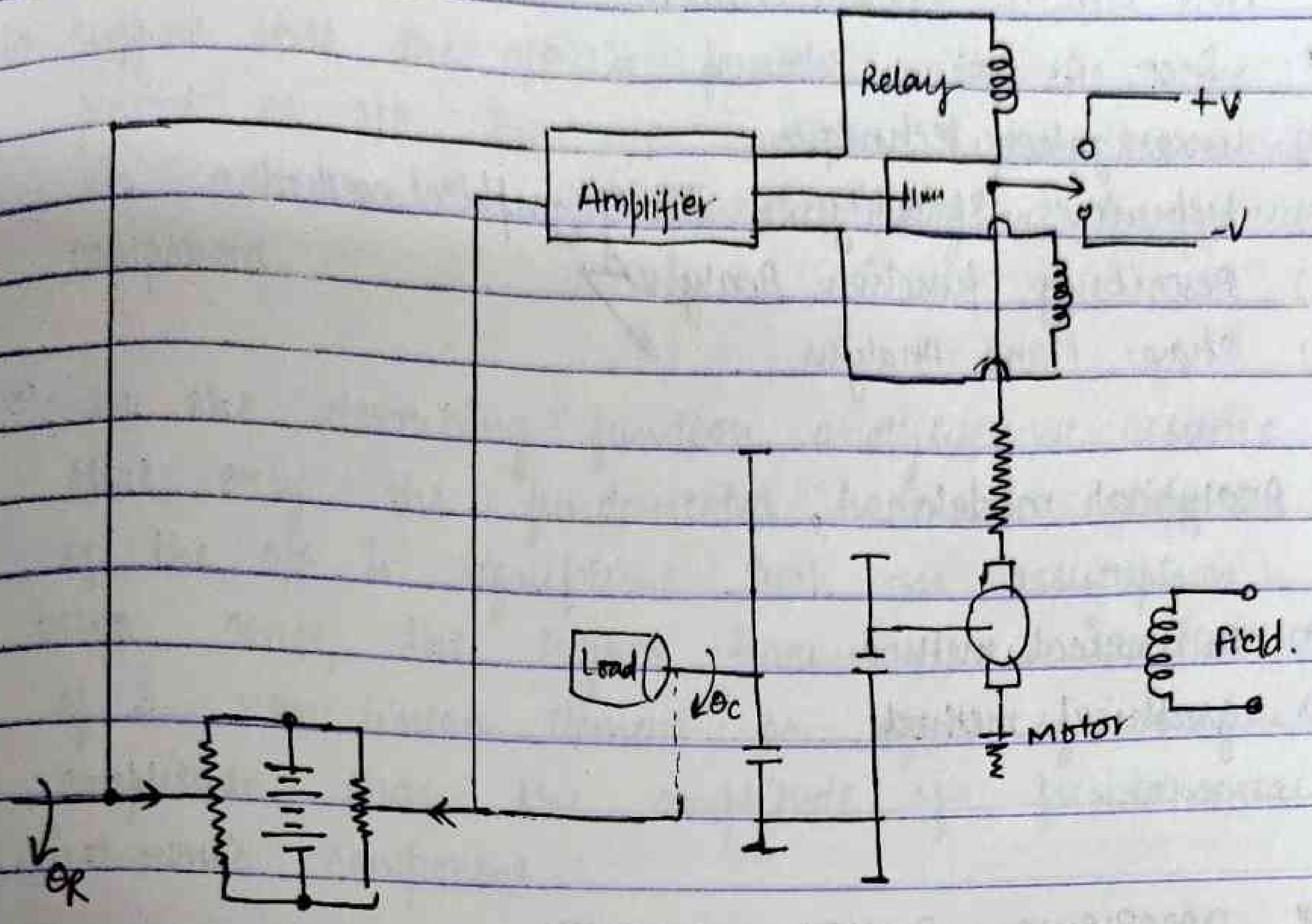
(2) Intentional Non- linearity-

- They are intentionally introduced to improve system performance.
- Non-linearity is the behaviour of a circuit, in which the O/P signal strength doesn't vary in direct proportion to I/P signal strength. Ex- diode
- Intentional non linearity inserted in the system to modify system characterstics.

(a) Relay-

- It is intentional nonlinearity, Ideal relay is the extreme case of saturation. Ideal characterstic can be nearly achieved by SCR switching.

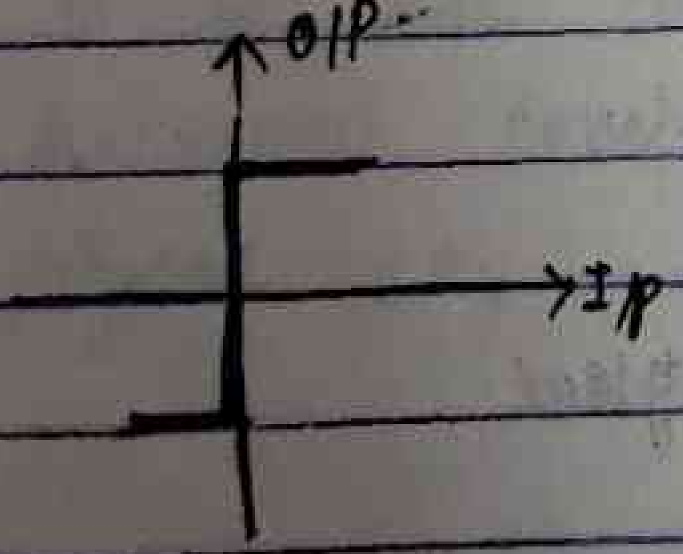
- A relay is a non-linear power amplifier which can provide large power amplification



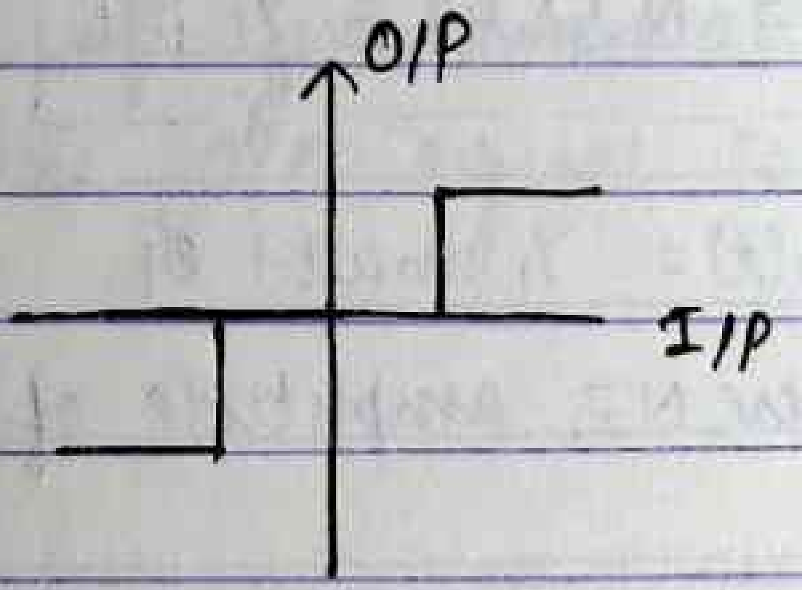
A relay servo system

- The error signal controls the relay current which moves the solenoid so as to make the contact in one direction or other.

Relay characterstics-

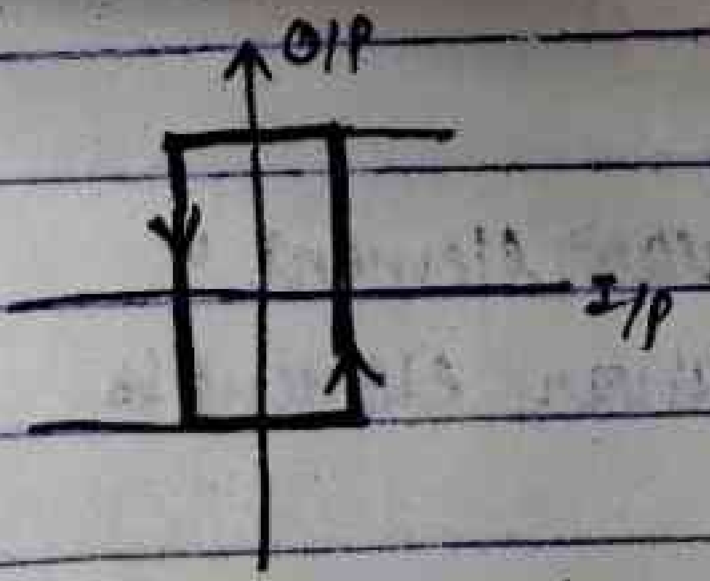


Ideal relay

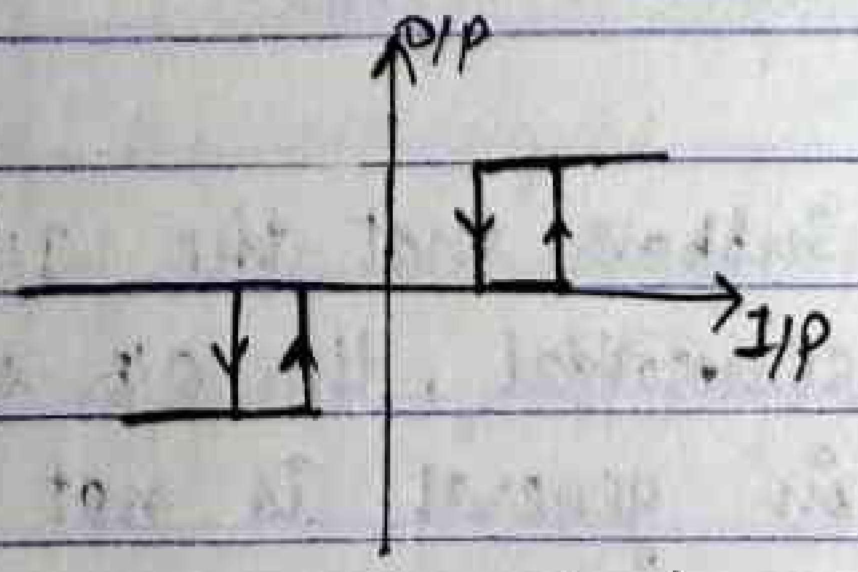


Relay with deadzone.

RELAY NONLINERITY



Relay with pure hysteresis



Relay with dead zone & hysteresis

NON- LINEAR SYSTEM ANALYSIS-

(a) Linearization technique
(b) Lyapunov's stability
(c) Describing function Analysis
(d) Phase Plane Analysis

graphical method.

Analytical model-

(a) Numerical method
(b) graphical method

* DESCRIBING FUNCTION ANALYSIS-

$$x(t) \longrightarrow \boxed{\text{Non-linear element}} \longrightarrow y(t)$$

$x(t) = X \sin \omega t$

- $y(t)$ express into fourier series
- If I/P is sinusoidal, O/P is not sinusoidal
- $\boxed{N = N(X, \omega) = \frac{Y_1}{X} \angle \phi_1}$
- $y_1(t) = Y_1 \sin \omega t + \phi_1$

where N = amplitude of frequency signal.

→ First harmonic component of O/P signal complex function is equal to first harmonic component of O/P signal.

→ Suppose that the I/P of non-linear element is sinusoidal, the O/P of the non-linear element in general is not sinusoidal.

- Suppose that the O/P is periodic with the same period as I/P. The O/P contains higher harmonics in addition to fundamental higher harmonics component.

- In the describing function analysis, we assume that only the fundamental harmonic component of the O/P is significiant. Such an assumption is valid since, the higher harmonics in the O/P of a non-linear element or often of smaller amplitude than the amplitude of fundamental harmonic component.

- In addition most control system are low pass filters, with the result that the higher harmonics are very much attenuated, compared with the fundamental harmonic component.

- The describing function or sinusoidal describing function of a non-linear element is defined to be the complex ratio of fundamental harmonic component of the O/P signal to I/P signal i.e;

$$N = \frac{Y_1}{X} \angle \phi_1$$

where, N = describing function
X = amplitude of I/P sinusoidal
Y_1 = amplitude of fundamental harmonic component of the O/P signal
ϕ_1 = Phase shift of the fundamental harmonic component of O/P signal.

- If no energy storage element is included in non-linear element then N is a function only of the amplitude of the I/P to the element on the other hand if an energy storage element is included then N is a function of both amplitude and frequency of I/P signal. In calculating the describing func for a given non-linear element, we need to find the fundamental harmonic component of the O/P.

- For the sinusoidal I/P $\boxed{x(t) = X\sin\omega t}$, the non linear element the O/P, $y(t)$ may be expressed as a fourier series as:-

$$y(t) = A_0 + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t)$$

$$\boxed{y(t) = A_0 + \sum_{n=1}^{\infty} Y_n \sin(n\omega t + \phi_n)}$$

Where

$$A_n = \frac{1}{\pi}\int_0^{2\pi} y(t) \cos n\omega t \, d(\omega t)$$

$$B_n = \frac{1}{\pi}\int_0^{2\pi} y(t) \sin n\omega t \, d(\omega t)$$

$$Y_n = \sqrt{A_n^2 + B_n^2}$$

$$\phi_n = \tan^{-1}\left(\frac{A_n}{B_n}\right)$$

$$A_0 = \frac{1}{2\pi}\int_0^{2\pi} y(t)\, dt$$

$$\boxed{\omega = \frac{2\pi}{T}, \quad a_0 = \frac{1}{T}\int_0^T y(t)\,dt \Rightarrow a_n = \frac{2}{T}\int_0^T y(t)\cos\omega t\,dt \quad \Rightarrow b_n = \frac{2}{T}\int_0^T y(t)\sin n\omega t\,dt}$$

(c) If the non-linearity is skew symmetric then $\boxed{A_0 = 0}$
The fundamental harmonic component of O/P is

$$y_1(t) = A_1 \cos\omega t + B_1 \sin\omega t$$
$$= Y_1 \sin(\omega t + \phi_1)$$

The describing function is then given by

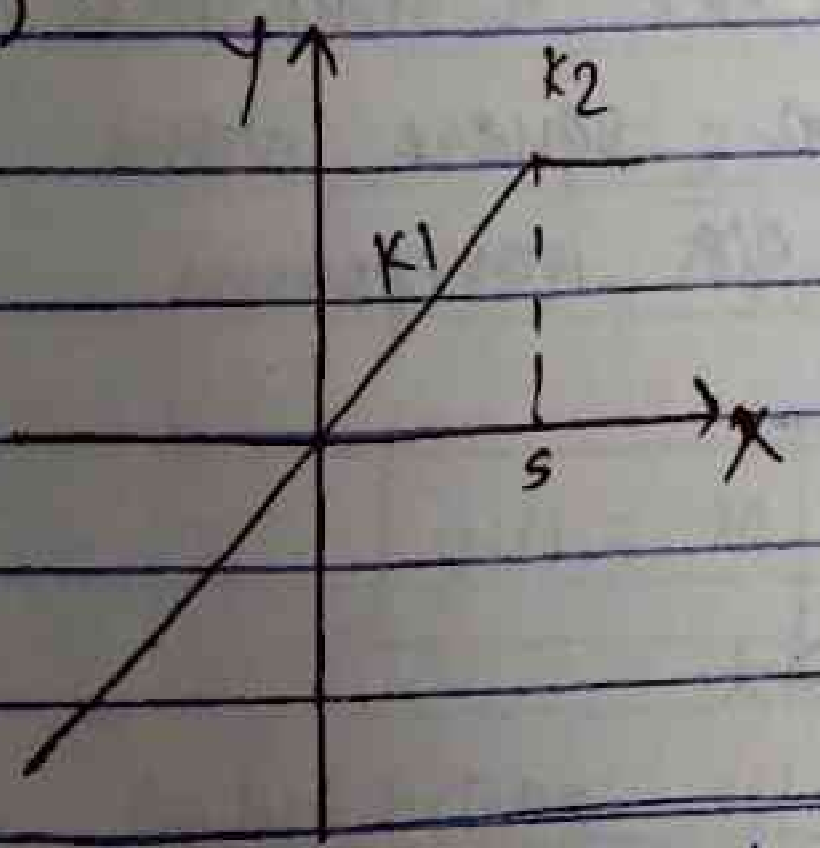
$$N = \frac{Y_1}{X} \angle \phi_1$$

$$= \frac{\sqrt{A_1^2 + B_1^2}}{X} \angle \tan^{-1}\left(\frac{A_1}{B_1}\right)$$

Here it is clear that N is a complex quantity when ϕ_1 is non-zero.

Example- characteristic function of describing function analysis

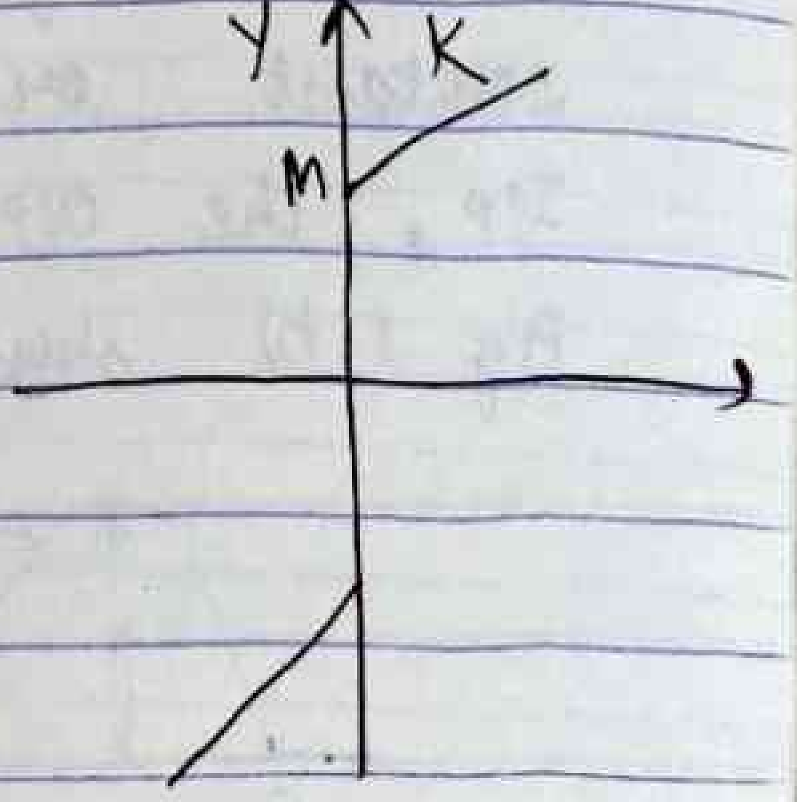
(a)



$$\boxed{N = k_2 + \frac{2(k_1 - k_2)}{\pi}\left(\sin^{-1}\frac{S}{X} + \frac{S}{X}\sqrt{1 - \frac{S^2}{X^2}}\right)}$$

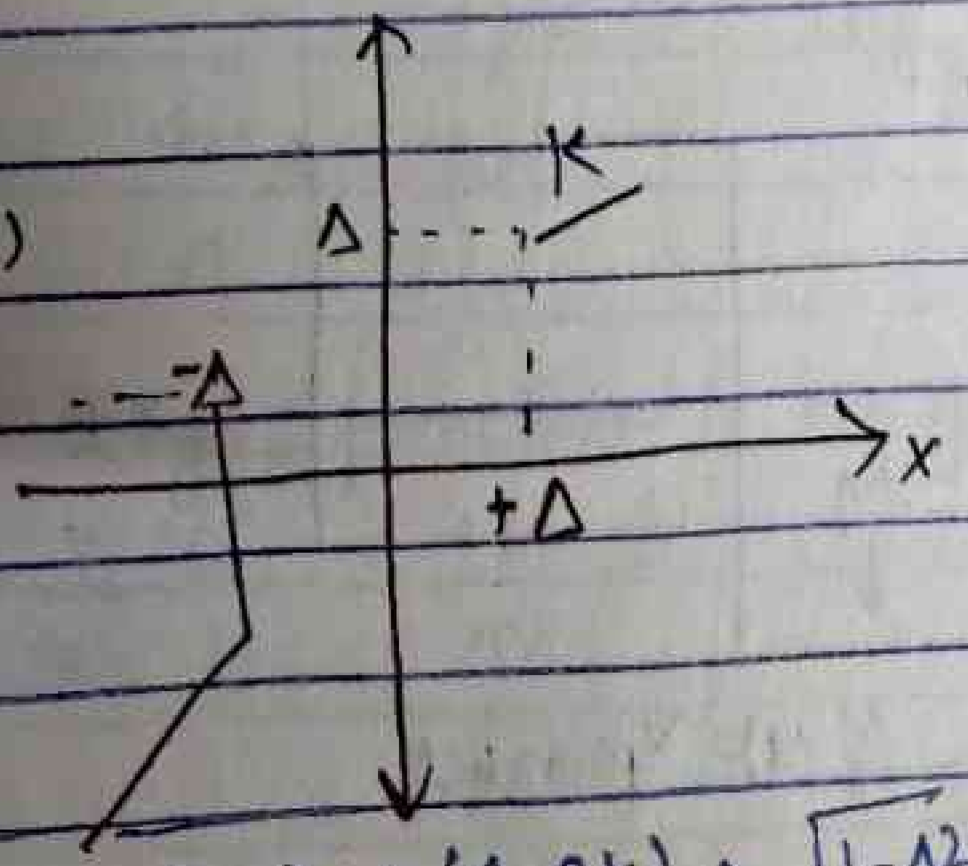
$(x \geq S)$

(b)



$$\boxed{N = K + \frac{4M}{\pi X}}$$

(c)



$(x \geq \Delta)$

$$\boxed{N = K - \frac{2K}{\pi}\sin^{-1}\left(\frac{\Delta}{\pi}\right) + \frac{(4 - 2K)\Delta}{\pi X}\sqrt{1 - \frac{\Delta^2}{X^2}}}$$

(vvi) ·

* Describe the function of ON-OFF Non-linearity -

The ON-OFF non-linearity is often called a two position non-linearity. Consider the on-off element whose I/P, O/P characteristic curve is shown in fig (1.a) given below.

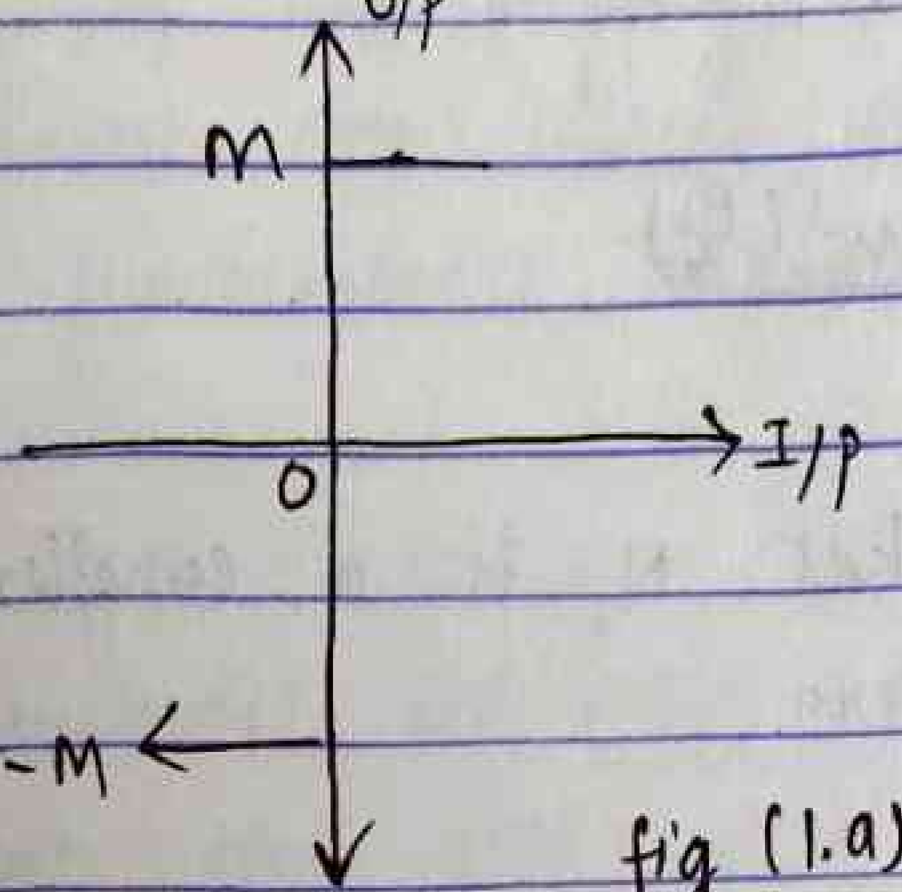
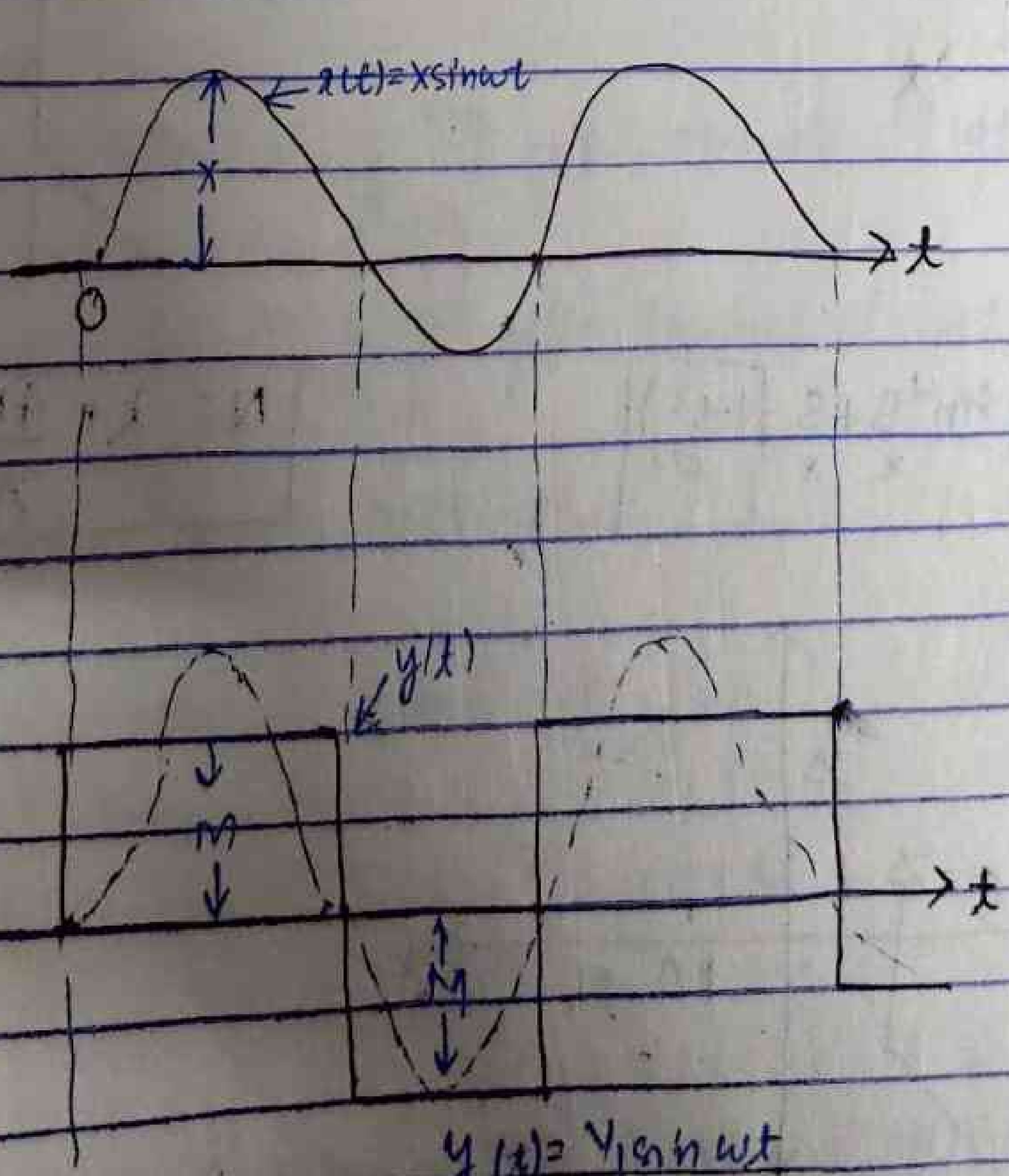


fig (1.a).

The O/P of this element is either a +ve constant or a -ve constant. For a sinusoidal I/P, the O/P signal becomes a square wave. Fig 1(b) shows the I/P and O/P waveforms.



y(t) = $Y_1 \sin \omega t$

let us obtain the Fourier series expansion of the o/p $y(t)$ is of such an element.

$$\boxed{y(t) = A_0 + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t)}$$

The o/p is an odd function, for any odd function we have $A_n = 0$ for $n = 0, 1, 2, 3 \ldots$

i.e;

$A_n = 0, \quad (n = 0, 1, 2 \ldots)$

Hence, $y(t) = \sum_{n=1}^{\infty} B_n \sin \omega t$

The fundamental harmonic component $y(t)$ is

$y(t) = B_1 \sin \omega t = Y_1 \sin \omega t$

Where $Y_1 = \frac{1}{\pi} \int_0^{2\pi} y(t) \sin \omega t \, d(\omega t)$

$= \frac{2}{\pi} \int_0^{\pi} y(t) \sin \omega t \, d(\omega t)$

$$\boxed{y(t) = M}$$

Substituting $y(t) = M$, in the last equation is

$y_1(t) = \frac{2M}{\pi} \int_0^{\pi} \sin \omega t \, d(\omega t) = \frac{4M}{\pi}$

Thus,

$y_1(t) = \frac{4M}{\pi} \sin \omega t$

The describing function N is given by

$N = \frac{Y_1}{X} \angle 0^\circ = \frac{4M}{\pi X}$

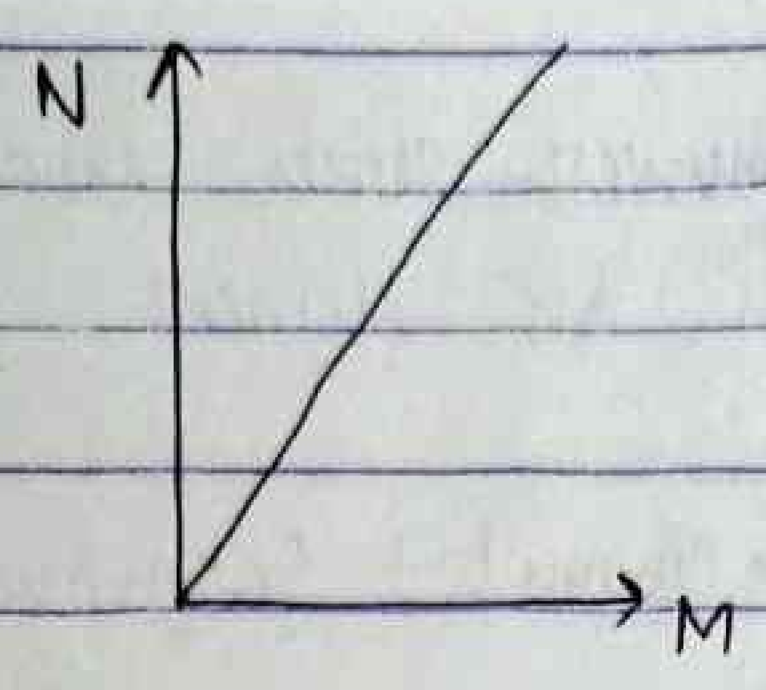


fig (2)

clearly, the describing function for, the ON-OFF element is a real quantity and is a function only of the I/P, amplitude X. Such non-linearity is commonly called an amplitude dependent non-linearity.

* ON-OFF Non-linearity with hysteresis-

Consider the ON-OFF element with hysteresis whose I/P, O/P characterstic curve is shown in fig 3(a). For a sinusoidal I/P, the O/P becomes a square wave with a phase lag of the amount $\boxed{\omega t_1 = \sin^{-1}\frac{h}{X}}$ as shown in fig 3(b)

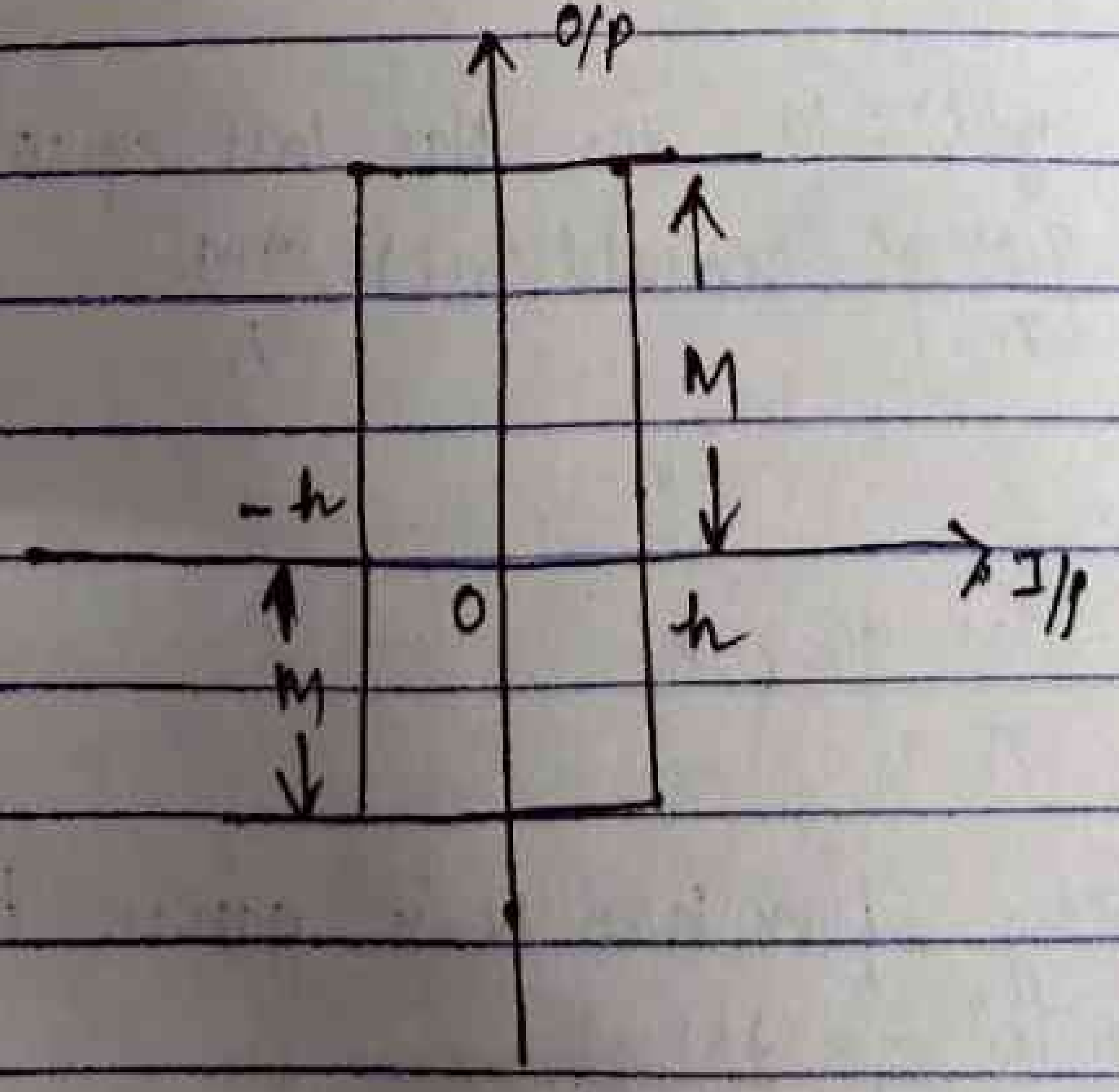
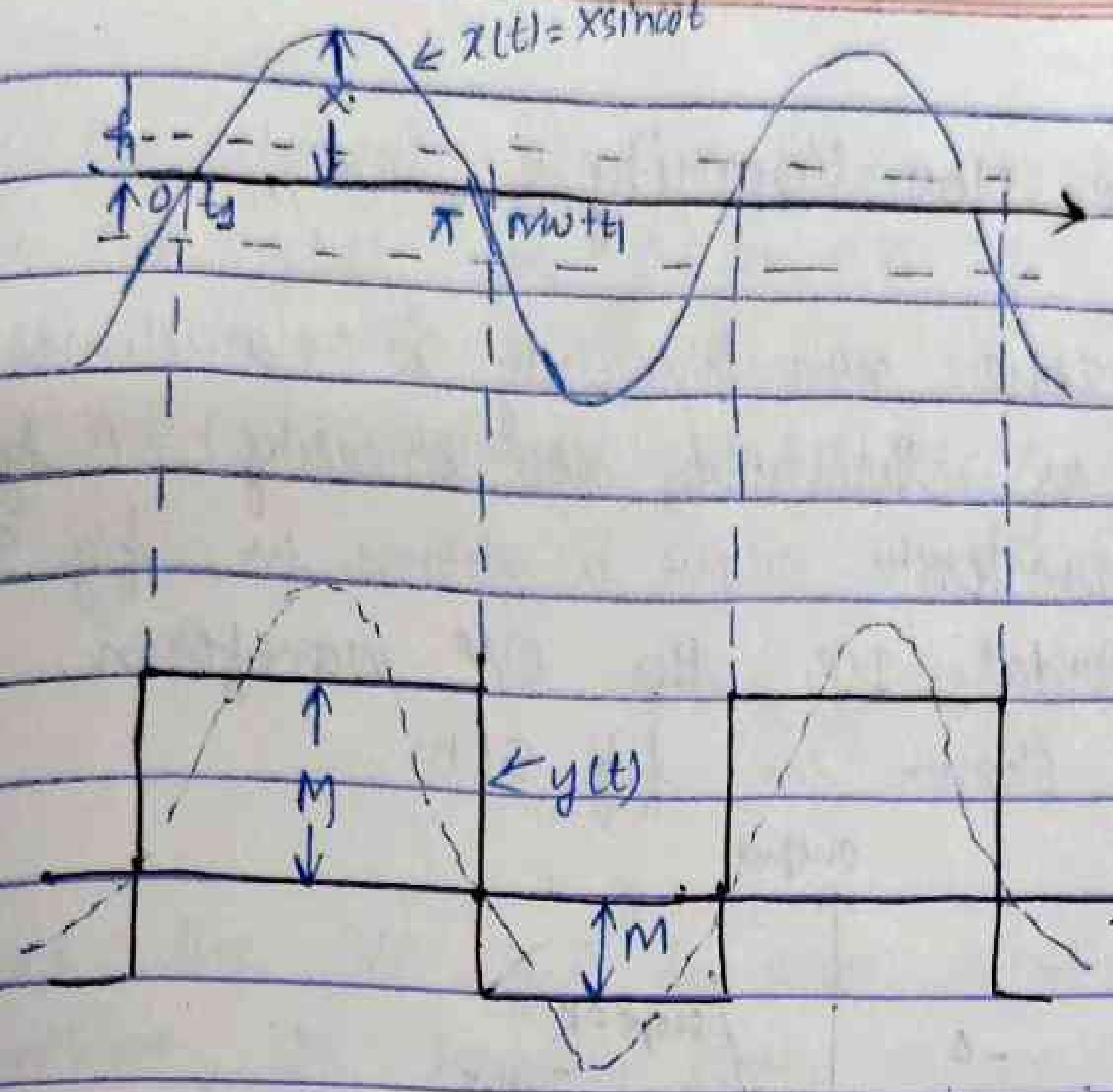


fig (3a) I/P - O/P characterstic of ON-OFF non linearity with hysteresis

$\omega t_1 = \sin^{-1}\left(\frac{h}{X}\right)$

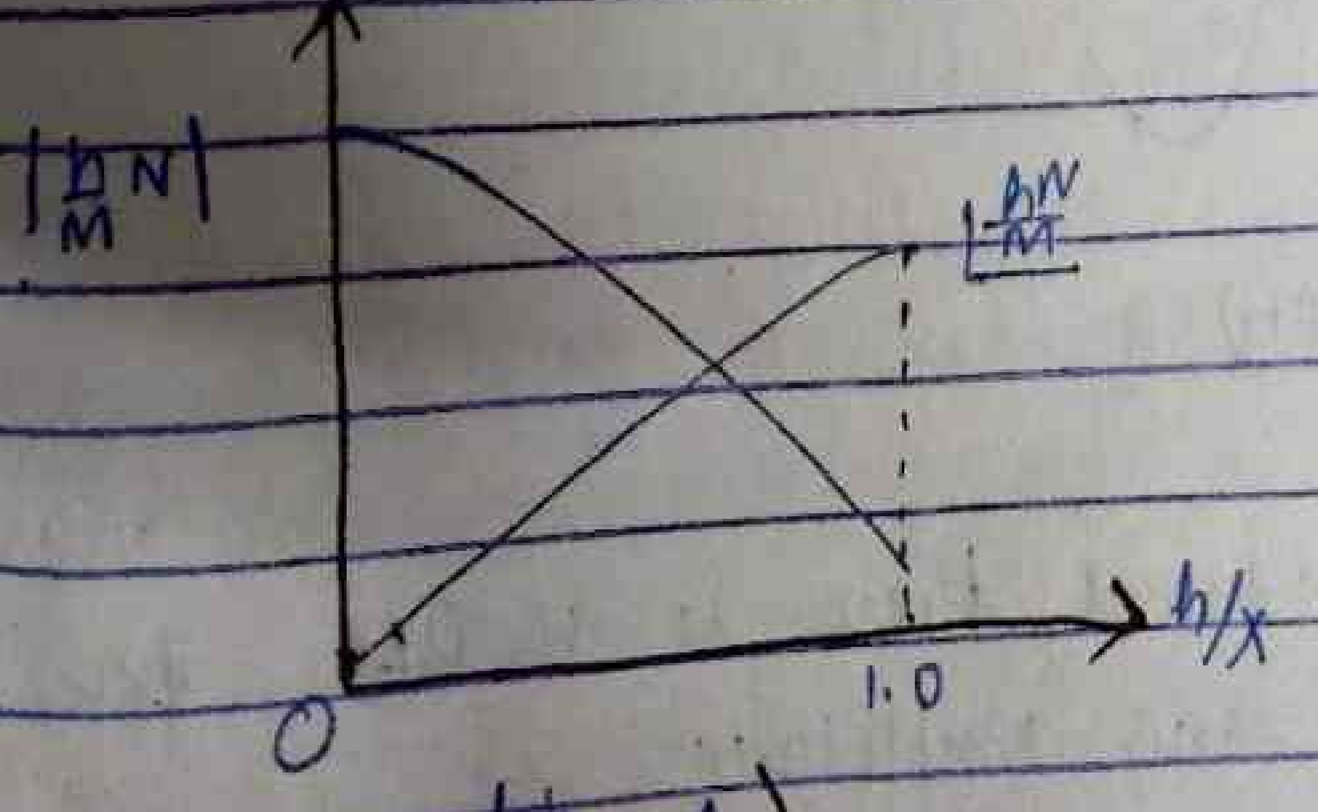


$$y_1(t) = Y_1 \sin(\omega t + \phi)$$

fig 3(b)

- Hence, the describing function for this non-linear element is $\boxed{N = \frac{4M}{\pi X} \angle -\sin^{-1}\left(\frac{h}{X}\right)}$

- It is convenient to plot $\frac{h}{M}N = \frac{4h}{\pi X} \angle -\sin^{-1}\left(\frac{h}{X}\right)$ vs $\left(\frac{h}{X}\right)$ rather than N vs $\left(\frac{h}{X}\right)$ because $\frac{h}{M}N$ is a function only by $\left(\frac{h}{X}\right)$. A plot of $\left(\frac{h}{M}N \text{ vs } \frac{h}{X}\right)$ is shown in figure.4



(fig 4)

* Dead zone Non-linearity -

The dead zone non-linearity is sometimes referred to as threshold non-linearity. A typical I/P, O/P characterstic curve is shown in fig 5(a). For a sinusoidal I/P, the O/P waveform becomes as shown in fig 5(b).

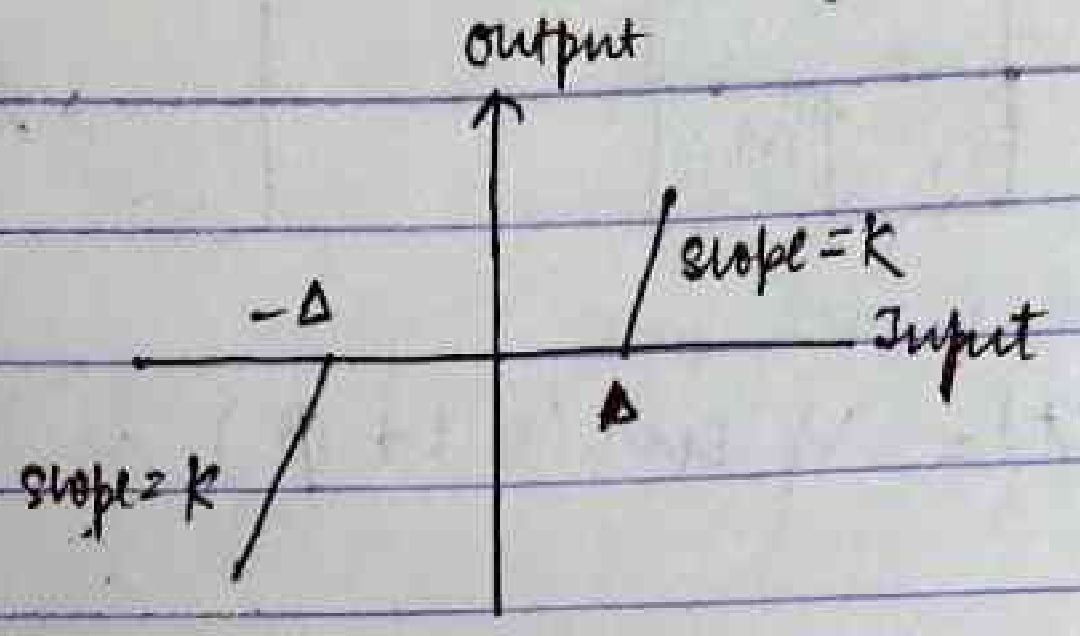


fig 5(a)

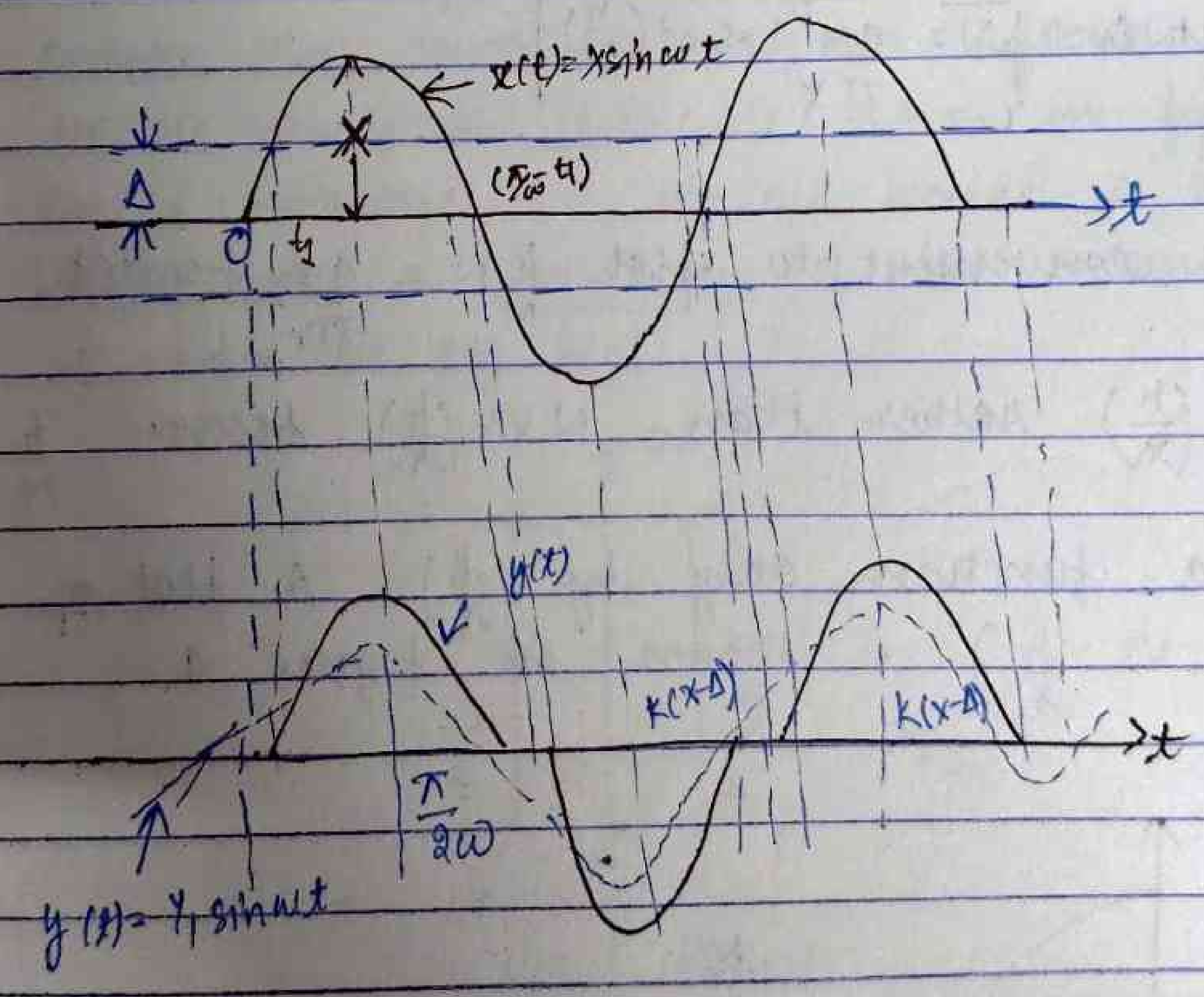


fig 5(b)

For the dead zone element, there is no O/P for I/p within the dead zone amplitude.

for the element with dead zone shown in fig 5(a) the O/P $y(t)$ for $0 \le \omega t \le \pi$ is given by

$$\begin{cases} y(t) = 0 & \text{for } 0 < t \le t_1 \\ = K(X \sin\omega t - \Delta) & \text{for } t_1 < t < \left(\frac{\pi}{\omega} - t_1\right) \\ = 0 & \text{for } \left(\frac{\pi}{\omega} - t_1\right) < t < \frac{\pi}{\omega} \end{cases}$$

Since, the O/P, $y(t)$ is once again an odd function, its fourier series expansion has only sine terms. The fundamental harmonic component of the O/P is given by $\boxed{y_1(t) = Y \sin\omega t}$

where $Y_1 = \frac{1}{\pi}\int_0^{2\pi} y(t) \cdot \sin\omega t \, d(\omega t)$

$$= 4\pi \int_0^{\pi/2} y(t) \sin\omega t \, d(\omega t)$$

$$= \frac{4K}{\pi}\int_{\omega t_1}^{\pi/2} (X\sin\omega t - \Delta)\sin\omega t \, d(\omega t)$$

where $\Delta = X \sin\omega t_1$

or

$\omega t_1 = \sin^{-1}\left(\frac{\Delta}{X}\right)$

Hence, $Y_1 = \frac{4XK}{\pi}\left[\int_{\omega t_1}^{\pi/2} \sin^2\omega t \, d(\omega t) - \sin\omega t_1 \int_{\omega t_1}^{\pi/2} \sin\omega t \, d(\omega t)\right]$

$$= \frac{2XK}{\pi}\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{\Delta}{X}\right) - \frac{\Delta}{X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}\right]$$

The describing function for an element with dead zone can be obtained as $N = \frac{Y_1}{X}\angle 0^\circ$

$$= K - \frac{2K}{\pi}\left[\sin^{-1}\left(\frac{\Delta}{X}\right) + \frac{\Delta}{X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}\right]$$

UNIT-I

THE CONTROL SYSTEM

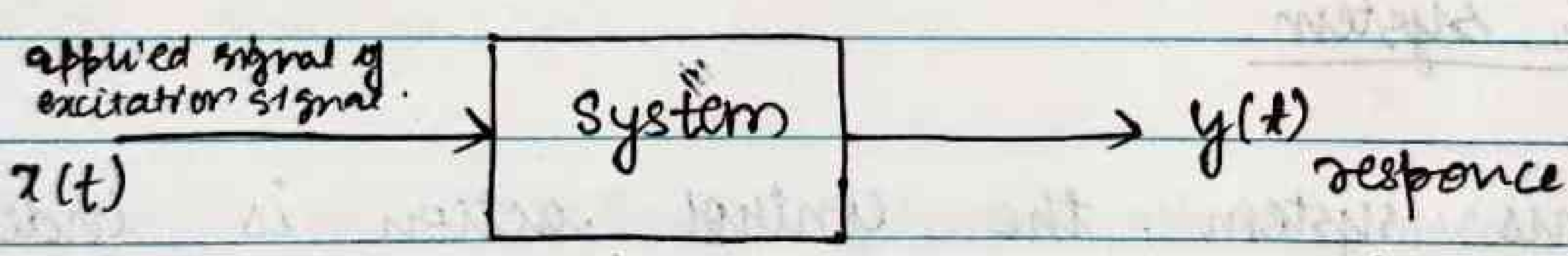
Notes-

* SYSTEM-

- A system is a combination of different physical components which set together to achieve desired output.

Or

- when a no. of elements or components are connected in a sequence to perform a specific function the group thus formed is called a system.



block representation of signal.

Eg- classroom, lamp, etc.

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* CONTROL SYSTEM-

To control mean to regulate, to direct or to command.
" A system which consist of no. of components connected together to perform a specific function in which the O/P is controlled by I/P called control system.

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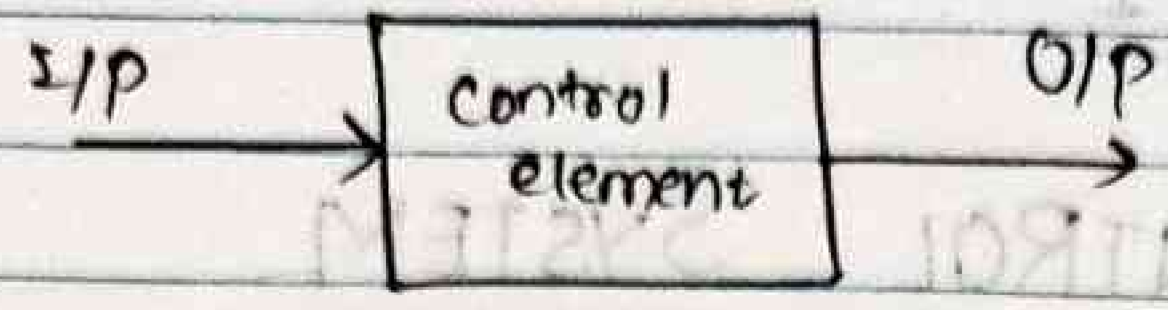
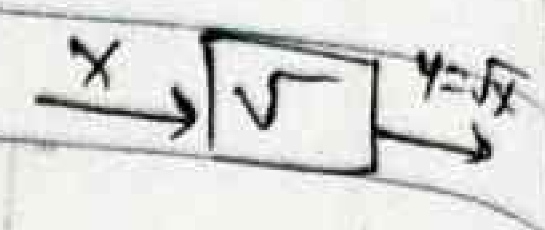


$\Rightarrow$



block diag of control system

eg- kite, lecture, class, professor, etc.

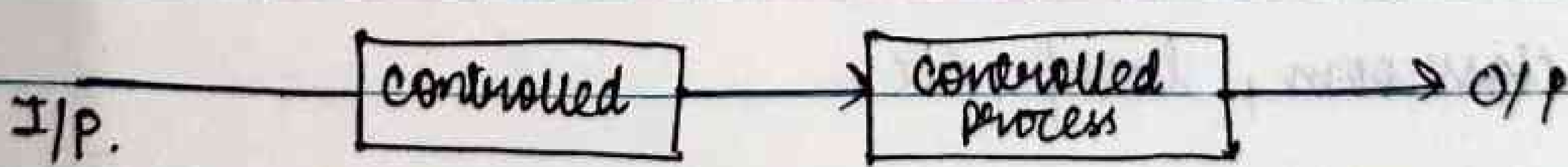
* TYPES OF CONTROL SYSTEM-

Basically there are two types of system-

- Open loop ~~closed~~ control system
- Closed loop ~~closed~~ control system

(a) Open loop control system-

- The open loop control system is also known as control system without feedback or non feedback control system.
- In this system the control action is independent of desired O/P. and the O/P is not compared with the reference I/P.



example-

- Automatic washing machine
- Immersion Rod
- A field control dc motor
- Automatic control of traffic light

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→ Advantages -

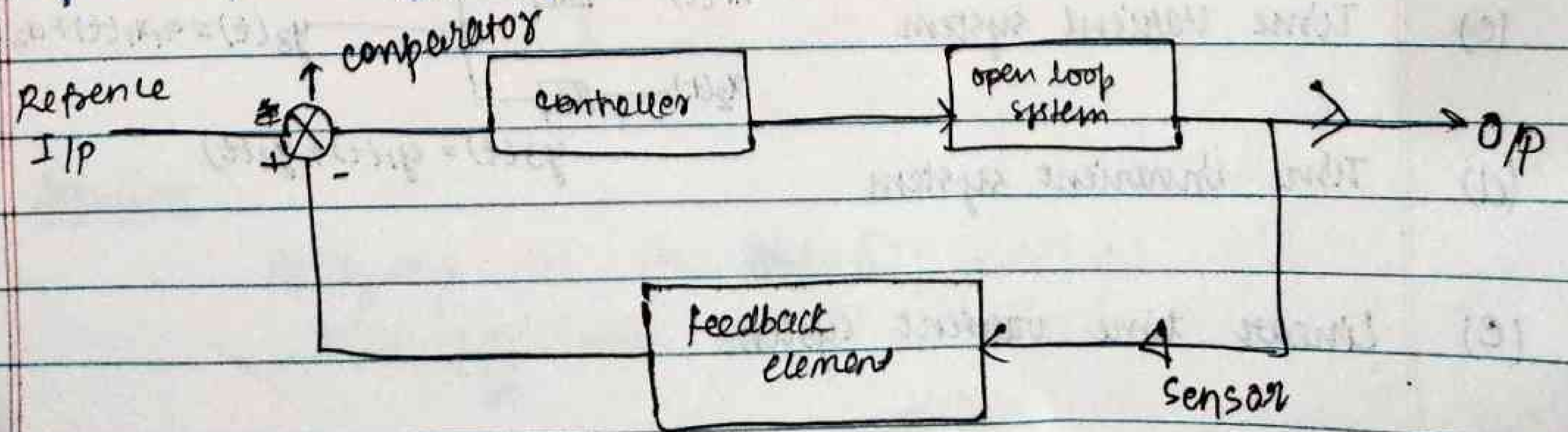
- Simple and economical
- easier to construct
- stable

→ Disadvantages -

- Inaccurate
- Changes in O/P are not checked automatically.

(b) Close-loop control system -

- A close loop system is also known as feedback control system.
- The type of control system which uses feedback signals to both control and adjust itself is called a close loop system.
- The reference to "feedback" simply means that some portion of O/P is returned "back" to the I/P to form part of system excitation.



example-

- Thermostat heater
- Voltage stablizer
- Inverter AC
- Automatic toaster

→ Advantages-

- Accurate
- less addicted by noise

→ Disadvantages-

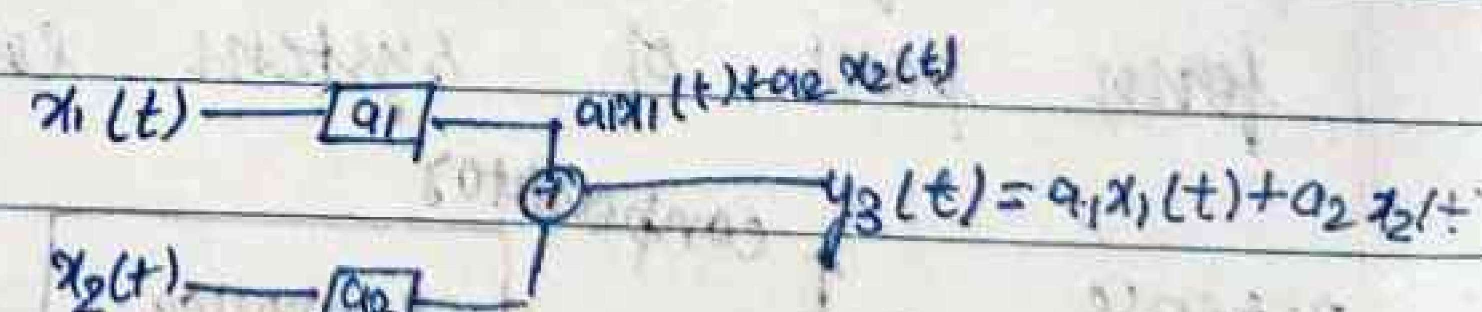
- Complex and controller.
- Feedback reduces the overall gain at system
- Stability is a major problem.

→ Some other types of control system are-

(a) Linear system $x_1(t) \longrightarrow \boxed{a_1} \longrightarrow y_1(t) = a_1 x_1(t)$

(b) Non-linear system $x_2(t) \longrightarrow \boxed{a_2} \longrightarrow y_2(t) = a_2 x_2(t)$

(c) Time varient system



$y_3(t) = y_1(t) + y_2(t)$

(d) Time invarient system

(e) Linear time varient system

(f) linear time invarient system.

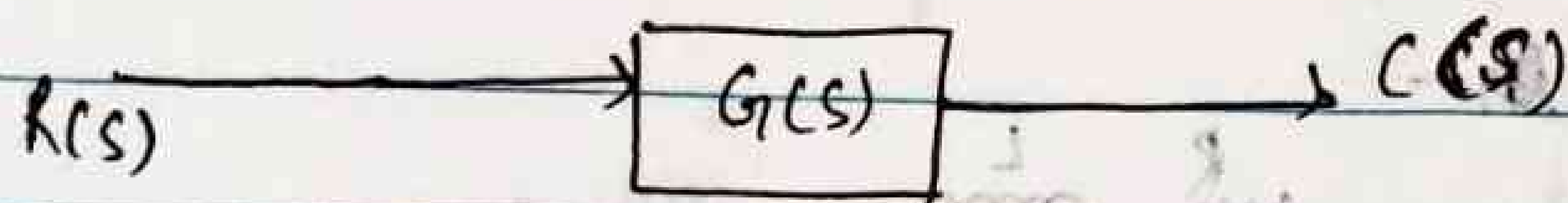
* TRANSFER FUNCTION - Methods for TF

- KVL, KCL
- Signal flow ~~block diag~~ graph
- block diag reduction.

It is defined as - The ratio of laplace ~~TF~~ transform of the O/P to the Laplace transform of I/P with all initial condition are 'zero'

Consider the block diagram of open loop control system where,

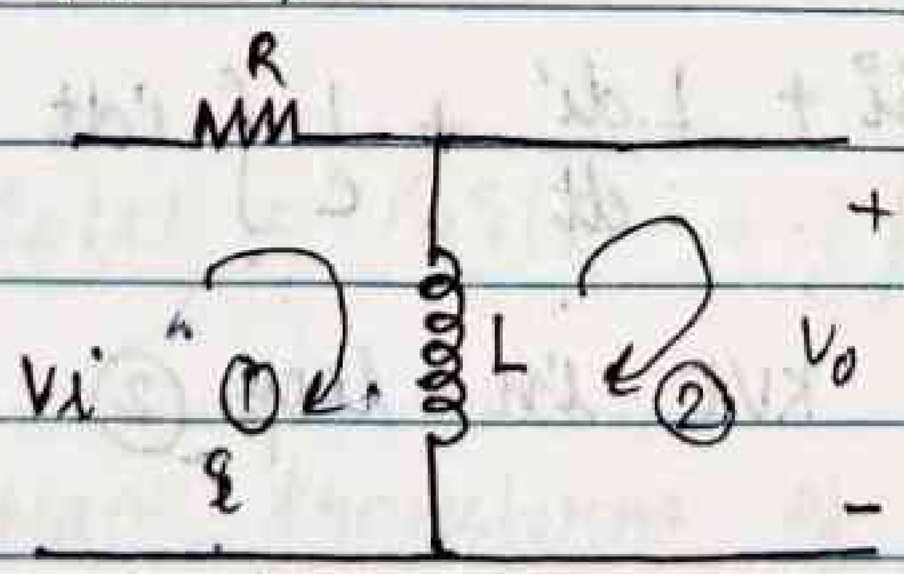
$R(S)$ = Laplace Transform of I/P
$C(S)$ = Laplace " " O/P
$G(S)$ = transfer function.



$$\boxed{G(S) = \frac{C(S)}{R(S)}} = \frac{Lc(t)}{Lr(t)}$$

* Some numericals - • (KVL KCL → applying this rule)

① Find the TF →



Solution-

Applying KVL in loop ①

$$Ri + L\frac{di}{dt} = V_i \quad \text{——— ①}$$

Applying KVL in loop ②

$$V_0 = L\frac{di}{dt} \quad \text{——②}$$

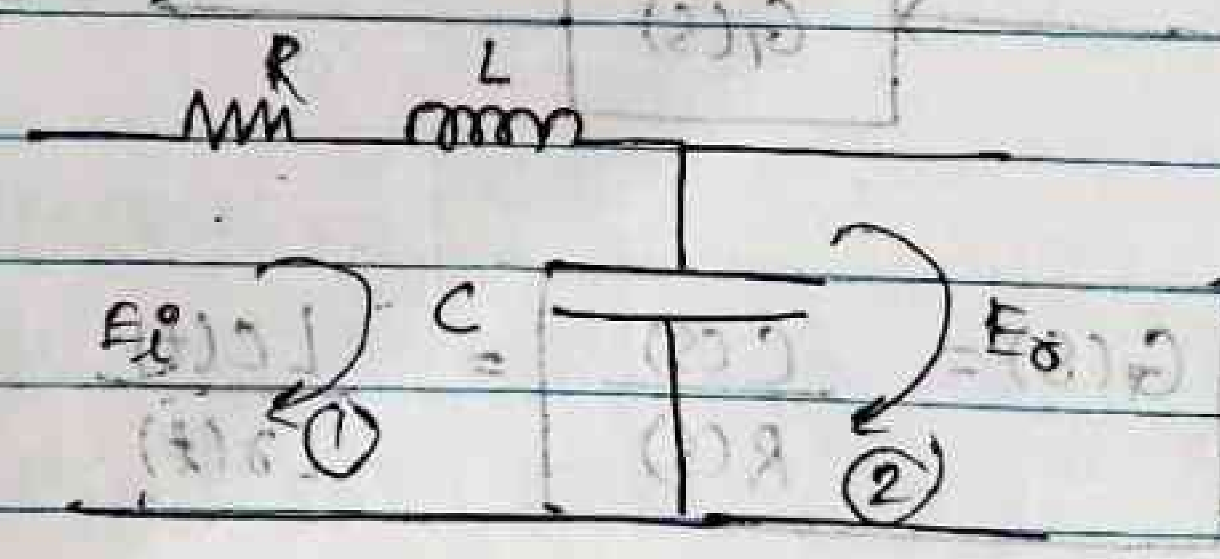
Taking LT of both eq ①, ②

$$RI(S) + SLI(S) = V_i(S)$$
$$SLI(S) = V_0(S)$$

Now, the TF is ⇒ $\frac{V_0(S)}{V_i(S)} = \frac{SLI(S)}{RI(S) + SLI(S)}$

$$\boxed{\frac{V_0(S)}{V_i(S)} = \frac{SL}{R+SL}}$$

② Find TF :-



② solution,

Applying KVL in loop ①

$$Ei = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \text{——①}$$

Applying KVL in loop ②

$$E_0 = \frac{1}{C}\int i\,dt \quad \text{——②}$$

Taking LT of eq ① & ②

$$E_i(S) = RI(S) + LS\,I(S) + \frac{1}{CS}I(S)$$
$$E_0(S) = \frac{1}{CS}I(S)$$

$I(S)(R+LS)+\frac{1}{CS}I(S)$

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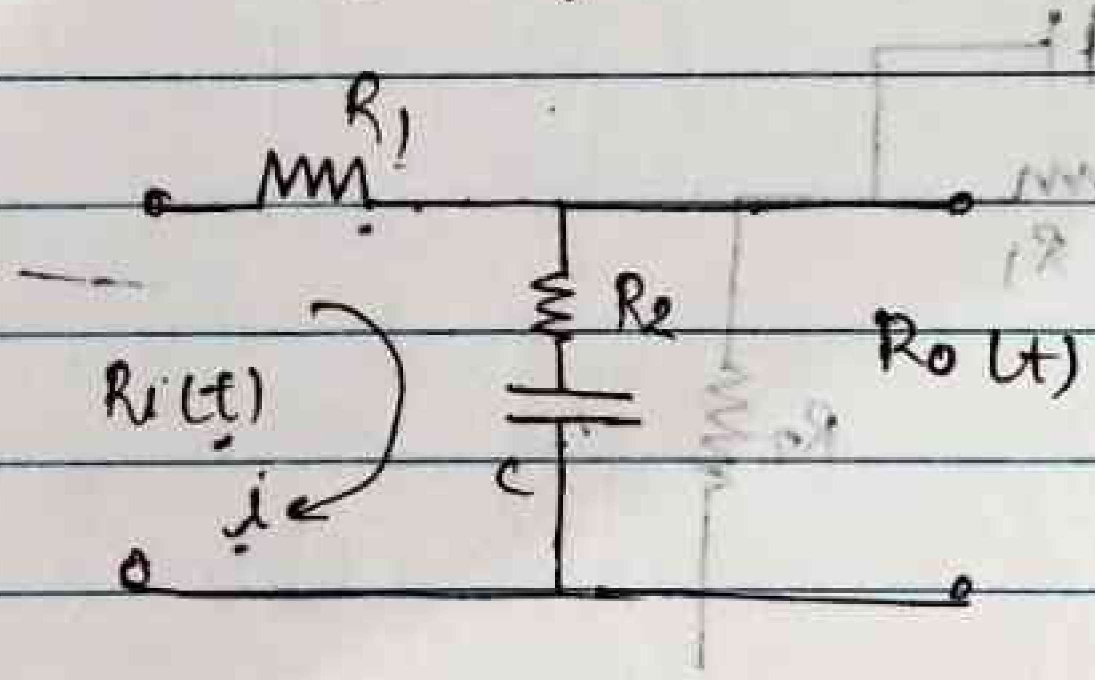
Now TF = $\frac{E_o(S)}{E_i(S)} = \frac{\frac{1}{CS}I(S)}{RI(S)+LSI(S)+\frac{1}{CS}I(S)}$

$$= \frac{\frac{1}{CS}}{R+LS+\frac{1}{CS}}$$

$$\frac{E_o(S)}{E_i(S)} = \frac{1}{CS} \times \frac{CS}{RCS+LCS^2+1}$$

$$\boxed{\frac{E_o(S)}{E_i(S)} = \frac{1}{S^2LC+SRC+1}}$$

③ find the TF of lag network



③ Soln Applying KVL in both Mesh -

$$e_i(t) = R_1 i(t) + R_2 i(t) + \frac{1}{C}\int i(t)\,dt \quad \text{—①}$$

$$e_o(t) = R_2 i(t) + \frac{1}{C}\int i(t)\,dt \quad \text{———②}$$

Taking laplace transform of above eqn -

$$E_i(S) = R_1 I(S) + R_2 I(S) + \frac{1}{CS} I(S)$$

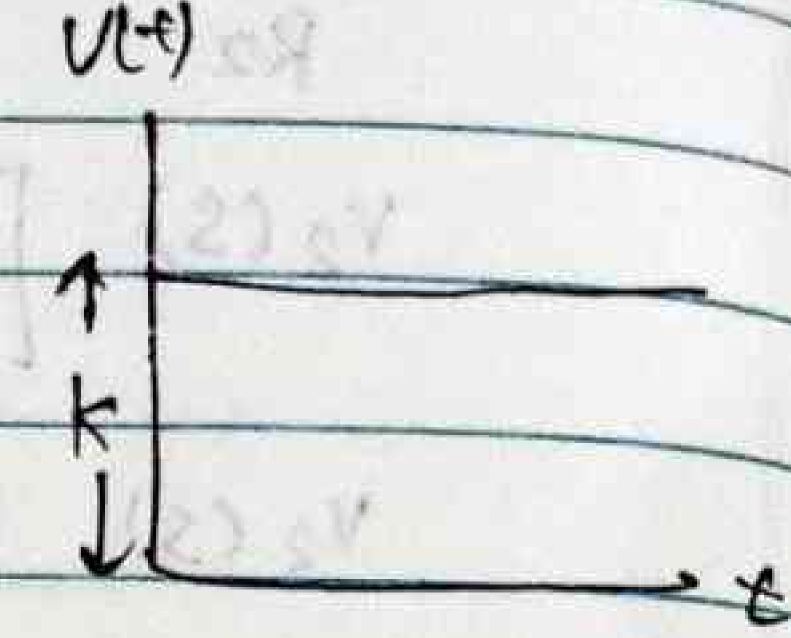
$$E_o(S) = R_2 I(S) + \frac{1}{CS} I(S)$$

Now, $\frac{E_o(S)}{E_i(S)} = \frac{I(S)\,[R_2+\frac{1}{CS}]}{I(S)\,[R_1+R_2+\frac{1}{CS}]}$

(unit-2)

SIGNALS -

(i) **Step Function -**



- It mathematically express as -

$$V(t) = \begin{cases} 0 & \text{for } t < 0 \\ k & \text{for } t \geq 0 \end{cases}$$

- A unit step function is denoted by $u(t)$ & is defined as -

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

- LT of such input is $\frac{1}{s}$

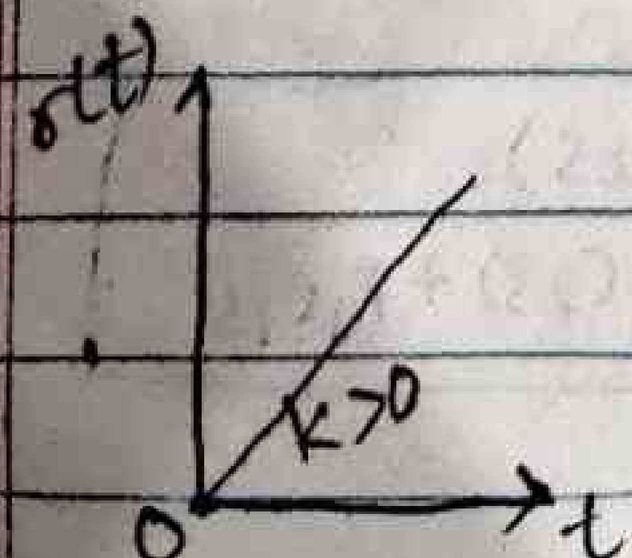
(ii) Ramp function -

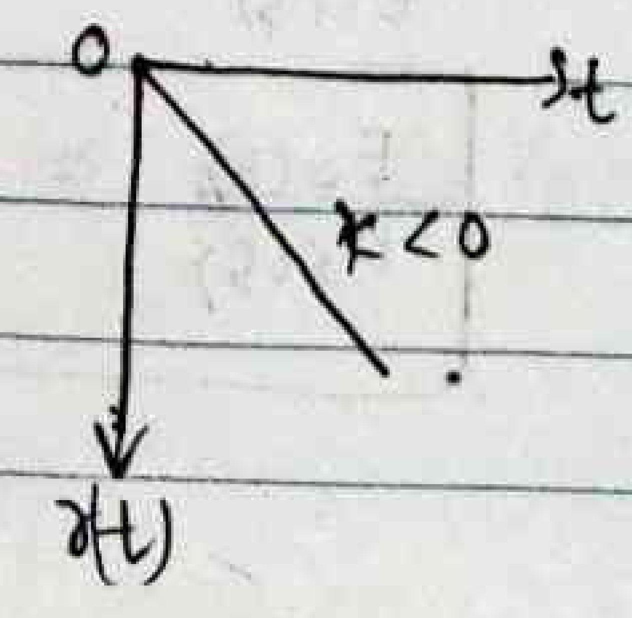
- Ramp function starts from origin and increase or decrease linearly with time, as shown in fig -

- A ramp function denoted by $r(t)$ -

$$r(t) = \begin{cases} 0 & t < 0 \\ kt & t > 0 \end{cases}$$

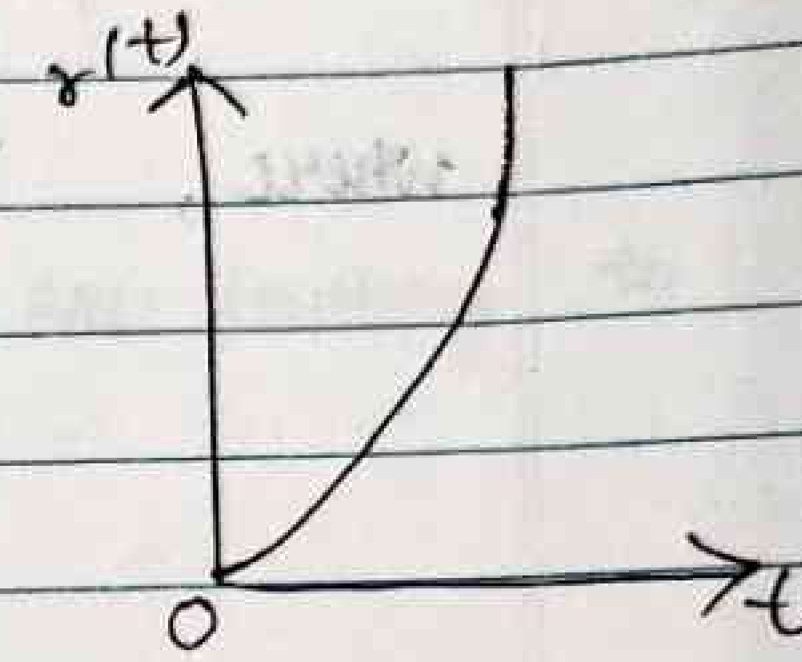
$$LT = \frac{k}{s^2}$$





(iii) Parabolic function-

- The value of $r(t)$ is zero for $t<0$ & is quadratic function of time for $t>0$.



- The parabolic function defined as-

$$r(t) = 0; \quad t \le 0$$
$$= \frac{Kt^2}{2}, \quad t > 0$$

where, 'K' is constant.

- For parabolic function, $K=1$, The unit parabolic function is defined as-

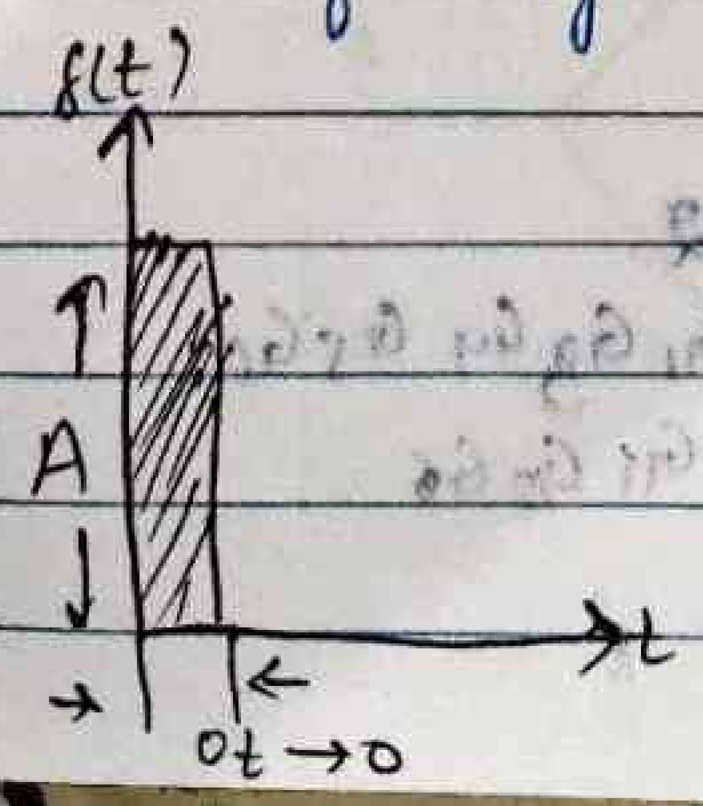
$$r(t) = 0, \quad t<0$$
$$= \frac{t^2}{2}, \quad t>0$$

$$L.T = \frac{1}{s^3}$$

(iv) Impulse function-

- $\delta(t) = A$ for $t=0$
  $\quad\quad\quad 0$ for $t \neq 0$

- It is the I/P applied instantaneously for short duration of time of very high amplitude.



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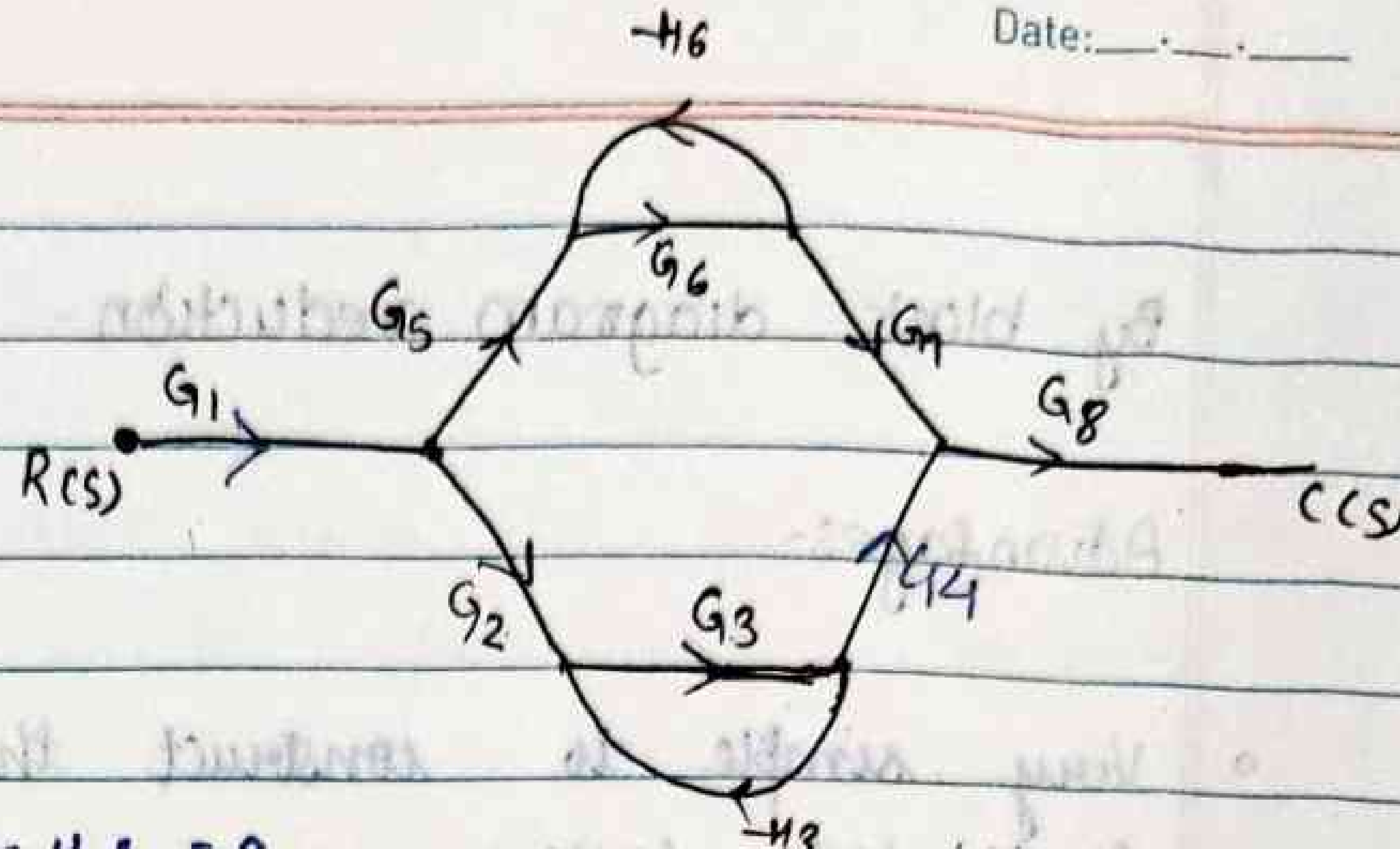
(3) Find $\frac{C(s)}{R(s)} = ?$



(3) soln -

No of forward paths = 2.

$$T_1 = G_1 G_5 G_6 G_7 G_8$$
$$T_2 = G_1 G_2 G_3 G_4 G_8$$

$$\therefore \frac{C(s)}{R(s)} = \frac{\sum_{k=1}^{2} T_k \Delta_k}{\Delta} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$$

Individual loops are -

$$L_1 = -G_6 H_6$$
$$L_2 = -G_3 H_3$$
$$L_3 \; G$$

Both L_1, L_2 are non-touching to each other -

$$\Delta = 1 - [L_1 + L_2] + [L_1 L_2]$$
$$= 1 + G_6 H_6 + G_3 H_3 + G_3 H_6 G_6 H_3 H_6$$

Consider T_1, L_2 is non-touching

$$\Delta_1 = 1 - L_2 = 1 + G_3 H_3$$

Consider T_2, L_1 is non-touching

$$\Delta_2 = 1 - L_1 = 1 + G_6 H_6$$

$$\frac{C(s)}{R(s)} \Rightarrow \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_5 G_6 G_7 G_8 (1 + G_3 H_3) + G_1 G_2 G_3 G_4 G_6 (1 + G_6 H_6}{\Delta}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_5 G_6 G_7 G_8 (1 + G_3 H_3) + G_1 G_2 H_3 G_4 G_8 (1 + G_6 H_6)}{1 + G_6 H_6 + G_3 H_3 + G_3 G_6 H_3 H_6}$$

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* BLOCK DIAGRAM ALGEBRA-

- As far as we know, the I/P/O/P behaviour of a linear system or element of linear system is given by its TF.

$$\boxed{G(S) = \frac{C(S)}{R(S)}}$$

where,

$R(S)$ = LT of I/p variable
$C(S)$ = LT of O/p variable

- A convenient block graphical representation of this behaviour is block diag shown in below fig -

$$R(S) \xrightarrow{I/P} \boxed{G(S)} \longrightarrow C(S) \text{ O/P}$$

where, $R(S)$ = signal into the block represents the I/P
$C(S)$ = signal out of the block represents the O/P
$G(S)$ = block itself stand for TF.

- The flow of information (signal) is unidirectional from I/P to O/P with the O/P being equal to the I/P multiplied by T/F of block.

- A complex system comprising of several non-loading elements is represented by interconnection of blocks for individual element.

- The block are connected by lines with arrows indicating the unidirectional flow of info from O/P of one block to I/P of other.

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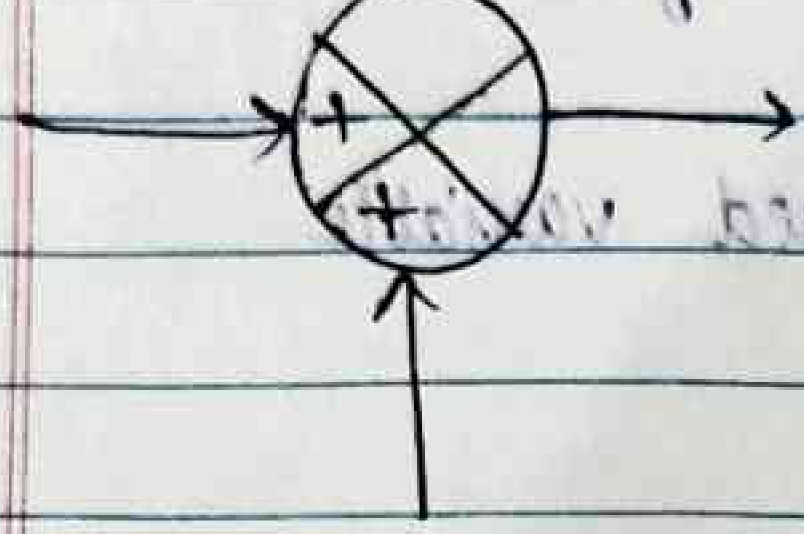
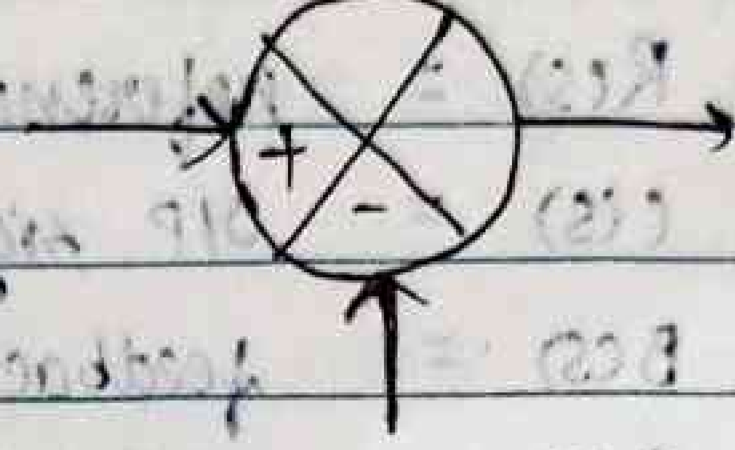
- In addition, to this, summing or differencing of signal is indicated by symbols

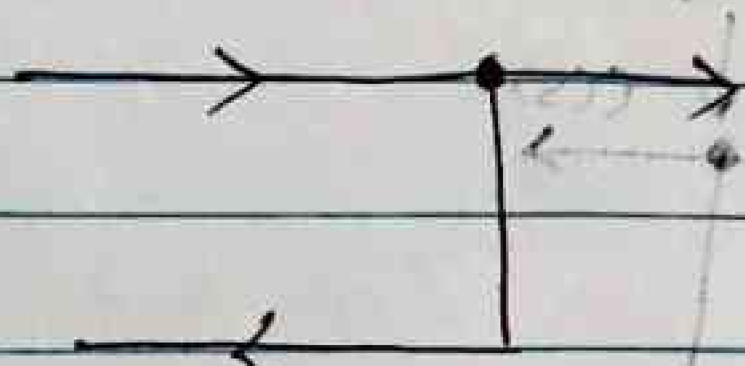


(a) summing



(b) differencing

- While the take-off point of signal -



(c) take-off point.

---

imp Note -

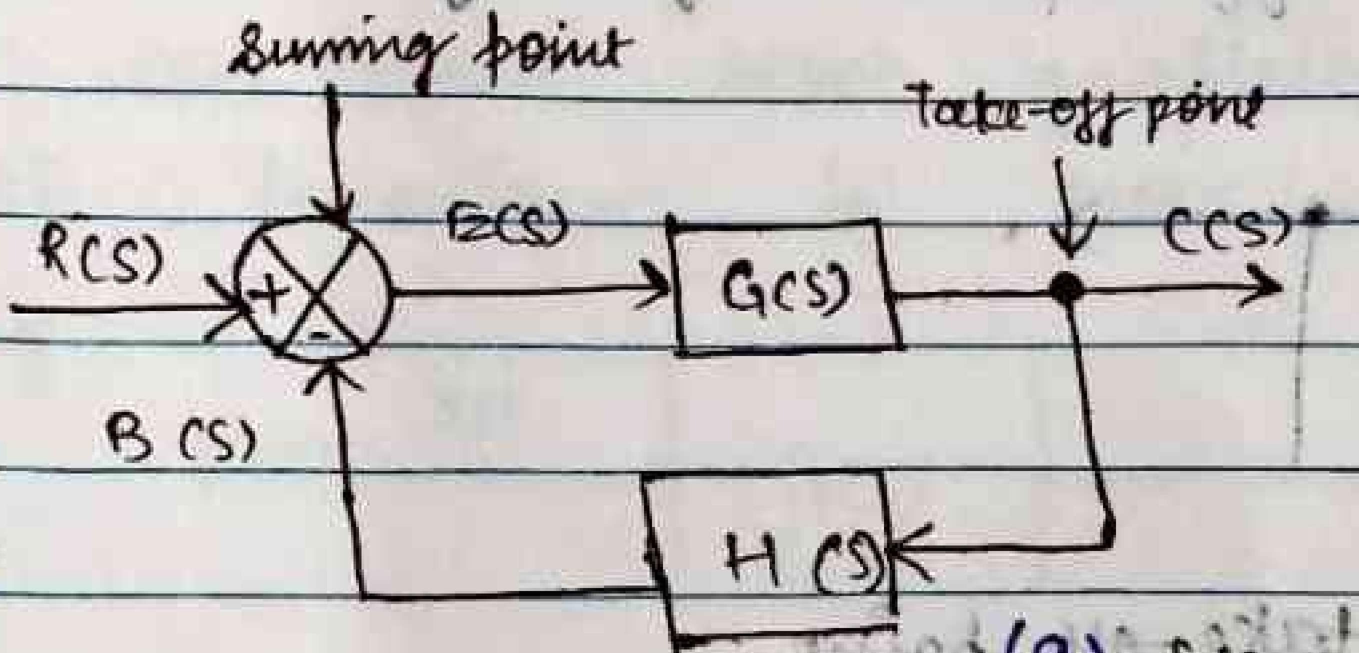
Block diagram of some of the control system turn out to be very complex such that the evaluation of their performance requires simplification of block diag which is carried out by block diag rearrangements.

Block diag of closed loop system -

→

Fig (a) shows the block diag of a -ve feedback system. With reference to this the terminology used in block diag control system are -

$R(s)$ = reference point
$C(s)$ = O/P signal or controlled variable
$B(s)$ = feedback signal
$E(s)$ = actuating signal
$G(s) = C(s)/E(s)$ = forward path TF
$H(s)$ = TF of feedback elements.
$G(s)H(s) = B(s)/E(s)$ = loop TF
$T(s) = C(s)/R(s)$ = closed-loop TF



(a)-fig

From above fig -

$$C(s) = \frac{G(s)}{E(s)} \quad \text{—— (1)}$$

$$E(s) = R(s) - B(s) = R(s) - H(s)C(s) \quad \text{—— (2)}$$

eliminating $E(s)$ from eq (1) & (2) -

$$C(s) = G(s)R(s) - G(s)H(s)C(s)$$

Or

$$\frac{C(s)}{R(s)} = T(s) = \frac{G(s)}{1+G(s)H(s)} \quad \text{—— (3)}$$

∴ The system in fig (a) can be reduced as single block as shown in fig (b)

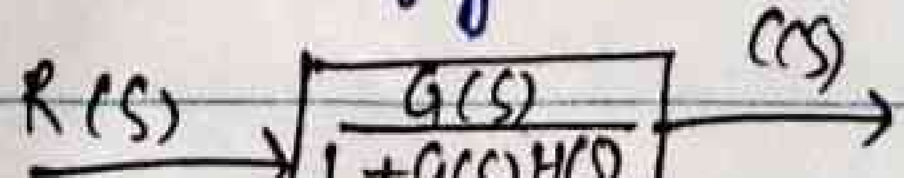


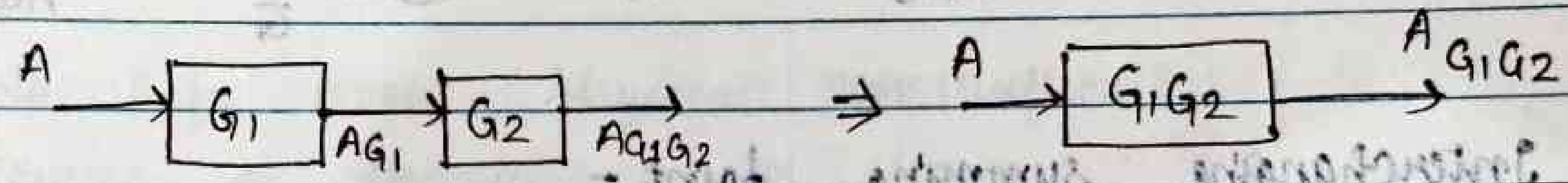
fig - (b)

③ <u>By block diagram reduction:</u>

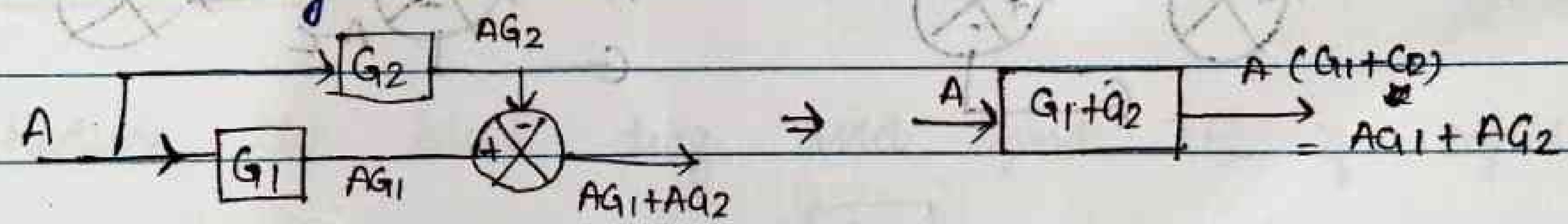
Block diag reduction is a pictorial representation of each and every component of control system

<u>Rules -</u>

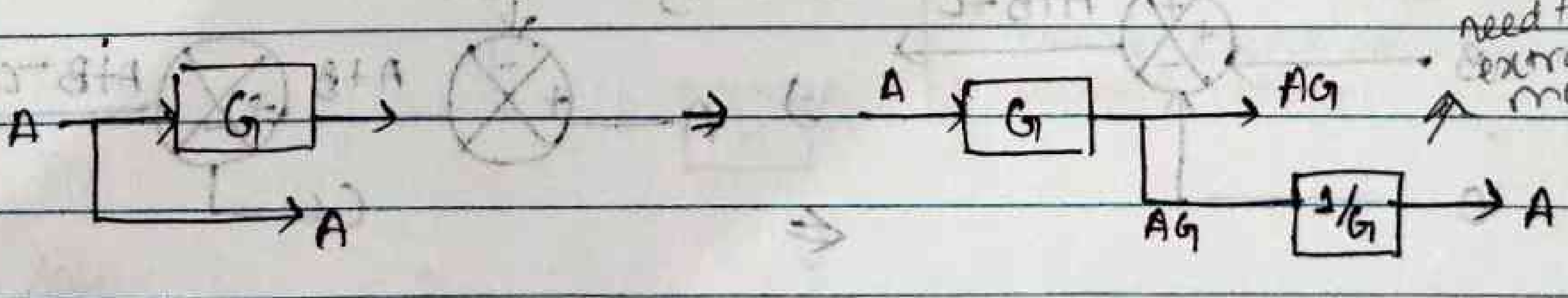
(1) Combining the blocks in cascade



(2) Combining || blocks -

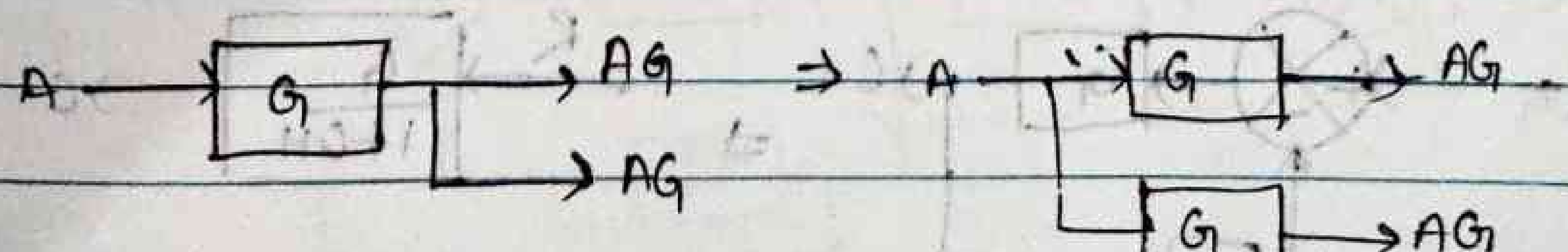


(3) Moving the branch point ahead of the block.

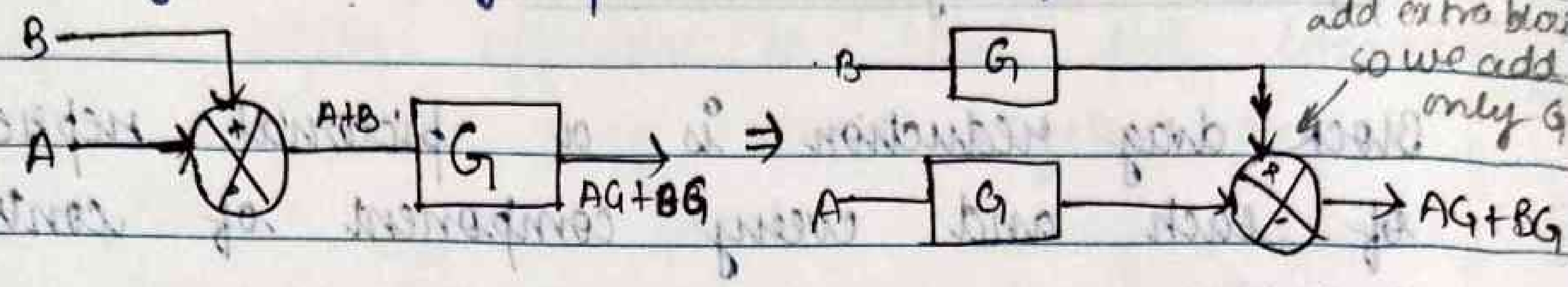


need to add an extra block in moving the branching point

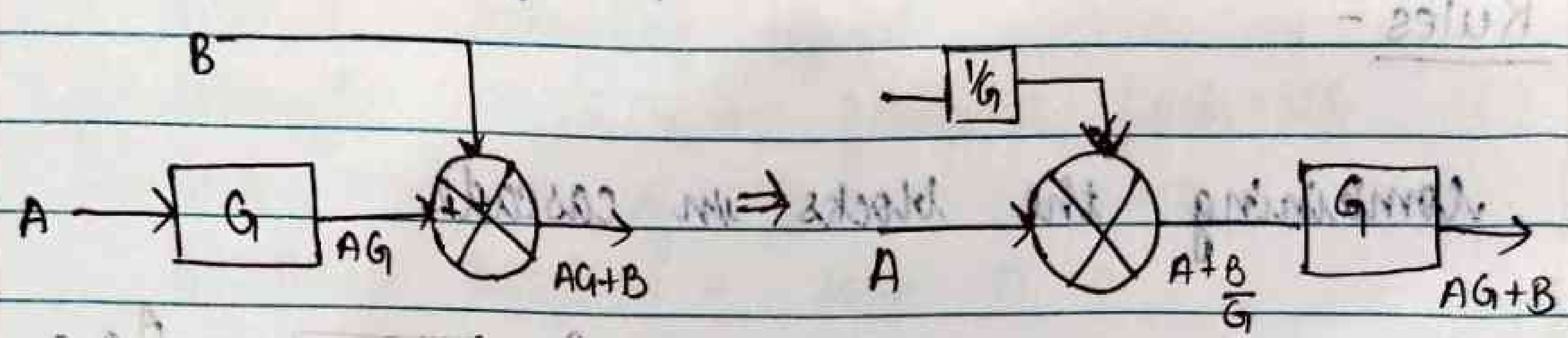
(4) Moving the branch point before the block.



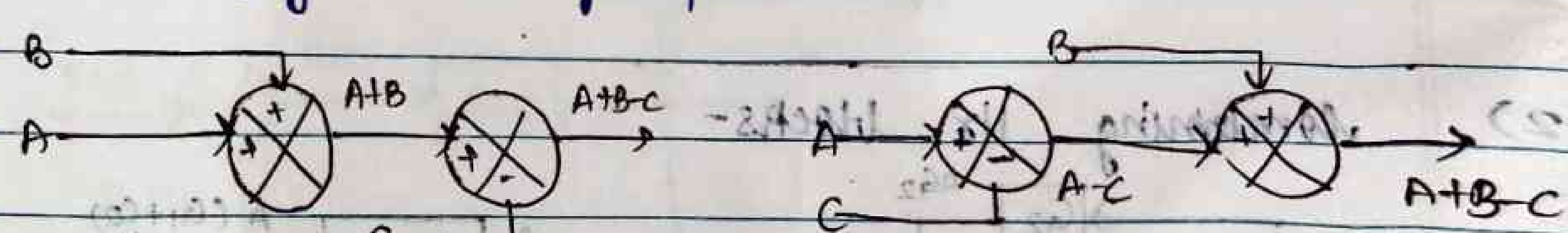
(5) Moving summing point ahead of block - Here no need to add extra block so we add only G



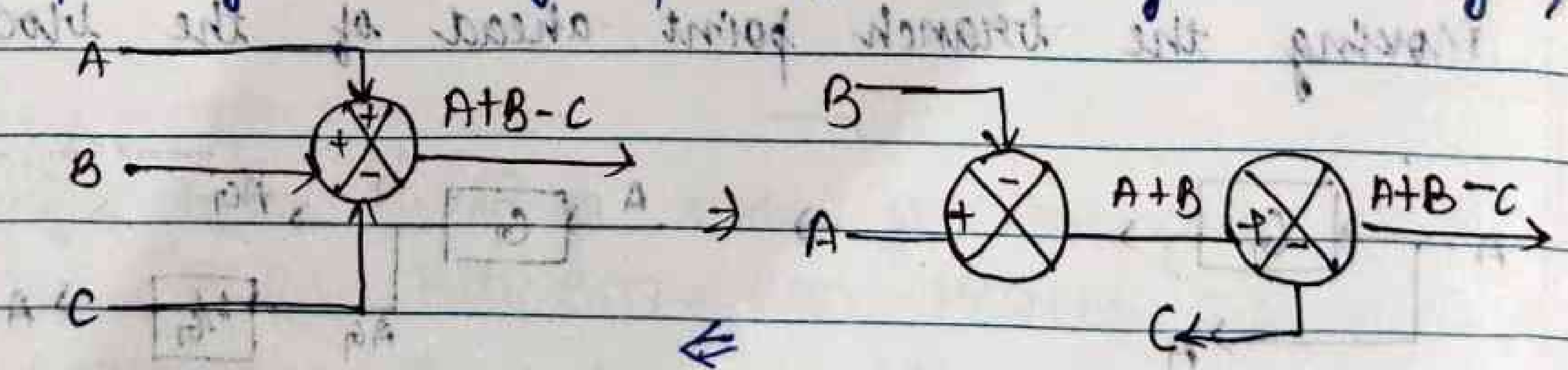
(6) Moving summing point before block -



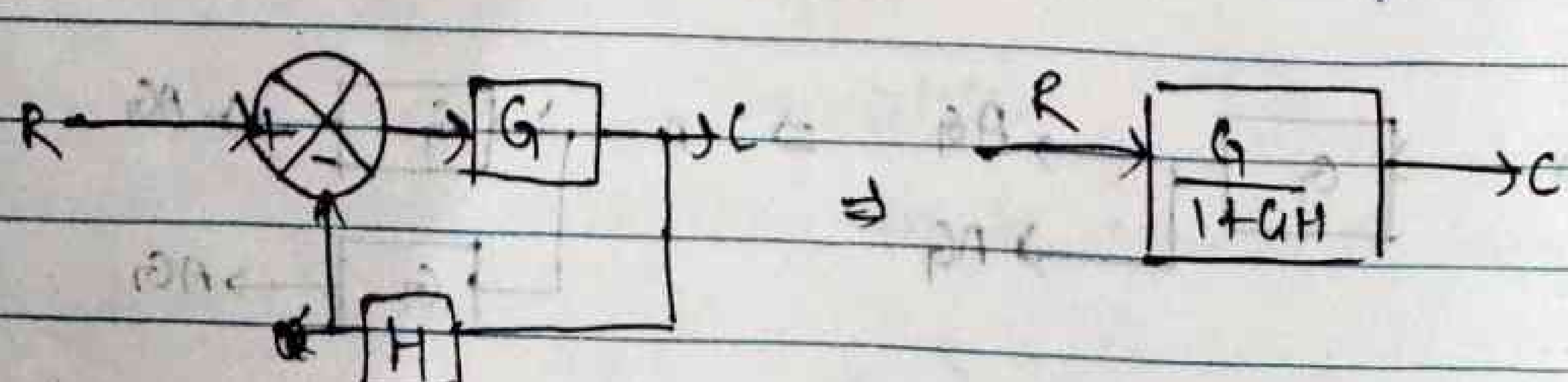
(7) Interchanging summing point -



(8) Splitting summing points | combining summing points



(9) Elimination of feed back loop -



Advantages - of block diagram reduction -

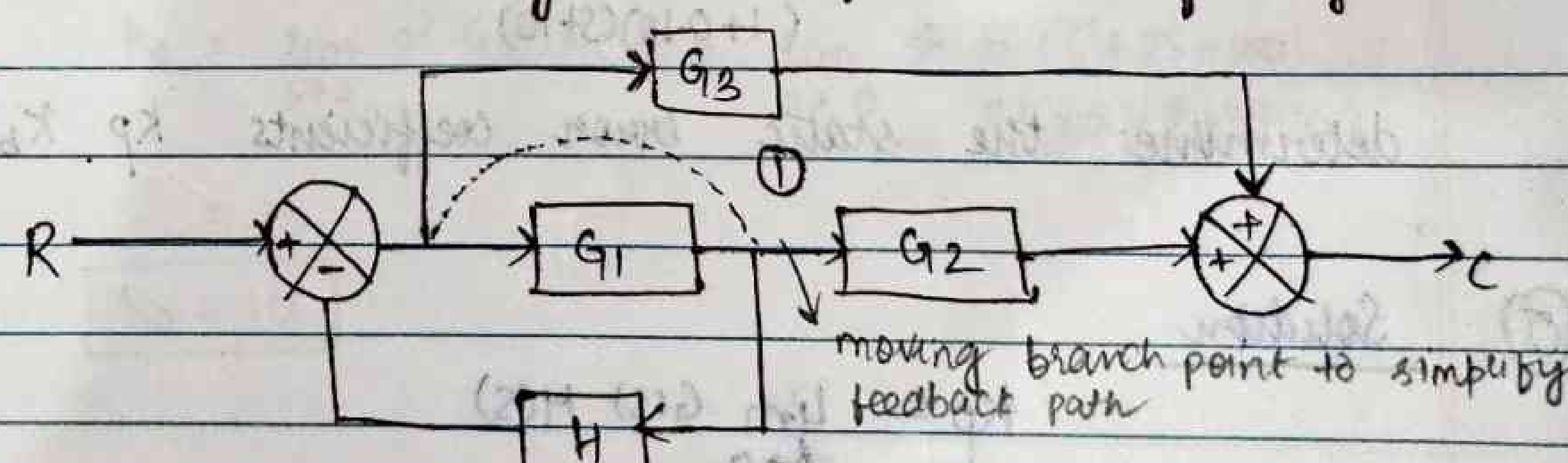
- Very simple to construct block diagram for a complicated system.
- Functions of individual element can be visualised.
- Individual and overall performance can be studied.
- Overall TF can be calculated easily.

Disadvantages of block diagram reduction -

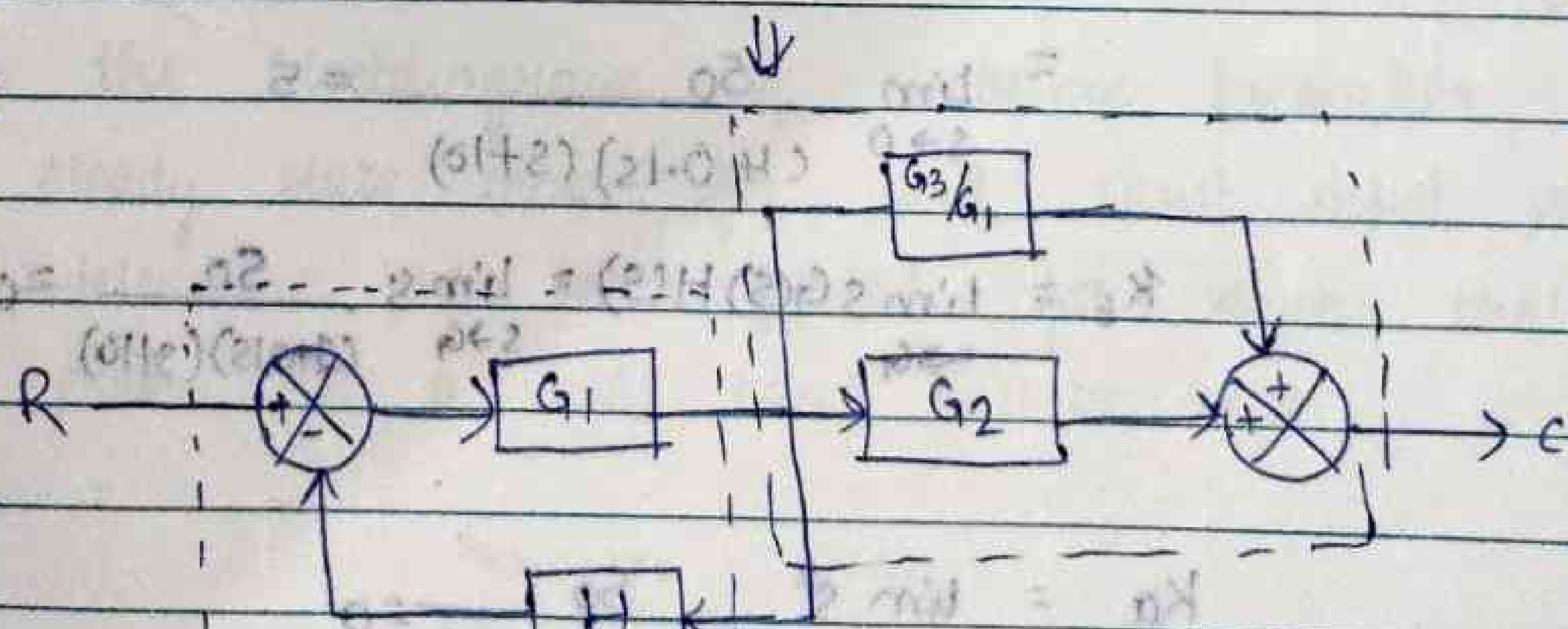
- No info about physical construction
- Source of energy is not shown

Numericals -

(1) Reduce the block diag and find TF of system.



(4) Soln



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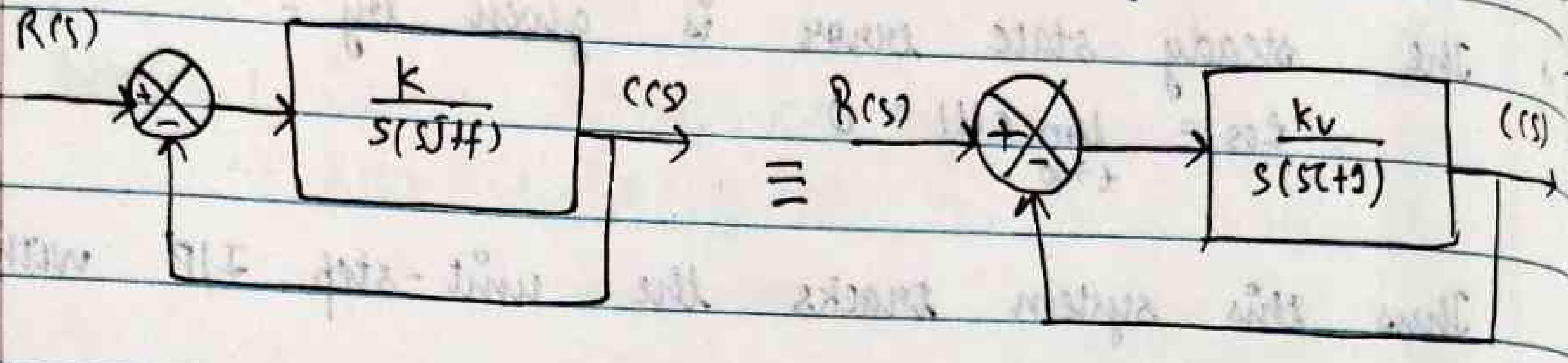
(b) Second order system-

Let us consider the second order system with



Simplified block diagram Of 2nd Order System.

- The order of a control system is determined by power of 's' in the denominator of its T/F.

- If the power of s in the denominator of TF of a control system is 2. then system is said to be second order system.

- The general expression of second order system is given as-

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}}$$

Where, ζ & ω_n are damping ratio

◉ Response to the unit step -

From above fig - The overall TF of system is

$$\frac{C(s)}{R(s)} = \frac{K_v}{\tau s^2+s+K_v} = \frac{K_v/\tau}{s^2+\frac{1}{\tau}(s)+\frac{K_v}{\tau}} \quad \text{—(1)}$$

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It can also be written in the standard form -

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2} = \frac{P(S)}{q(S)} \quad \text{—— (2)}$$

Where, ξ = damping factor $= \frac{1}{2\sqrt{(K_v \tau)}} = \frac{f}{2\sqrt{(KJ)}}$

ω_n = undamped natural frequency $= \sqrt{K_v/\tau} = \sqrt{(K/J)}$

→ The denominator polynomial called characteristic polynomial i.e.

$q(S)=0$ —— (3)

→ The characteristic eqn of system under consideration

$$S^2 + 2\xi\omega_n + \omega_n^2 = 0 \quad \text{—— (4)}$$

The roots of this characteristic eq are given by -

$$(S^2 + 2\xi\omega_n S + \omega_n^2) = (S-S_1)(S-S_2)$$

For $\xi<1$

$$S_1, S_2 = -\xi\omega_n \pm j\omega_n\sqrt{(1-\xi^2)} = -\xi\omega_n \pm j\omega_d$$

where $\omega_d = \omega_n\sqrt{(1-\xi^2)}$, is called damped natural frequency

→ From eq (1) $\frac{C(S)}{R(S)} = \frac{K_v}{\tau S^2+S+K_v} = \frac{K_v/\tau}{S^2+\frac{1}{\tau}(S)+\frac{K_v}{\tau}}$ for the unit step I/P, the O/P response given by -

$$C(S) = \frac{\omega_n^2}{S[S+\xi\omega_n - j\omega_n\sqrt{(1-\xi^2)}][(S+\xi\omega_n + j\omega_n\sqrt{(1-\xi^2)}]}$$

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* TIME RESPONSE SPECIFICATIONS -

(a) Delay time td:

It is the time required for the response to reach 50% of the final value in first attempt.

(b) Rise time tr:

It is time required for the response to rise from 10% to 90% of the final value for overdamped system and 0-100% of the final value for underdamped system

(c) Peak time tp:

It is the time required for the response to reach the peak of time response or the peak overshoot.

(d) Peak overshoot Mp:

It indicates the normalised difference b/w time response peak and stedy o/p and is defined as -

$$\text{Peak percent overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

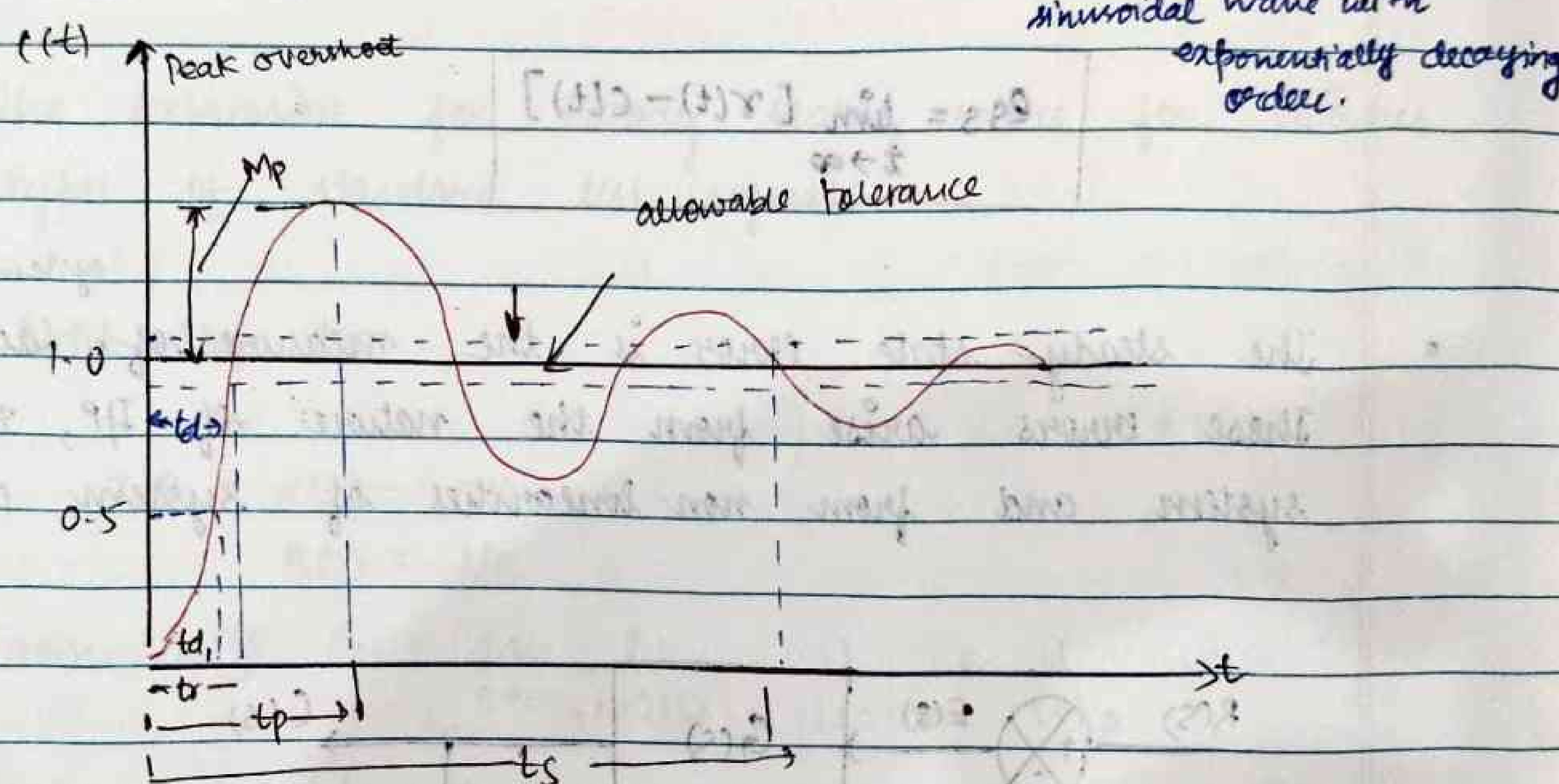
(e) Setting time ts -

It is time required for the response to reach and stay within a specified tolerance band of its final value

(f) ⇒ Steady state error ess -

It indicates the error b/w the actual o/p and desired o/p as t tends to infinity, i.e;

$$e_{ss} = \lim_{t\to\infty} [r(t) - c(t)]$$



Time response specifications.

From above fig it's seen that by specifying td, tr, tp, Mp, ts and ess, the shape of unit-step time response curve is virtually fixed.

Note-

- The performance of a control system are express in terms of transient response to a unit step I/P becoz its easy to generate.

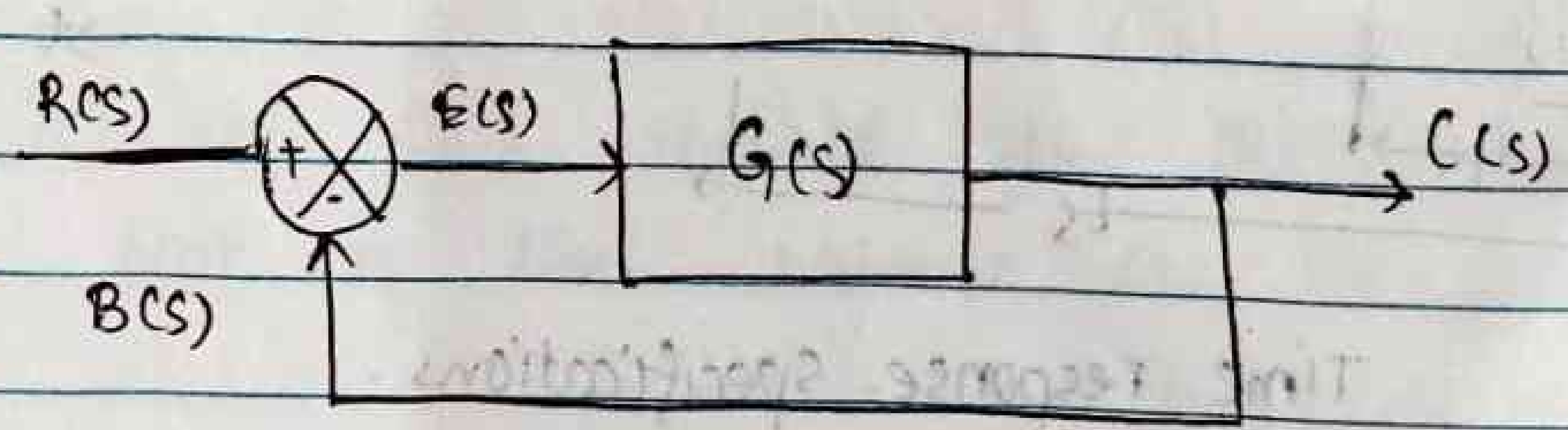
IMP • The transient response of a control system to a unit step I/P depends upon the initial condition.

* STEADY STATE ERROR [ess]

- It is the difference b/w actual O/P and desired O/P as time 'd' tends to infinity.

$$\boxed{e_{ss} = \lim_{t\to\infty} [r(t) - c(t)]}$$

- The steady state error is the measure of system accuracy. These errors arise from the nature of I/P, type of system and from non-linearities of system componen



(a) unit feedback system.

Consider a unit feedback system (shown in fig). The I/P is R(S), O/P is C(S), feedback signal B(S) and the difference b/w I/P and O/P is the error signal E(S).

From above fig -

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)}$$

$$C(S) = E(S)\,G(S)$$

$$\therefore E(S) = \frac{1}{1+G(S)} R(S) \quad \text{--- (1)}$$

- The steady-state error e_{ss}, may now be found by use of final value theorem

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{sR(s)}{1+G(s)}} \quad \text{--- (2)}$$

From eq (2), it shows the steady-state error depends upon I/P ~~(Rs)~~ $R(s)$ and forward T/F $G(s)$.

→ The expression for steady state errors for various types of standard test signals-

(a) Unit-Step I/P-

$$\text{I/P : } r(t) = u(t)$$
$$R(s) = 1/s$$

From eq (2) $e_{ss} = \boxed{\lim_{s \to 0} \frac{1}{1+G(s)} = \frac{1}{1+G(0)}} = \frac{1}{1+K_p}$

Where $K_p = G(0) \rightarrow$ position error constant. --- (3)

(b) Unit-ramp ~~I/P~~ (Velocity) I/P-

$$\text{I/P } r(t) = t \text{ or } r(t) = 1$$
$$R(s) = 1/s^2$$

From eq (2),

$$e_{ss} = \lim_{s \to 0} \frac{1}{s + sG(s)} = \lim_{s \to 0} \frac{1}{sG(s)} = \frac{1}{K_v}$$

Where $K_v = \lim_{s \to 0} sG(s) \rightarrow$ velocity error constant --- (4)

(c) Unit-parabolic (acceleration) I/P-

$$\text{I/P : } r(t) = t^2/2 \text{ or } r(t) = 1$$

$R(s) = \frac{1}{s^3}$

From eq (2)

$$e_{ss} = \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s)}$$

$$= \lim_{s \to 0} \frac{1}{s^2 G(s)} = \frac{1}{K_a} \quad \text{—— (6)}$$

Where $K_a = \lim_{s \to 0} s^2 G(s) \rightarrow$ acceleration error constant.

* ERROR CONSTANTS-

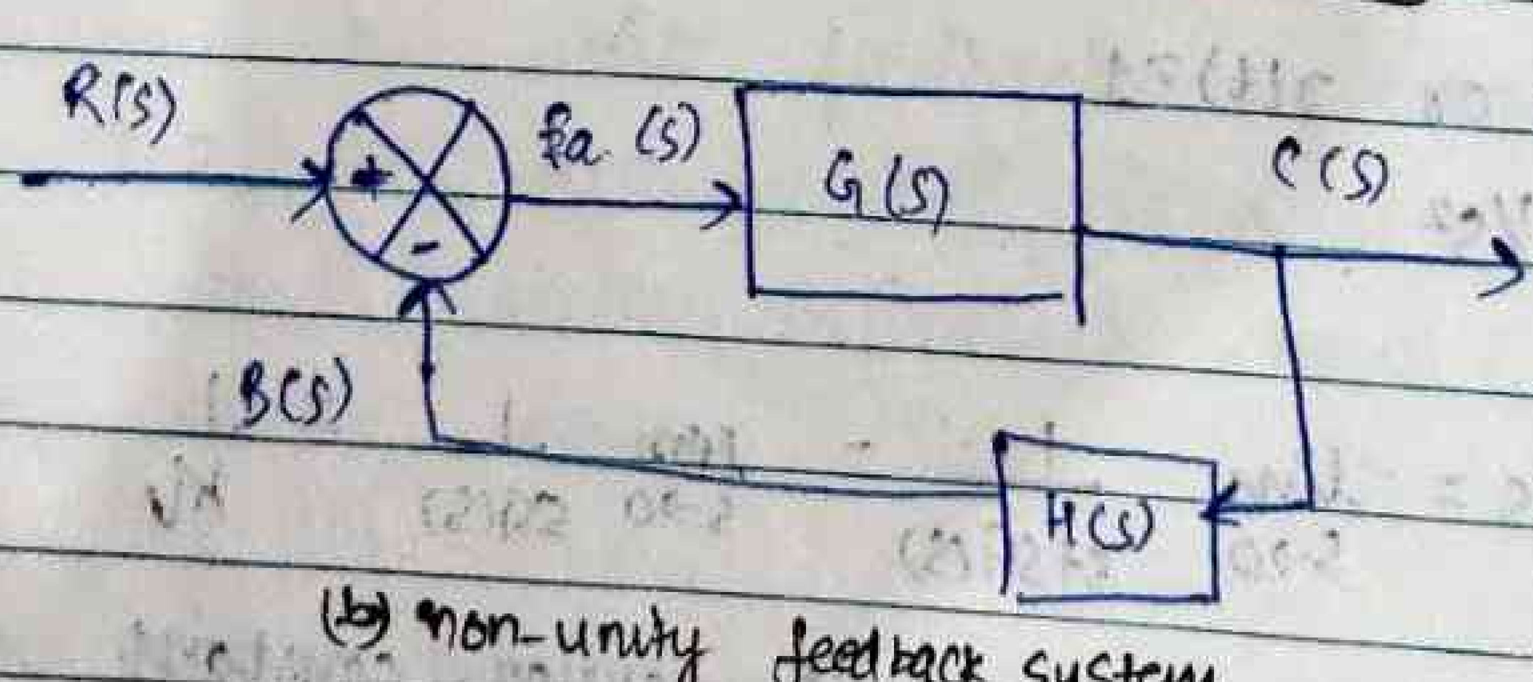
For non-unity feedback systems the diff b/w the I/P signal R(s) and feedback signal B(s) is the actuating error signal $E_a(s)$ which is given by-

$$\boxed{E_a(s) = \frac{1}{1 + G(s)H(s)} R(s)} \quad \text{—— (7)}$$

∵ The ~~$E_a(s)$~~

∴ The steady-state actuating error is-

$$\boxed{e_{ass} = \lim_{s \to 0} \frac{s R(s)}{1 + G(s)H(s)}}$$



(b) non-unity feedback system

- The error constants for non-unity feedback signal system may be obtained by replacing G(s) by G(s) H(s)
- The error constants K_p, K_v & K_a describe the ability of a system to reduce or eliminate steady-state error

Time response with step I/P.

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* TIME RESPONSE SPECIFICATION OF SECOND ORDER-

Derivation of Rise time (t_r)-

$\{c(t)_{t} = T_r\} = 1$ — (Theeta) reads as 'fai'

$\therefore \quad 1 = 1 - \dfrac{e^{-\xi\omega_n T_r}}{\sqrt{1-\xi^2}} \sin(\omega_d T_r + \theta)$ (for unit step I/P.

$\therefore \quad -\dfrac{e^{-\xi\omega_n T_r}}{\sqrt{1-\xi^2}} \sin(\omega_d T_r + \theta) = 0$

equation will get satisfied only if, $\sin(\omega_d T_r + \theta) = 0$.
Trignometrically this is true only if,

$\omega_d T_r + \theta = n\pi$ where $n = 1, 2 \ldots$

As we are interested in first attempt use $n=1$

$\omega_d T_r + \theta = \pi$

$$\boxed{T_r = \frac{\pi - \theta}{\omega_d} \text{ sec}}$$

Derivation of Peak time (T_p)-

Transient response of second order underdamped system is given by

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

where $\theta = \tan^{-1} \dfrac{\sqrt{1-\xi^2}}{\xi^2}$

As at $t = T_p$ $c(t)$ will achieve its maxima, Acc. to maxima theorem,

$$\left.\frac{d\,c(t)}{dt}\right|_{t=t_p} = 0$$

So differentially $c(t)$ w.r.t 't' we can write

i.e; $$\frac{-e^{-\xi\omega_n t}(-\xi\omega_n)\sin(\omega_d t+\theta)}{\sqrt{1-\xi^2}} - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\omega_d\cos(\omega_d t+\theta) =$$

$$\therefore \xi\sin(\omega_d t+\theta) - \sqrt{1-\xi^2}\cos(\omega_d t+\theta) = 0$$

$$\therefore \xi\sin(\omega_d t+\theta) - \sqrt{1-\xi^2}\cos(\omega_d t+\theta) = 0$$

$$\tan(\omega_d t+\theta) = \frac{\sqrt{1-\xi^2}}{\xi}$$

Now, $$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

$$\frac{\sqrt{1-\xi^2}}{\xi} = \tan\theta$$

Peak time (T_p) - for trignometric formula

$$\tan(n\pi+\theta) = \tan\theta$$

$$\therefore \omega_d t = n\pi \quad \text{where } n = 1,2,3$$

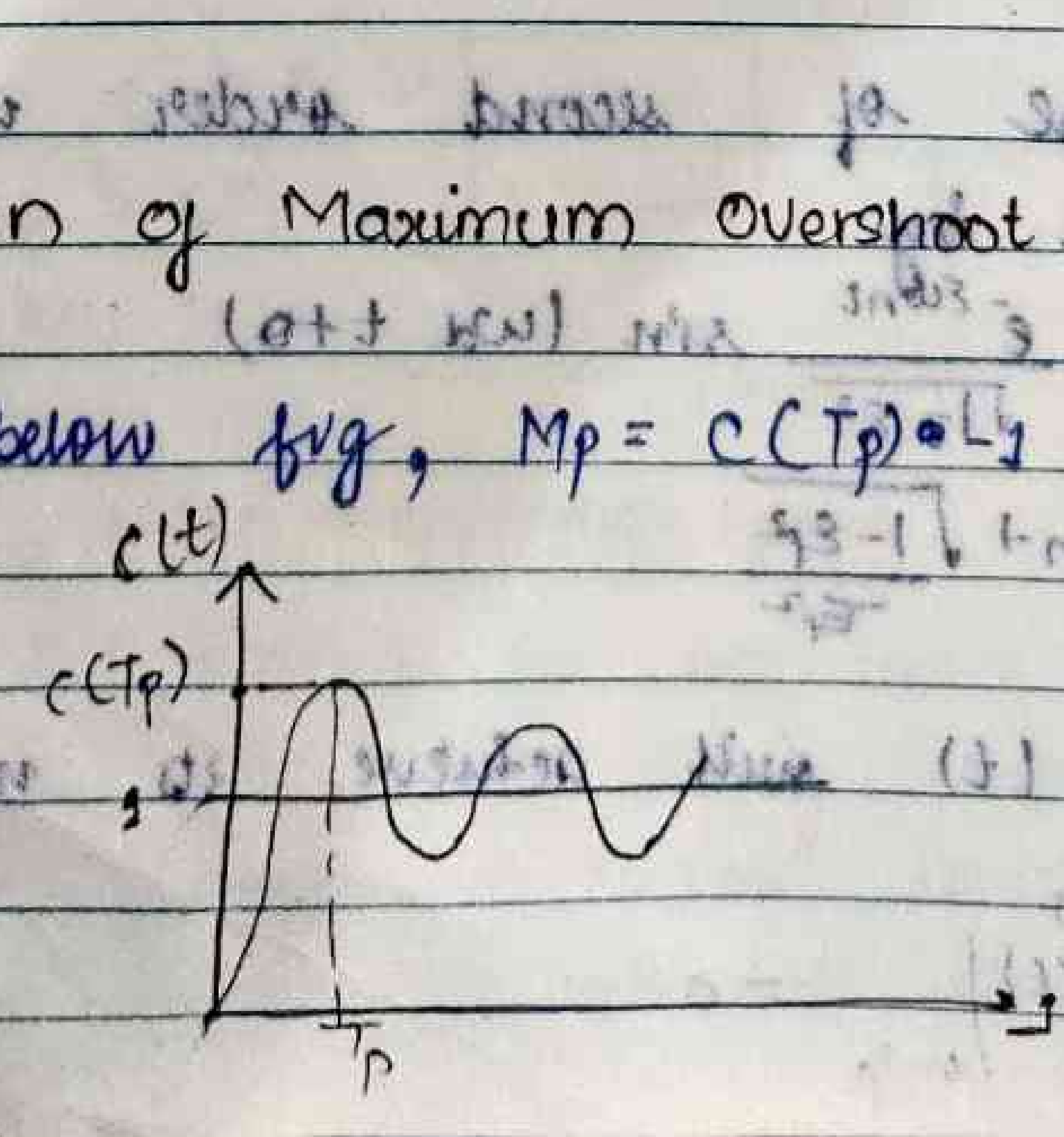
But T_p, the time required for first peak overshoot $\therefore n=1$

$$\omega_d T_p = \pi$$

$$\boxed{T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \text{ sec}}$$

Derivation of Maximum Overshoot (M_p) -

From below fig, $M_p = C(T_p) - 1$



$$M_p = \left\{ 1 - \frac{e^{-\xi \omega_n T_p}}{\sqrt{1-\xi^2}} \sin(\omega_d T_p + \theta) \right\} - 1$$

$$M_p = \frac{e^{-\xi \omega_n T_p}}{\sqrt{1-\xi^2}} \sin(\omega_d T_p + \theta)$$

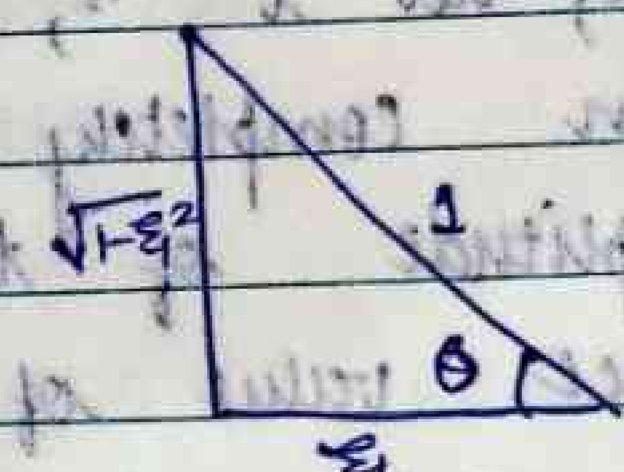
But $T_p = \frac{\pi}{\omega_d}$, substituting above we get,

$$M_p = \frac{e^{-\xi \omega_n T_p}}{\sqrt{1-\xi^2}} \sin \cancel{(\omega_d T_p + \theta)} (\pi + \theta)$$

Now,

$$\sin(\pi + \theta) = -\sin(\theta)$$

$$M_p = \frac{e^{-\xi \omega_n T_p}}{\sqrt{1-\xi^2}} \sin\theta$$



We know that, $\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}$ as shown in above fig.

$\sin\theta = \frac{\text{opposite site}}{\text{Hypotenuse}}$ but hypotenuse $= \sqrt{(1-\xi^2)^2 + \xi^2} = 1$

$$\sin\theta = \frac{\sqrt{1-\xi^2}}{1} = e^{-\xi \omega_n T_p}$$

Substitute $T_p = \frac{\pi}{\omega_d}$

$$M_p = e^{-\xi \omega_n \frac{\pi}{\omega_n \sqrt{1-\xi^2}}}$$

$$= e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}}$$

$$\boxed{\% M_p = 100\, e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}}}$$

② The forward path transfer function of a unity feedback control system is given by -

$$G(S) = \frac{S\ (S^2+2S+100)}{S^2\ (S+5)(S^2+3S+10)}$$

Determine the step, ramp and parabolic error coefficients. Also determine the type of the system.

② As

$$K_p = \lim_{S\to 0} G(S)\ H(S) = \lim_{S\to 0} \frac{S\ (S^2+2S+100)}{S^2(S+5)\ (S^2+3S+10)}$$

$$K_p = \infty \qquad \boxed{K_p = \infty}$$

$$K_v = \lim_{S\to 0} S\ G(S)\ H(S) = \lim_{S\to 0} S\ \frac{S\ (S^2+2S+100)}{S^2(S+5)\ (S^2+3S+10)}$$

$$K_v = \infty \qquad \boxed{K_v = \infty}$$

$$K_a = \lim_{S\to 0} S^2\ G(S)\ H(S) = \lim_{S\to 0} S^2\ \frac{(S^2+2S+100)}{S^2\ (S+5)\ (S^2+3S+10)}$$

$$= 10$$

$$\boxed{K_a = 10}$$

In denominator the value of $m = 2$, hence, the given system is type '2' system.

③ The block diagram of an electronic pacemaker is given in fig below. Determine the steady state error for unit ramp I/P when K = 400 also determine the value of k for which the steady state error to a unit ramp will be 0.02.

K_1, K_2 network

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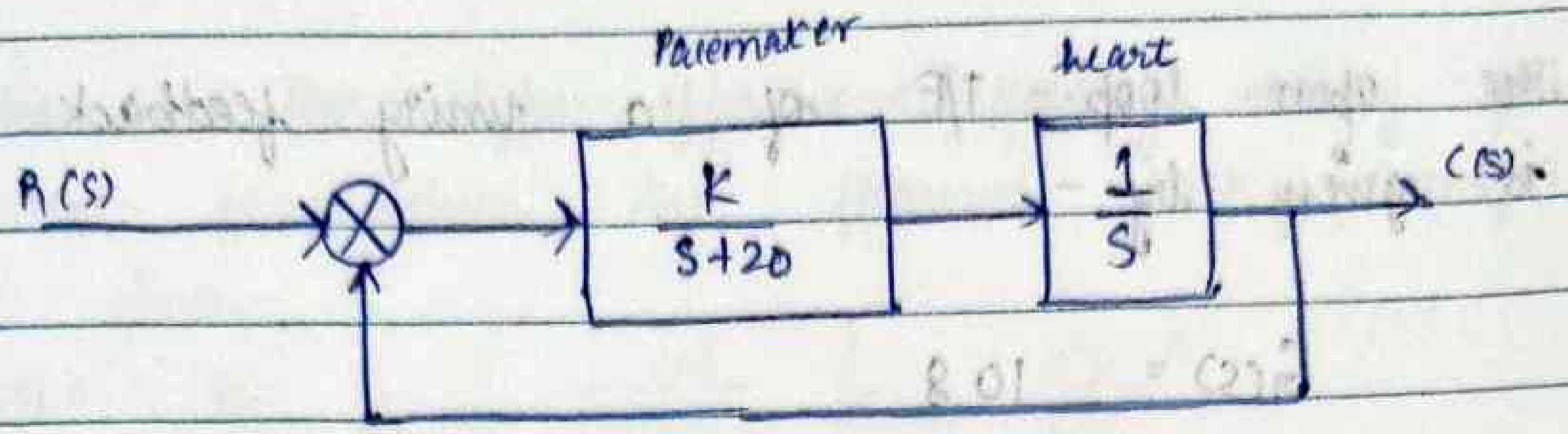
(3)



given that

$K = 400$

$R(S) = \frac{1}{s^2}$

$H(S) = 1$

$G(S)\,H(S) = \frac{K}{s(s+20)}$

Steady state error is given by $e_{ss} = \lim_{s \to 0} s.\, \frac{R(s)}{1+G(s)H(s)}$

$$R_{ss} = \lim_{s \to 0} s\, \frac{1}{s^2}\, \frac{1}{1 + \frac{K}{s(s+20)}}$$

$$\lim_{s \to 0} \frac{s+20}{s(s+20)+400} = 0.05 \quad \text{Ans}$$

Now $e_{ss} = 0.02$ (given)

$$e_{ss} = \lim_{s \to 0} \frac{s\,\frac{1}{s^2}}{1 + \frac{K}{s(s+20)}}$$

$$0.02 = \lim_{s \to 0} \frac{s+20}{s(s+20)+K}$$

$$0.02 = \frac{20}{K}$$

$\boxed{\therefore K = 1000}$ Ans

UNIT-03

CONTROL SYSTEM COMPONENTS

* CONTROLLER COMPONENTS-

- Sensors
- Actuators
- Electric System
- Differencing and amplification

* SERVOMOTORS-

They are of two types-
→ AC Servomotors
→ DC Servomotors.

AC Servomotor-

→ A type of servomotor that uses AC electrical I/P in order to produce mechanical O/P in the form of precise angular velocity known as AC servomotor.

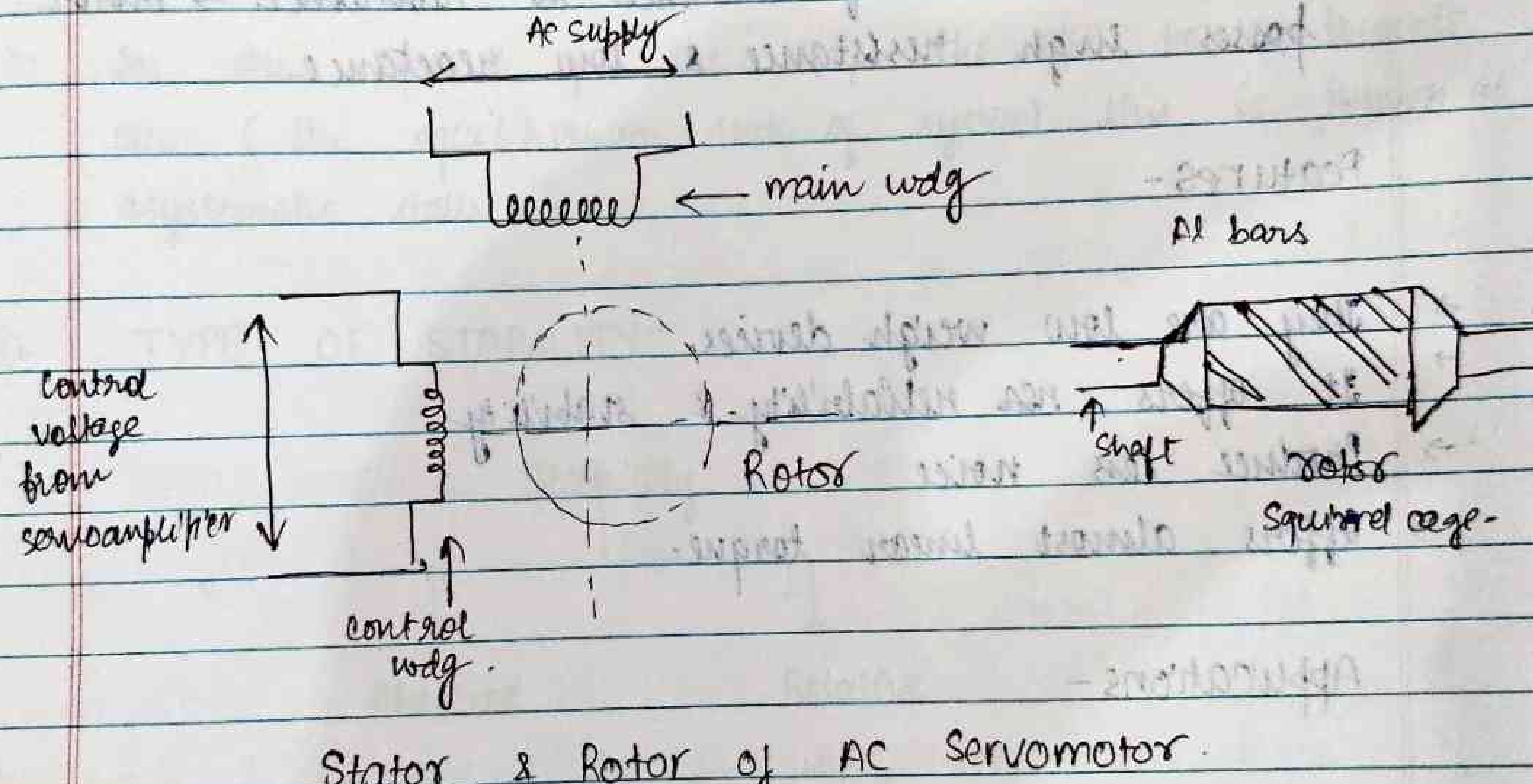
→ They are basically two-phase induction motor.

→ The O/P power achieved from ac servomotor ranges b/w some watt to a few hundred watts. While the operating freq ranges b/w 50 to 400 Hz.

→ It provides closed-loop control to the feedback system

Construction-

→ Mainly composed of two major units – stator and rotor.
→ Out of two windings i.e, stator and rotor, one is referred as- main or fixed wdg and other is
→ called control wdg.
→ There is also one special rotor- drag cup type rotor



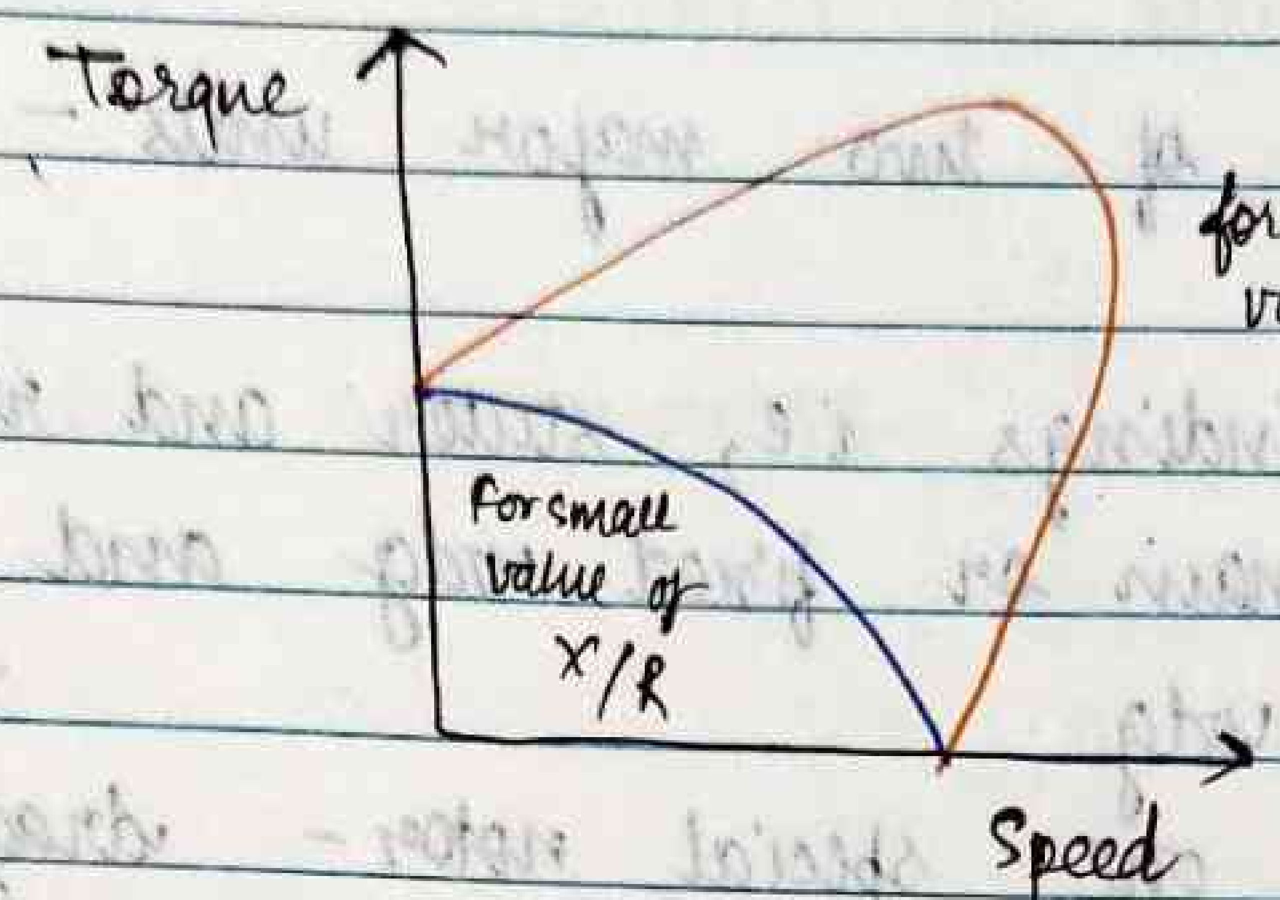
Stator & Rotor of AC Servomotor.

Working Principle-

→ AC two-phase induction motor that uses the principle of servomechanism. Initially a const. ac voltage is provided at main wdg of stator. & the other is connected to control wdg.

→ The shaft of the control T/F has a certain specific angular position

Torque speed characterstics -



→ Low value ratio of reactance to resistance ⇒ motor posses high resistance & low reactance

Features -

→ They are low weigh devices.
→ It offers ~~rea~~ reliability & stability
→ Produce less noice
→ Offers almost linear torque.

Applications -

→ Involve in automation
→ Involve in robotics, CNC machinery
→ Computers, CD/DVD, players.
→ Used in solar tracking system.
→ Used in textiles: to control spinning & weaving machines
→ Used in automatics door.

* STABILITY & ALGEBRIC CRITERIA →

Concept

→ The concept of stability is very imp and in analysing and designing the system.

→ A system is said to stable if -
The system is excited by a boundary I/P, the O/P is bounded [BIBO stability criteria]

→ In the absence of the I/P, the O/P tends towards zero (the equilibirium state of system) this is known as asystematic stable.

TYPE OF STABILITY -

Stability
- Absolute
- Relative

→ Absolute stability means whether the system is stable or not.

→ If the system is stable then we measure the degree of stability. and this is known as relative stability.

* Necessary but not sufficient condition for stability -

Consider a system with characterstic system

$$a_0 s^m + a_1 s^{m-1} + \cdots + a_m = 0$$

→ All the coefficients of the equation should have same sign.

→ There should be no missing term.

Note-

→ If above two conditions weren't satisfied the system will be unstable.

→ But if all the coefficients have same sign & there is no missing term we have no guarantee that the system will be stable.

* For stability we use Routh- Hurwitz Criteria

* EFFECT OF LOCATION OF POLES ON STABILITY-

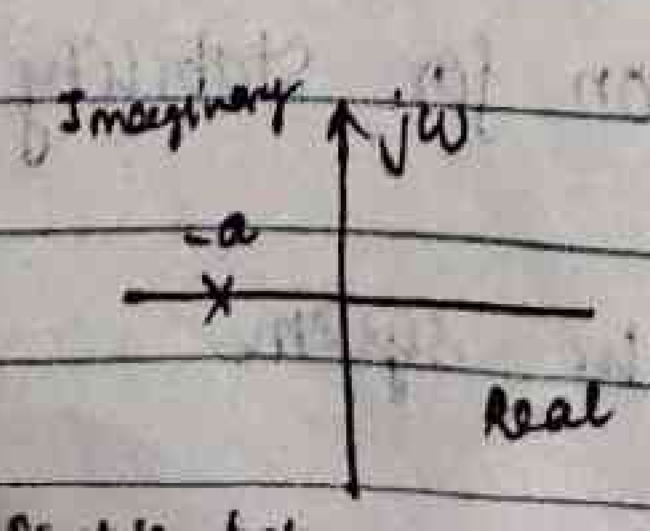
$$T/F = \frac{\text{Zeroe's}}{\text{Poles}}$$

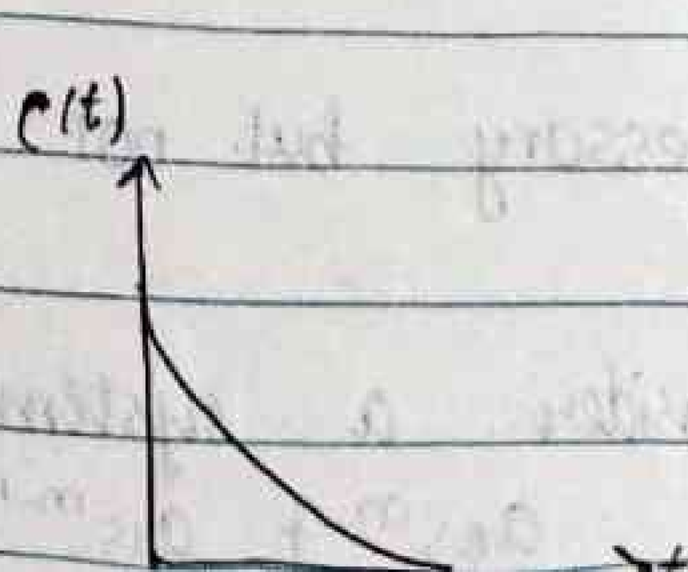
- Zeroes are those zeroes at which system T/F becomes zero
- Poles are those freq at which system T/F become ∞

(a) Poles on negative real axis

consider $G(s) = \frac{K}{S+a}$ $[S = -a]$

$$L^{-1}[G(s)] = g(t) = Ke^{-at}$$





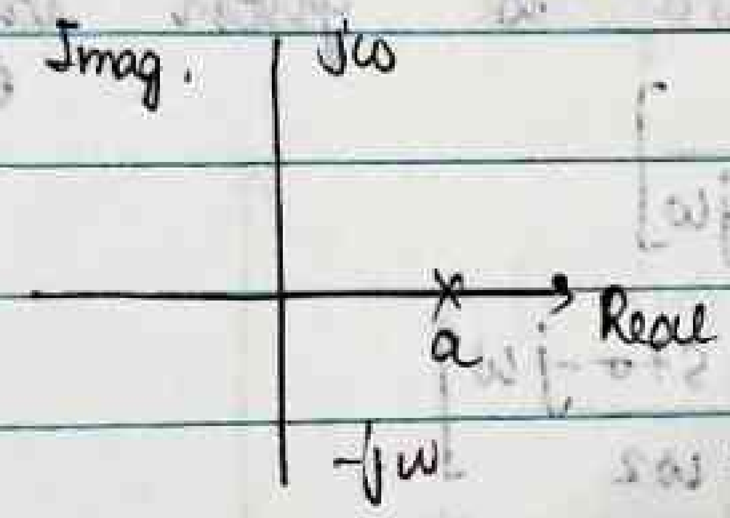
(a) simple pole on -ve real axis

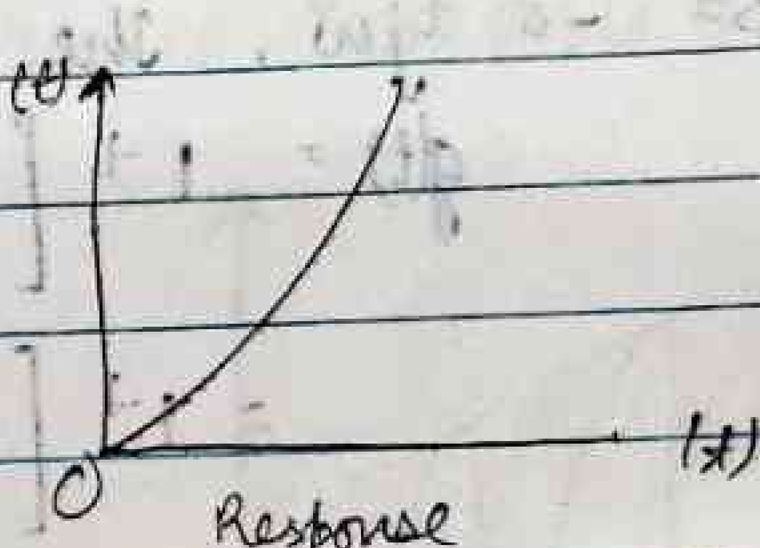
The time 't' increases, exponentially the response approaches zero & system is stable

(b) Poles on +ve real axis-

Consider a system having simple pole on +ve real axis at $s=a$, the corresponding impulse response is given by-

$$G(s) = \frac{k}{s-a} \Rightarrow \mathcal{L}^{-1}[G(s)] = ke^{at}$$





The response increases exponentially with time, then the system is unstable.

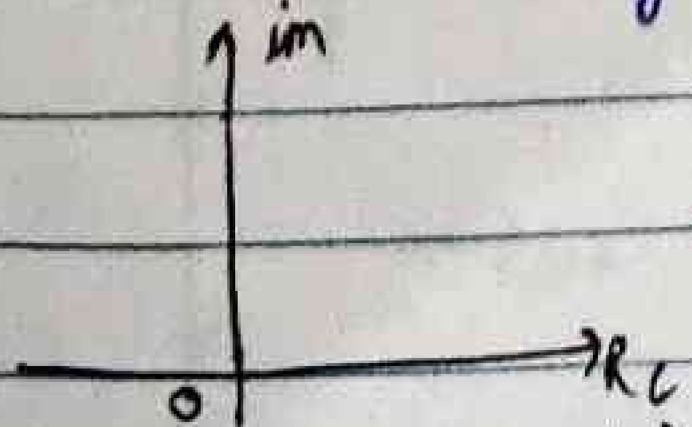
(c) Poles at origin-

Consider a pole at origin

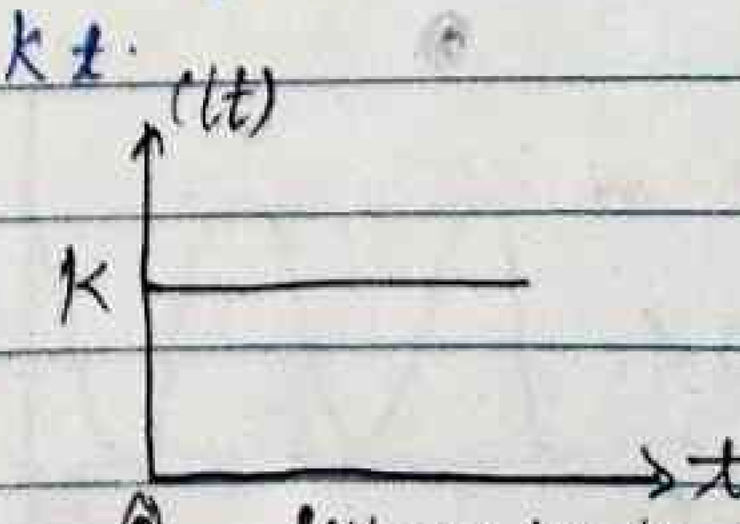
$$G(s) = \frac{k}{s} \Rightarrow g(t) = k$$

This is constant value hence the system is marginally stable.

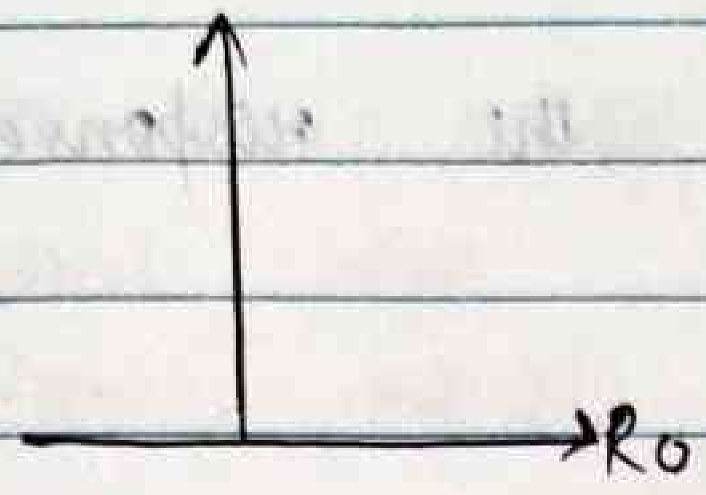
If there are two poles at origin, the time response should be $g(t) = \mathcal{L}^{-1}\frac{k}{s^2} = kt$.

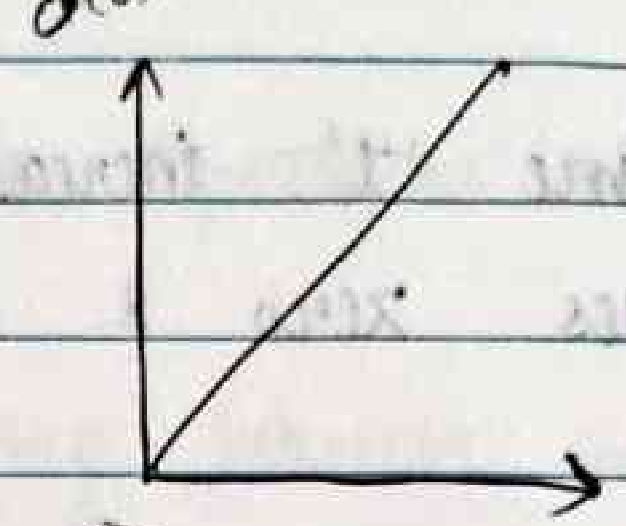


(a) single poles at origin



(b) Response due to single pole





(b) impulse response

(a) Double pole at origin

$\boxed{\text{unstable}}$

(d) Complex Poles in the left half of s-plane -

Let the T/F has a complex conjugate poles at $s = -\sigma \pm j\omega$. The time response is given by

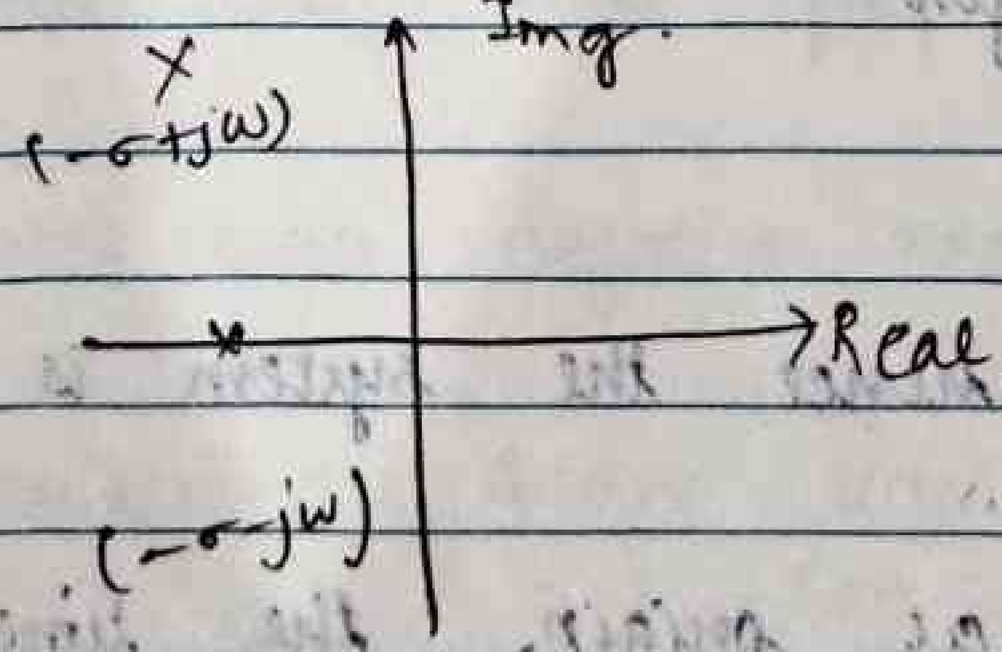
$$g(t) = L^{-1}\left[\frac{k}{s+\sigma-j\omega} + \frac{k}{s+\sigma+j\omega}\right]$$

$$= L^{-1}\left[\frac{k\,[s+\sigma+j\omega + s+\sigma-j\omega]}{(s+\sigma)^2+\omega^2}\right]$$

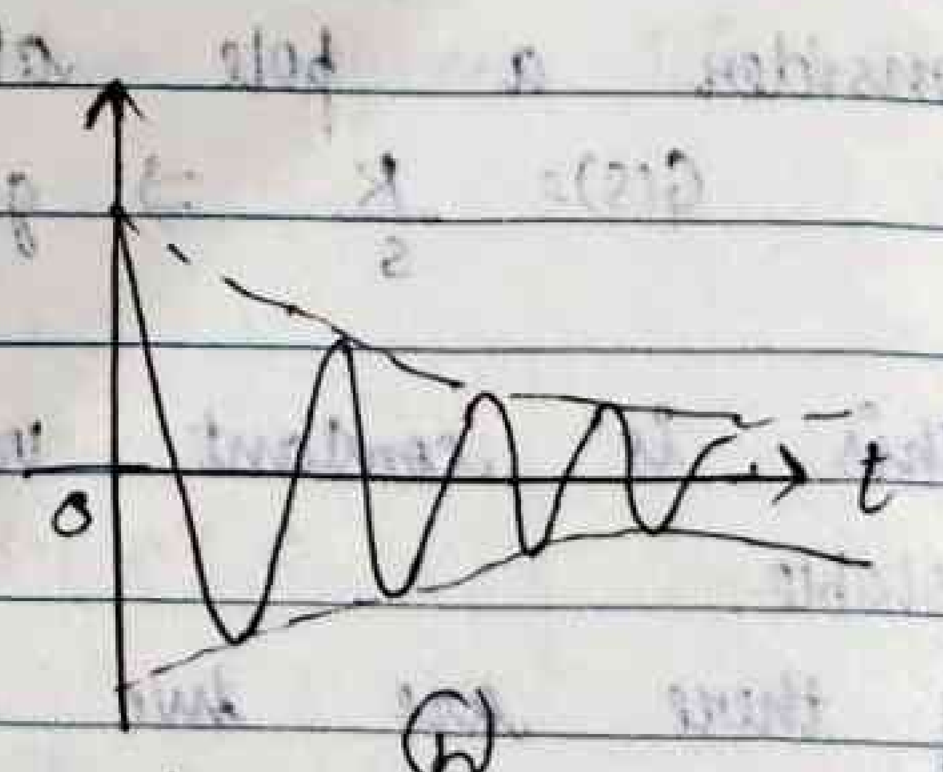
$$= L^{-1}\left[\frac{2k(s+\sigma)}{(s+\sigma)^2+\omega^2}\right]$$

$$g(t) = 2te^{-\sigma t}\cos\omega t$$

When t increases g(t) approaches zero & the system is stable.



(a)



(b)

(e) Complex poles in the right half of s-plane :-

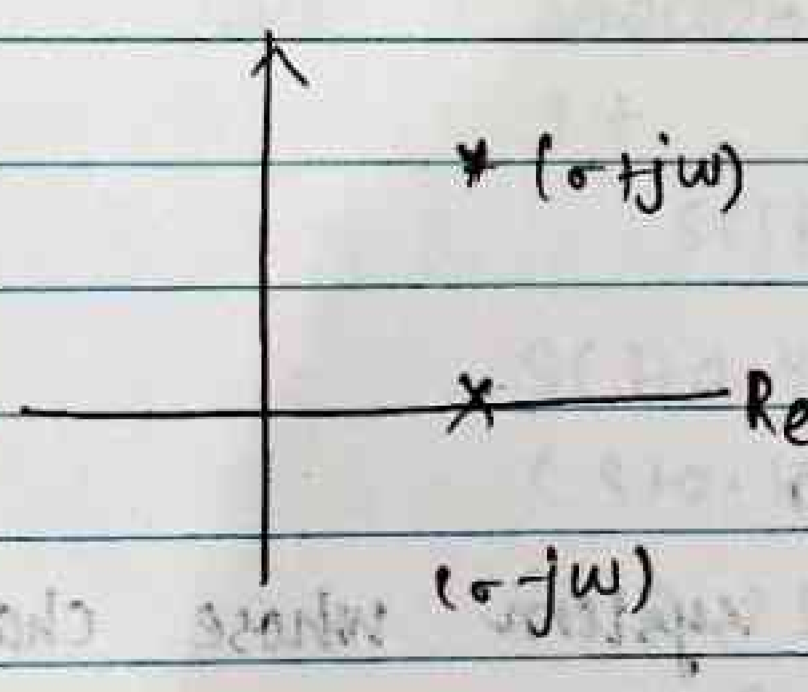
Consider at $s = \sigma \pm j\omega$

$$g(t) = L^{-1}\left[\frac{K}{s-\sigma+j\omega} + \frac{K}{s-\sigma-j\omega}\right]$$

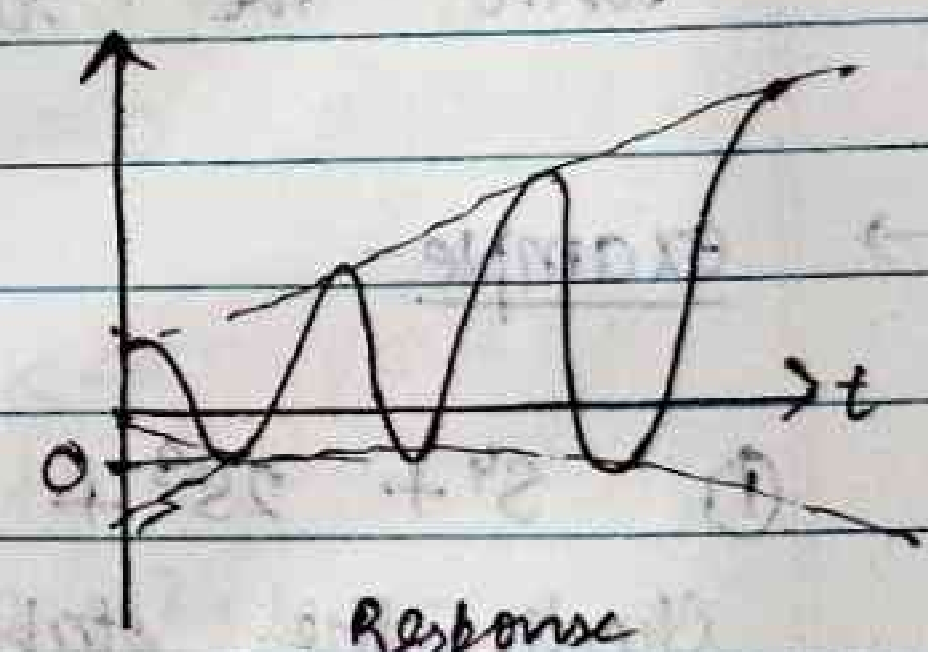
$$= L^{-1}\left[\frac{2K(s-\sigma)}{(s-\sigma)^2+\omega^2}\right]$$

$$g(t) = 2K e^{\sigma t}\cos\omega t$$

Hence, the response increase exponentially with time and therefore the system is unstable.



complex poles.

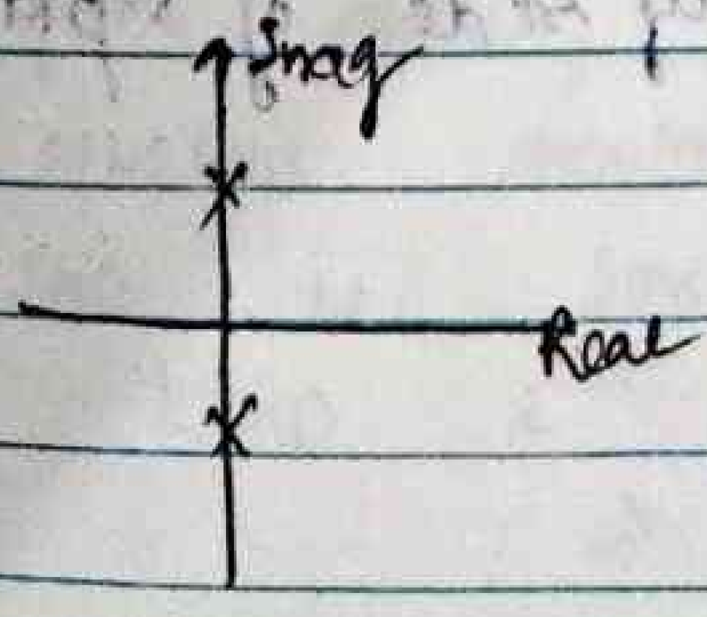


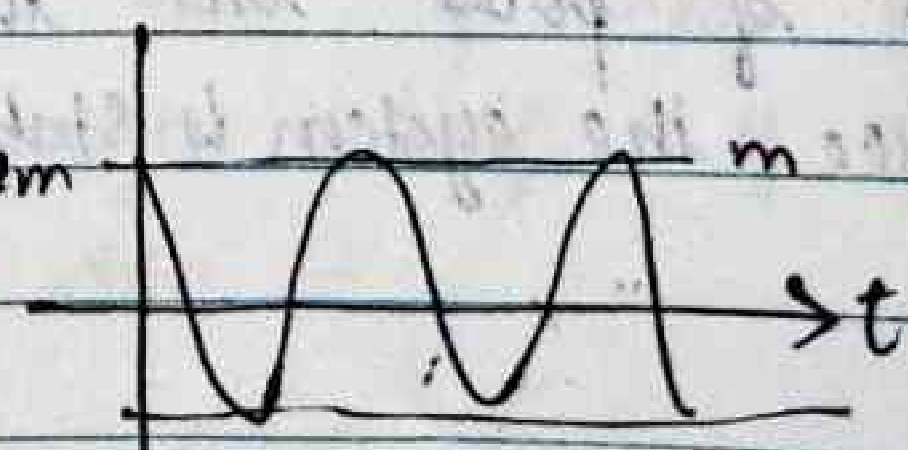
Response

(f) Poles on imaginary axis $[j\omega$ Axis$]$ $s = \pm j\omega$

$$g(t) = L^{-1}\left[\frac{K}{s+j\omega} + \frac{K}{s-j\omega}\right]$$

$$= L^{-1}\left[\frac{2K}{s^2+\omega^2}\right] = 2A\cos\omega t$$





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* ROUTH - HURWITZ CRITERION :-

- Routh - Hurwitz criterion states that the system is stable if and only if all the elements in the column have the same algebric expression.

- If all elements are not of not of the same sign then the no. of sign changes of the elements in first column equals the no. of roots of the characteristics eqn in the right half of the s-plane. (For equal to the no. of roots with +ve real ~~poi~~ part)

→ example-

① $s^4 + 2s^3 + 6s^2 + 4s + 1 = 0$

Check the stability of the system whose characteristic eqn is given by-

s^4	1	6	1
s^3	2	4	0
s^2	4		
s^1	3.5		
s^0	1		

Ans//

- From the above routh table no. of sign changes in first column = 0.
- No. of poles on the right half side of s-plane = 0

Hence the system is stable

* Application of ROUTH'S stability criterion to control system analysis -

(1) The open loop T/F of unity feedback system is $\frac{K}{S(1+0.4S)(1+0.25S)}$. Find the restrictions of K, so that the closed loop system is absolutely stable.

(1) Soln,

given,

$$G(S) = \frac{K}{S(1+0.4S)(1+0.25S)}, \quad H(S) = 1$$

The characteristic eqn $1 + G(S)\,H(S) = 0$

$$1 + \frac{K}{S(1+0.4S)(1+0.25S)} \cdot 1 = 0$$

$$S(1+0.4S)(1+0.25S) + K = 0$$

$$(S + 0.4S^2)(1 + 0.25S) + K = 0$$

$$S + 0.25S^2 + 0.4S^2 + 0.1S^3 + K = 0$$

$$0.1S^3 + 0.65S^2 + S + K = 0$$

$$\boxed{S^3 + 6.5S^2 + 10S + 10K = 0}$$

So,

S^3	1	10
S^2	6.5	10K
S^1	$\frac{65-10K}{6.5}$	0
S^0	10K	

For absolute stability should be no sign change the first column, that is no root of the characteristics equation should lie in right half of S-plane. This is possible only when -

$K > 0$ & $65 - 10K > 0$

$\Rightarrow 65 > 10K \Rightarrow 6.5 > K$

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* ROOT LOCUS TECHNIQUE -

Root locus analysis is a graphical method for examining how the roots of a system change with variation of a certain system parameter commonly a gain within a feedback system.

This a technique used as a stability criterion.

The root locus plots the ~~people~~ poles of the closed loop T/F in complex s-plane as a function of a gain parameter.

example -

Consider the system with $G(s) \cdot H(s) = \dfrac{K}{S(S+2)(S+4)}$

step 1 - Plot the poles.

$S=0,\ S=-2,\ S=-4$

No. of poles $= P = 3$

No. of zeroes $= Z = 0$

(if poles & imagn first angle of dep

step 2 -

No. of root locus $\Rightarrow N = 3$ $\quad (N = P - Z)$

Step 4 -

angle of asymptotes

$\phi = \left(\dfrac{2k+1}{P-Z}\right) \times 180$

draw angle of centroid of asy

$k=0, \quad \phi_0 = 60°$

$k=1, \quad \phi_1 = 180°$

$k=2, \quad \phi_2 = 300°$

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Step 3 - Centroid of asymtotes

$$\sigma_A = \frac{\text{sum of poles} - \text{sum of zeroes}}{P-Z}$$

$$\sigma_p = \frac{(0-2-4)-0}{3-0} = -2$$

Steps - Calculating break away point

$$1+G(s)H(s) = 0$$

$$1+\frac{k}{s(s+2)(s+4)} = 0$$

$$s(s+2)(s+4)+k = 0$$

$$(s^2+2s)(s+4)+k = 0$$

$$s^3+4s^2+2s^2+8s+k = 0$$

$$s^3+6s^2+8s+k = 0$$

$$k = -s^3-6s^2-8s$$

$$\frac{dk}{ds} = -3s^2-12s-8 = 0$$

$$\therefore 3s^2+12s+8 = 0$$

$$s = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$s = \frac{-12 \pm \sqrt{144-96}}{6}$$

$$= \frac{-12 \pm \sqrt{48}}{6}$$

$$s_1 = \frac{-12+\sqrt{48}}{6}$$

$$= -0.845$$

$s_2 = \frac{-12-\sqrt{48}}{6}$

$s_3 = -3.155$

Step 6 - Calculation of intersection point -

The characterstic eqn is given by -

$s^3 + 6s^2 + 8s + k = 0$

Now using Routh - Hurwitz criteria

s^3	1	8
s^2	6	k
s^1	$\frac{48-k}{6}$	0
s^0	k	

$b_1 = \frac{48-k}{6}$

$c_1 = \left(\frac{48-k}{6}\right)k - 0 \Big/ \frac{48-6}{6} = k$

$= \frac{48-k}{6} = 0$

$\Rightarrow k = 48$

For $k = 48$

$A(s) = 6s^2 + k$

$6s^2 + k = 0$

$6s^2 + 48 = 0$

$s^2 = \frac{-48}{6}$

$= -8$

$s = \pm j\,2.828$

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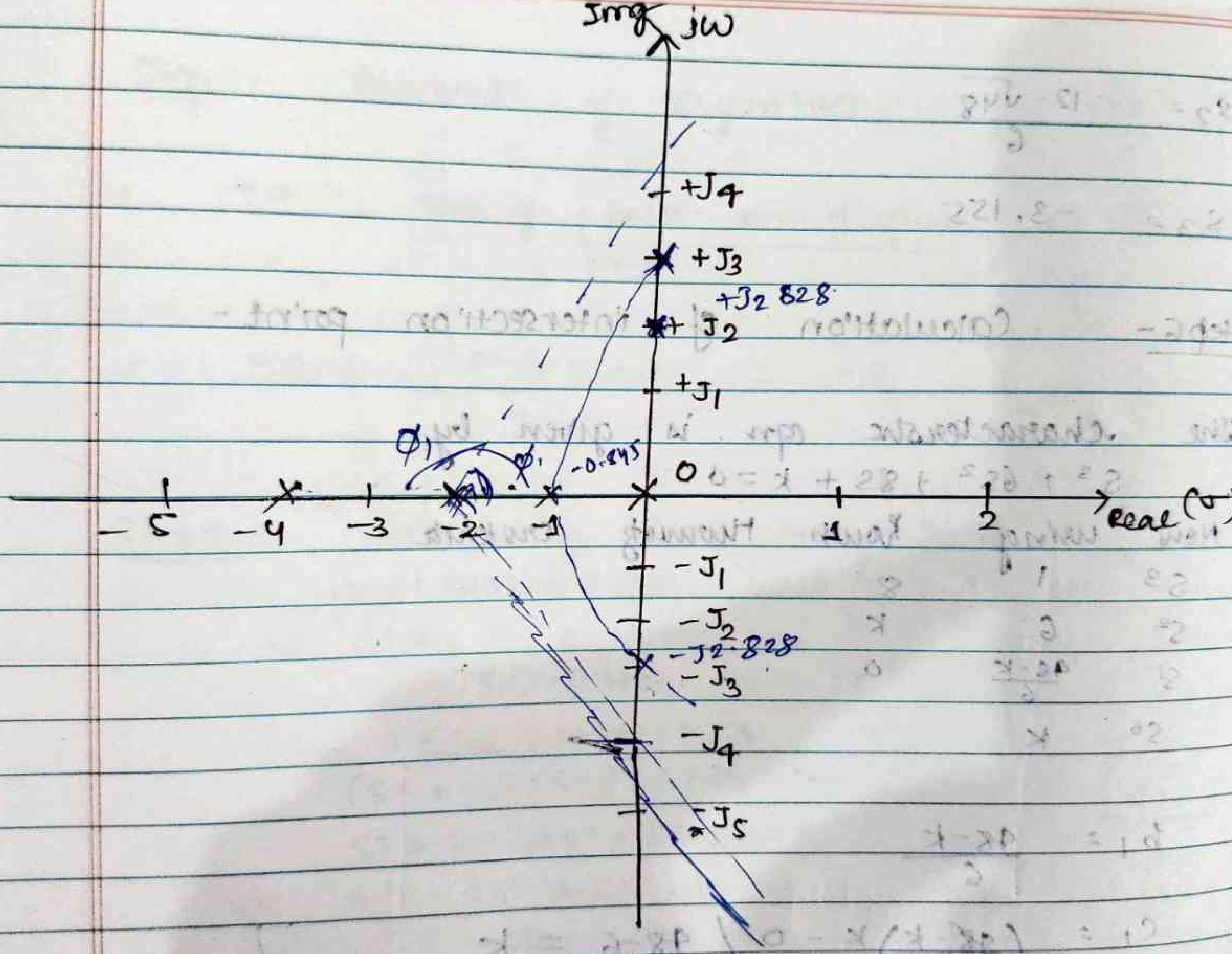
Img jw
+J4
+J3
+J2 828
+J2
+J1
0
-0.845
ϕ1
ϕ1
-5
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-1
1
2
Real (σ)
-J1
-J2
-J2.828
-J3
-J4
-J5