Course: Special Electrical Machines (KEE061)

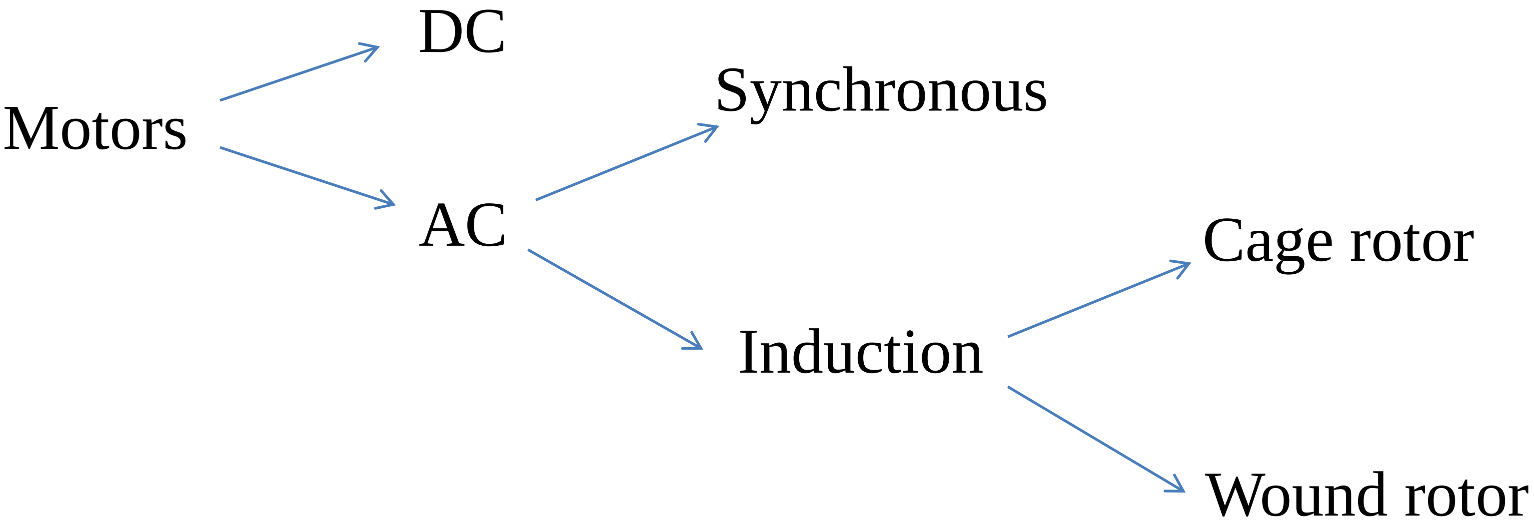
Unit 1
Induction machines

Electric Motors (A major power consumer in industries)

- Electromechanical device that converts electrical energy to mechanical energy.

- Motors in industry: 70% of electrical load.

General Classification of Motor



Motors

DC

AC

Synchronous

Induction

Cage rotor

Wound rotor

Why Induction Motor ?

(*Major Advantages*)

- Rugged structure
- Low maintance
- Better reliability
- Ability to operate in dirty and explosive environment
- Higher efficiency
- Available in wide power range

Uses of Induction Motor

- Industrial Applications:
  - Steel and cement mills
  - Paper and textile mills etc.

- Domestic uses:
  - Pumps
  - Fans
  - Other home appliances

- Fixed and variable speed applications

Constructional Details

Major parts:

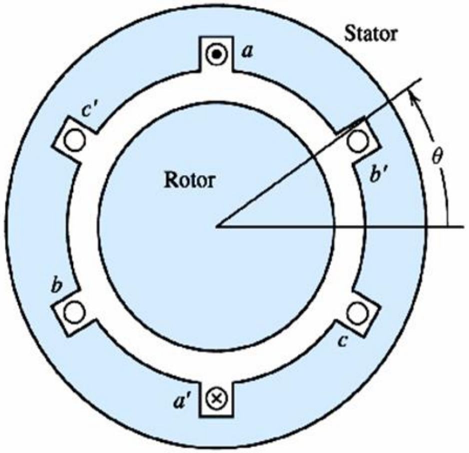
- Stator (stationary part)

  Three phase armature winding is embedded.

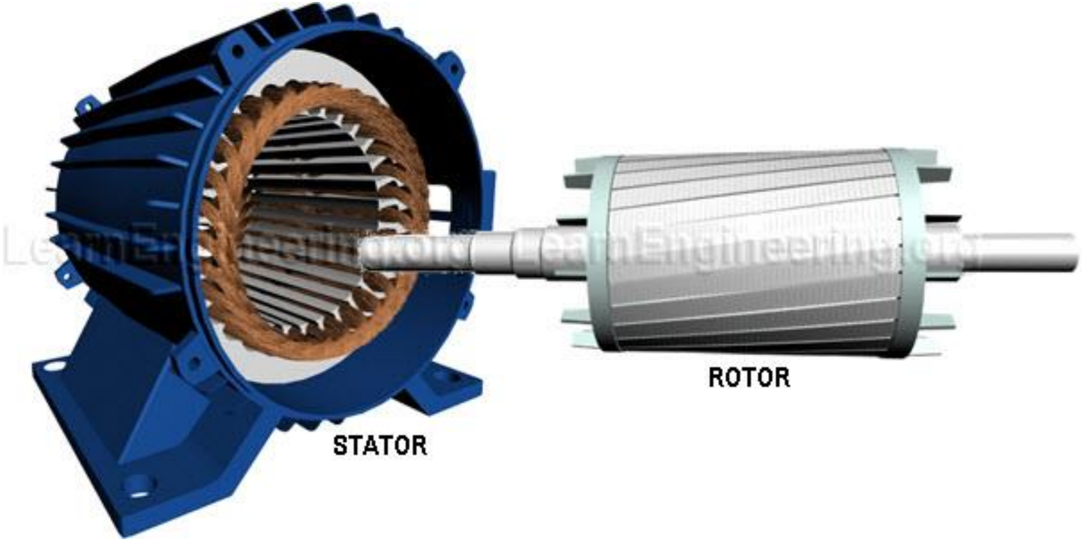
- Rotor (rotating part)

  Types: - Squirrel cage

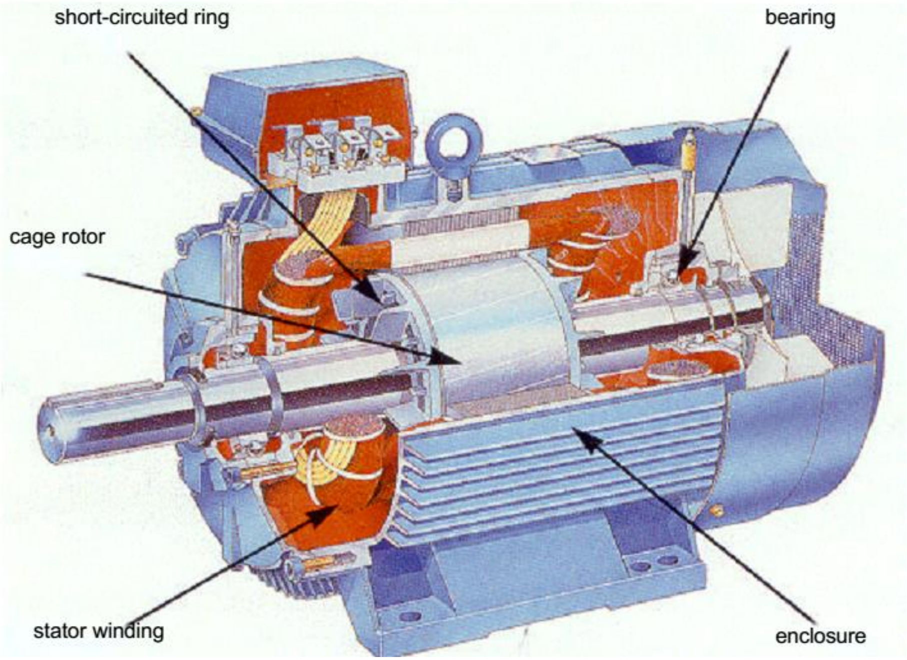
  - Wound rotor



Cross sectional view of Induction motor



General view of stator and rotor



Complete view of Induction motor

Operating Principle of Induction Motor

- Rotating Magnetic Field

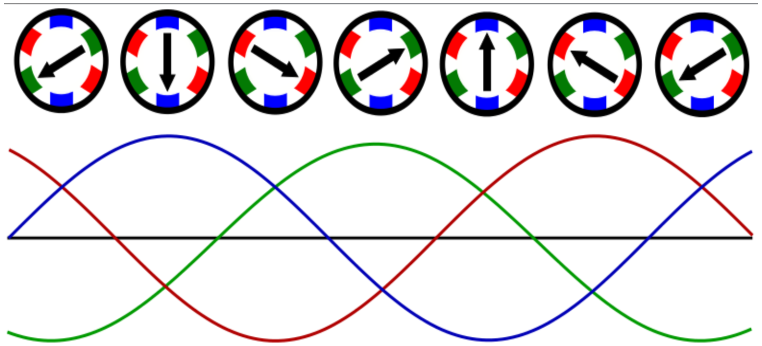
Input supply to stator

↓

Magnetic flux
in each phase

↓ Resultant Flux

Rotating flux of
constant magnitude ← Can be proved mathematically



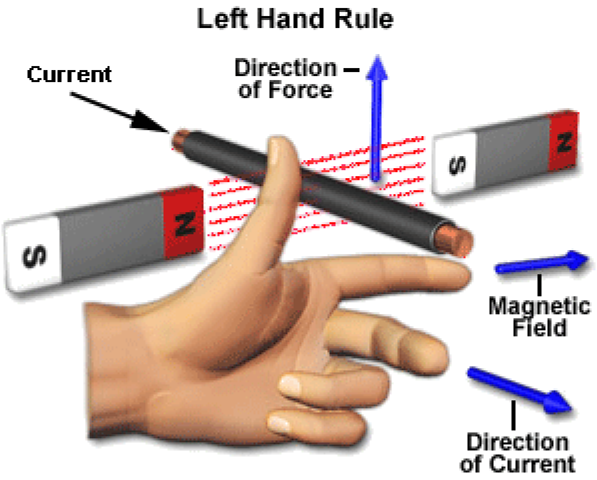
Representation of constant rotating flux

- Speed of rotating flux $\xrightarrow{\text{known as}}$ *Synchronous* Speed

How rotating torque is produced ?

- Induced e.m.f. in rotor conductor (theory of electromagnetic induction).
- Flow of rotor current due to its closed circuit.
- Presence of current carrying rotor conductor in magnetic field produced force.

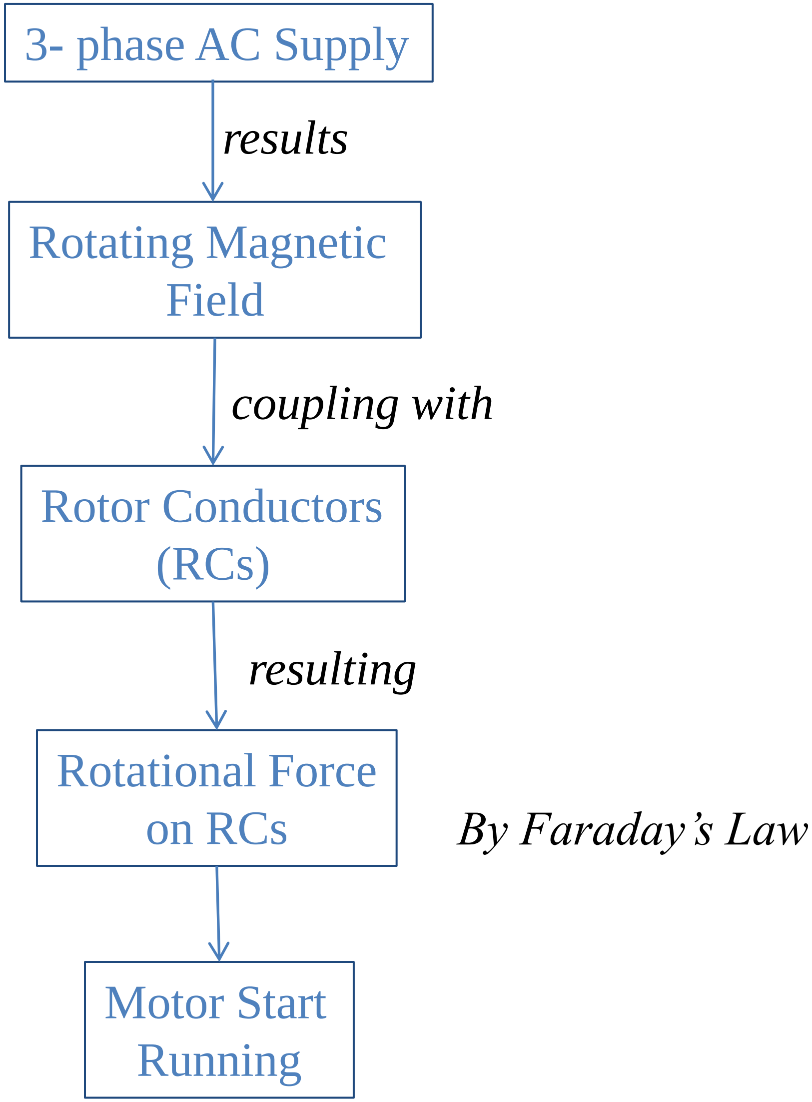
- Direction of force is governed by Faraday Left Hand Rule.



- Hence, induction motor is self starting.

- Speed of rotation is slightly less than synchronous speed.

Various Steps in motor operation



3- phase AC Supply

results

Rotating Magnetic Field

coupling with

Rotor Conductors (RCs)

resulting

Rotational Force on RCs

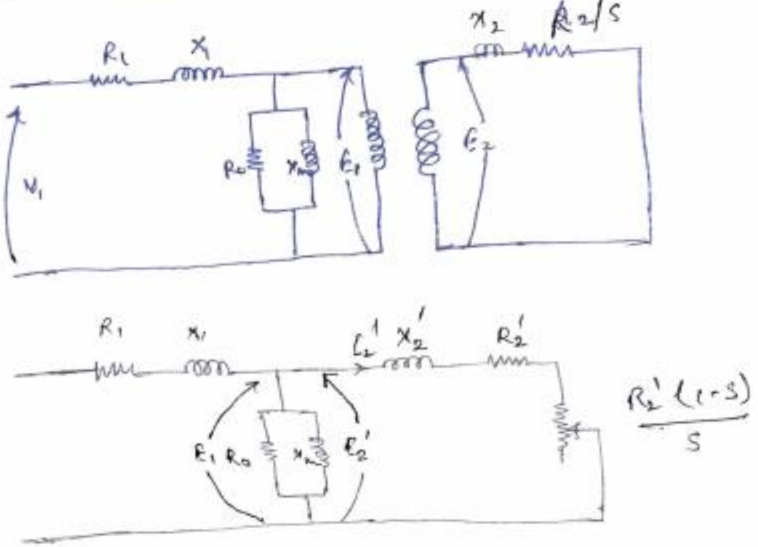
By Faraday's Law

Motor Start Running

Review of Induction motor

Review of Induction Motor

$$\frac{R_2}{s} = R_2 + \frac{(1-s)}{s} R_2$$



$$E_1 = E_2'$$

Now, $P_2 = I_2'^2 \frac{R_2}{s}$

$$P_{md} = P_2 - I_2'^2 R_2$$

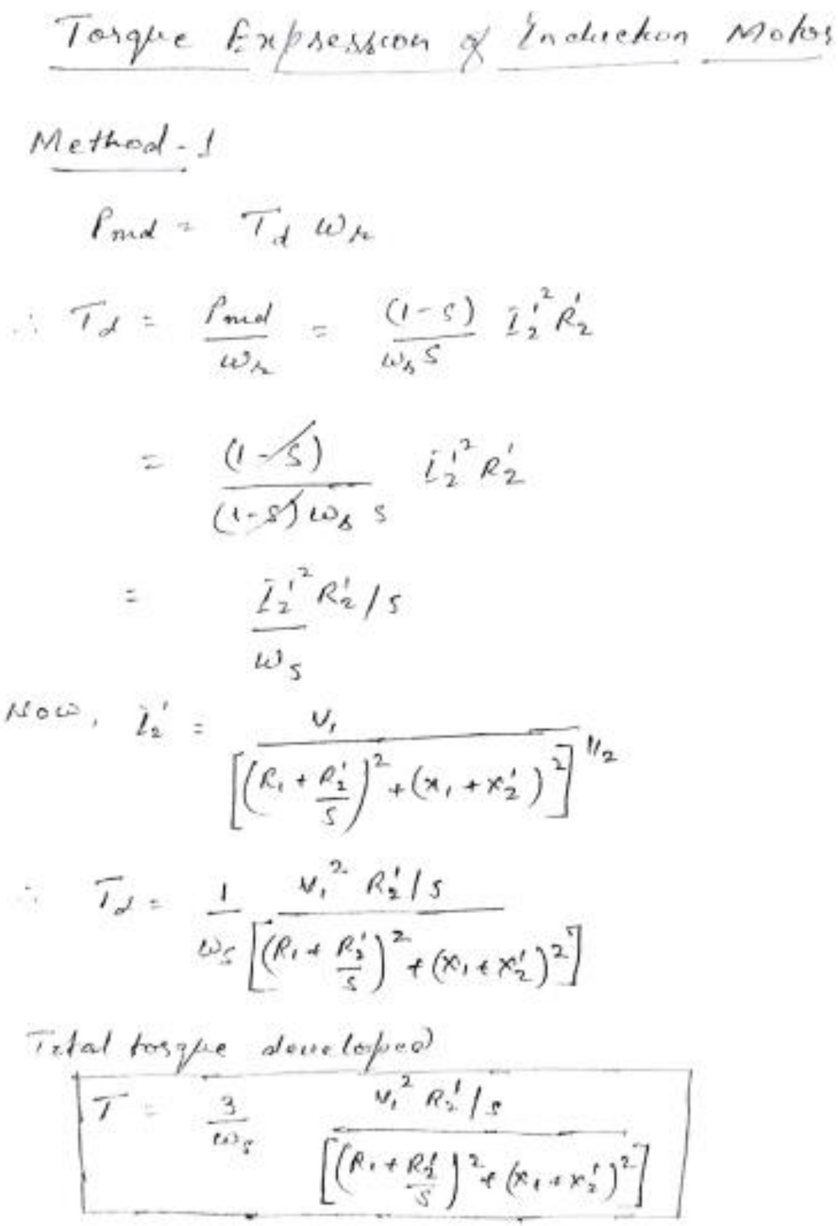
$$= I_2'^2 R_2 \left(\frac{1}{s} - 1\right)$$

$$P_{md} = \frac{(1-s)}{s} I_2'^2 R_2$$

$$= (1-s) P_2$$

$$\therefore \boxed{P_2 : P_{md} : P_{rc} = 1 : (1-s) : s}$$

Torque of induction motor



Torque Expression of Induction Motors

Method-1

$$P_{md} = T_d \omega_r$$

$$\therefore T_d = \frac{P_{md}}{\omega_r} = \frac{(1-s)}{\omega_s s} I_2'^2 R_2'$$

$$= \frac{\cancel{(1-s)}}{\cancel{(1-s)} \omega_s s} I_2'^2 R_2'$$

$$= \frac{I_2'^2 R_2'/s}{\omega_s}$$

Now, $$I_2' = \frac{V_1}{\left[\left(R_1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2')^2\right]^{1/2}}$$

$$\therefore T_d = \frac{1}{\omega_s} \frac{V_1^2 R_2'/s}{\left[\left(R_1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2')^2\right]}$$

Total torque developed

$$\boxed{T = \frac{3}{\omega_s} \frac{V_1^2 R_2'/s}{\left[\left(R_1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2')^2\right]}}$$

Method 2

Electrical Power generated in rotor circuit

$$= 3 E_{2s} I_{2s} \cos\phi_{2s}$$

$$= 3 \, s E_{20} \frac{E_{2s}}{Z_{2s}} \cdot \frac{R_2}{Z_{2s}}$$

$$= \frac{3 s^2 E_{20}^2 R_2}{(R_2^2 + s^2 X_{20}^2)} = s \times \omega_s T_d$$

$$\therefore T_d = \frac{3 s E_{20}^2 R_2}{\omega_s (R_2^2 + s^2 X_{20}^2)}$$

$$\text{or} \quad \boxed{T_d = \frac{3}{2\pi N_s} \frac{s E_{20}^2 R_2}{(R_2^2 + s^2 X_{20}^2)}}$$

* Maximum torque T_{max} is at S_m

where, $S_m = \frac{R_2}{X_{20}}$

$$\therefore T_{dmax} = \frac{3}{\omega_s} \frac{R_2}{X_{20}} \frac{E_{20}^2 R_2}{\left[R_2^2 + \frac{R_2^2}{\cancel{X_{20}^2}} \cdot \cancel{X_{20}^2}\right]}$$

$$= \frac{3}{\omega_s} \frac{\cancel{R_2}}{X_{20}} \frac{E_{20}^2 \cancel{R_2}}{2 \cancel{R_2^2}}$$

$$= \frac{3}{2\omega_s} \frac{E_{20}^2}{2 X_{20}}$$

$$= \frac{K}{2 X_{20}} \quad ; \quad K = \frac{3}{2\omega_s} E_{20}^2$$

Methods of Speed Control of Induction motors

Stator Side

1. Stator voltage Control
2. Stator Frequency Control (V/f control)
3. Pole changing

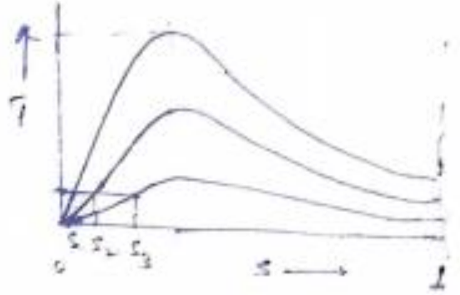
Rotor Side

1. Rotor circuit resistance variation
2. EMF injection in rotor circuit
3. Cascade connection method

Speed Control by Stator voltage Control

1 speed control by varying voltage

* $T \propto V_1^2$



1 * V ↓ s ↑ N ↓

2 * Range - From zero to rated speed.

3 * T_{max} ??

$T_{max} \propto E_2 \propto V_1$

∴ V ↓ T_{max} ↓

4 Rotor cu. loss = $s P_2$

V ↓ N ↓ s ↑ ⇒ (Rot cu loss) ↑

Speed Control by Stator Frequency Control (i.e. V/f control)

Frequency control of 3-phase Induction Motor

1 - $N_s = \frac{120f}{P}$

$N_r = N_s(1-s)$

$E = 4.44 K_w \phi f T \simeq V$

$E \propto \phi f$

$\therefore \phi \propto (V/f) \Rightarrow$ constant (V/f) control.

$f \downarrow \; \phi \uparrow \Rightarrow$ saturation $I_m \uparrow \Rightarrow$ iron losses

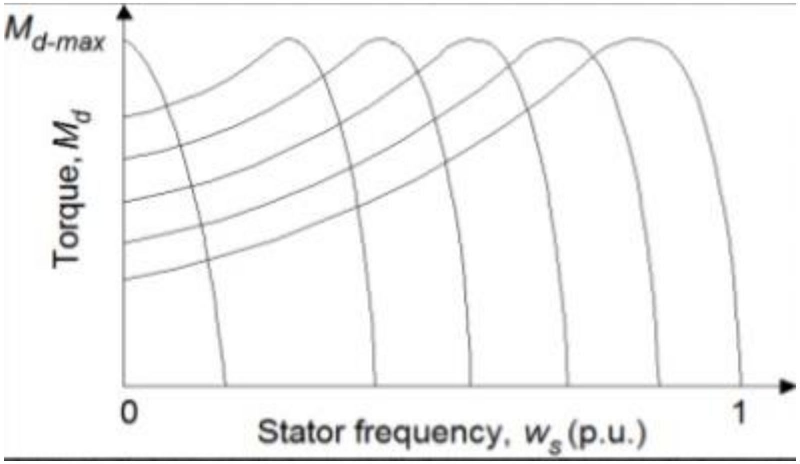
2 - $s_m = \frac{R_2}{X_2} \quad f \uparrow \; s_m \downarrow$

3 $T_{max} = \frac{K}{2X_2}$

$K = \frac{3 E_{20}^2}{2 \omega_s}$

$T_{max} \propto V^2 \cdot \frac{1}{2\pi f} \cdot \frac{1}{2 \omega_s L_{l2}} \propto \left(\frac{V}{f}\right)^2 = \text{constant}$

f3<f2<f1

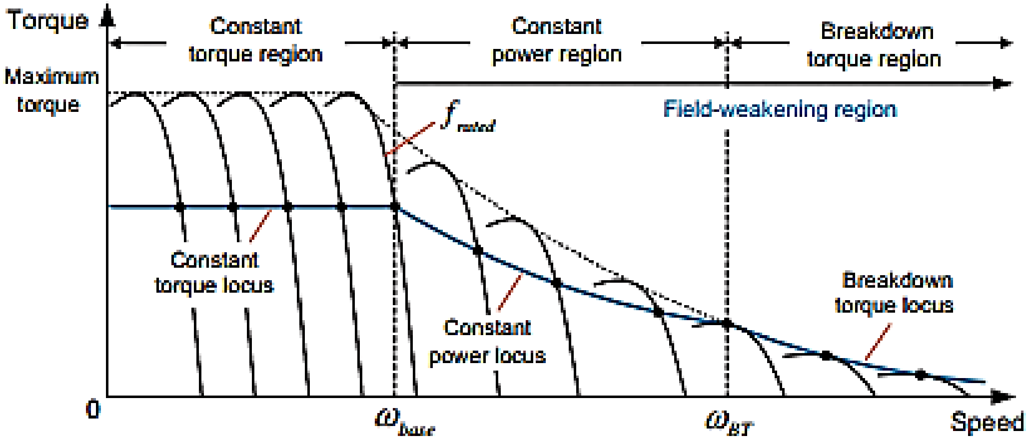


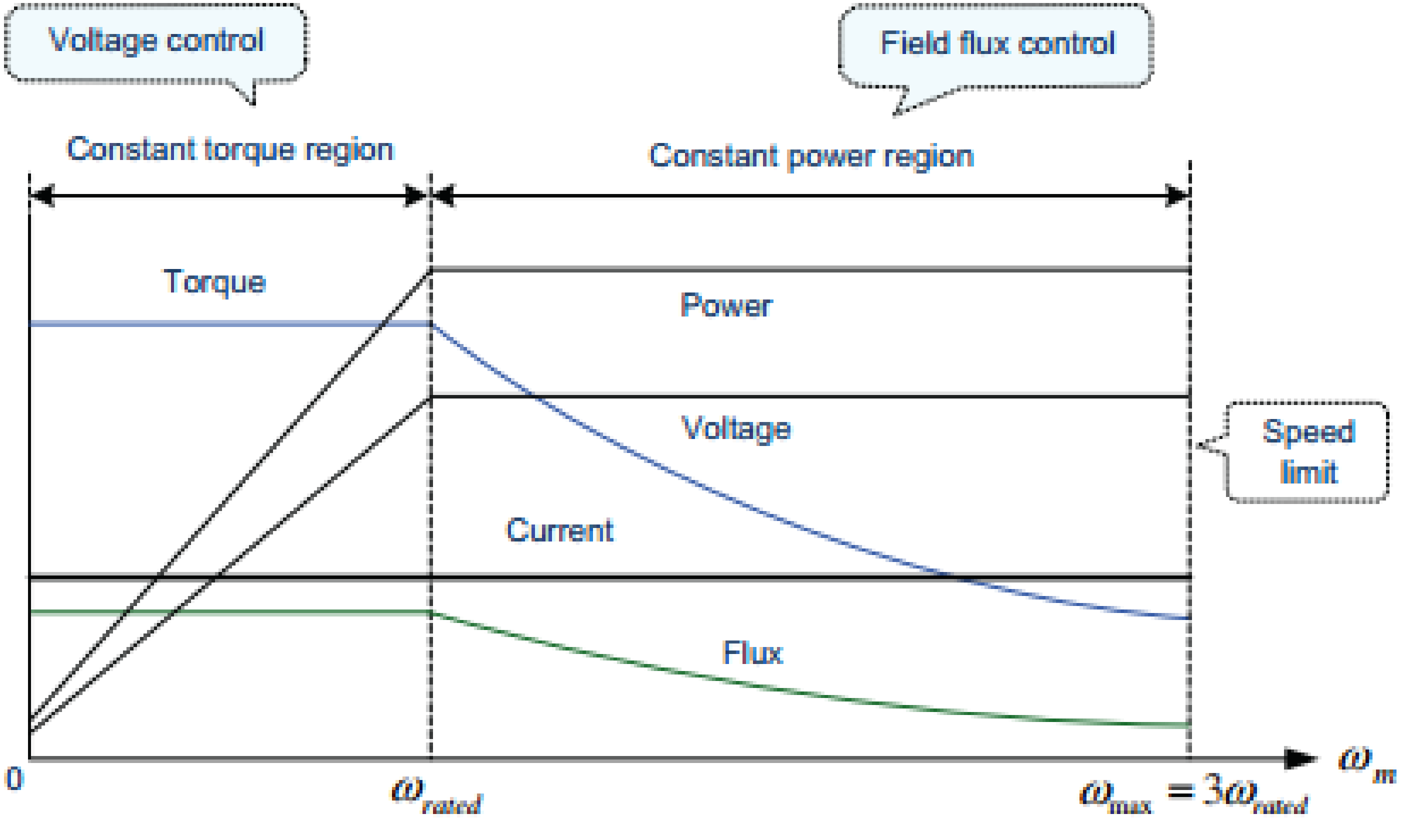
Operating region of Induction Motor

Constant torque region

Constant power region

Breakdown torque region



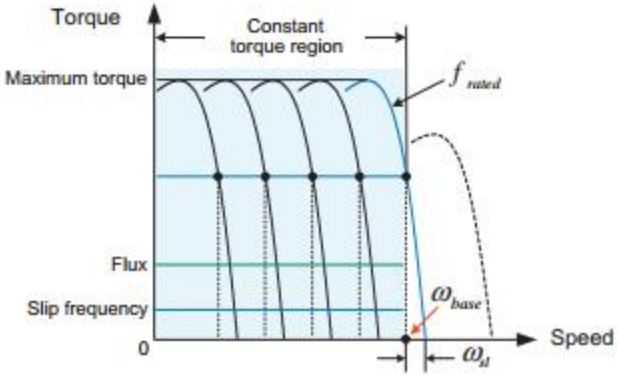


$$P = \mathrm{T}\,\omega$$

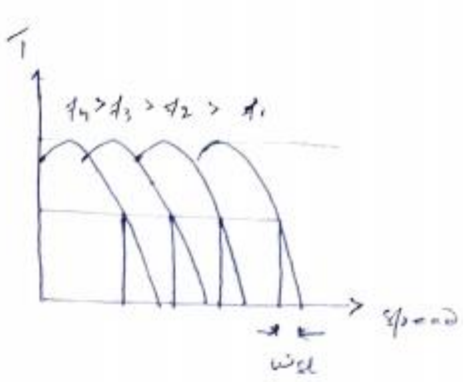
A) Constant torque region

- Motor speed is increased by increasing the stator frequency maintaining constant flux (according to the linear V/f relationship)

- In this control method, since the output torque is proportional to the slip frequency, the output torque can be produced equally at any speed for the same slip frequency. How?



constant torque region



$$T = \frac{3}{\omega_s} \frac{V_1^2 \, R_2/s}{\left(R_1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2')^2}$$

Neglecting stator drop

$$T = \frac{3}{\omega_s} \frac{V_1^2}{\left(\frac{R_2'}{s}\right)^2 + (X_2')^2} \cdot \frac{R_2}{s}$$

$s\omega_s = \omega_{sl}$

$$= \frac{3}{\omega_s} \frac{V_1^2 \, s R_2}{R_2'^2 + (s X_2')^2}$$

$$= 3\left(\frac{V_1}{\omega_s}\right)^2 \frac{R_2 \, (s\omega_s) \rightarrow \omega_{sl}}{R_2'^2 + (s X_2')^2 \rightarrow \omega_{sl} L_2}$$

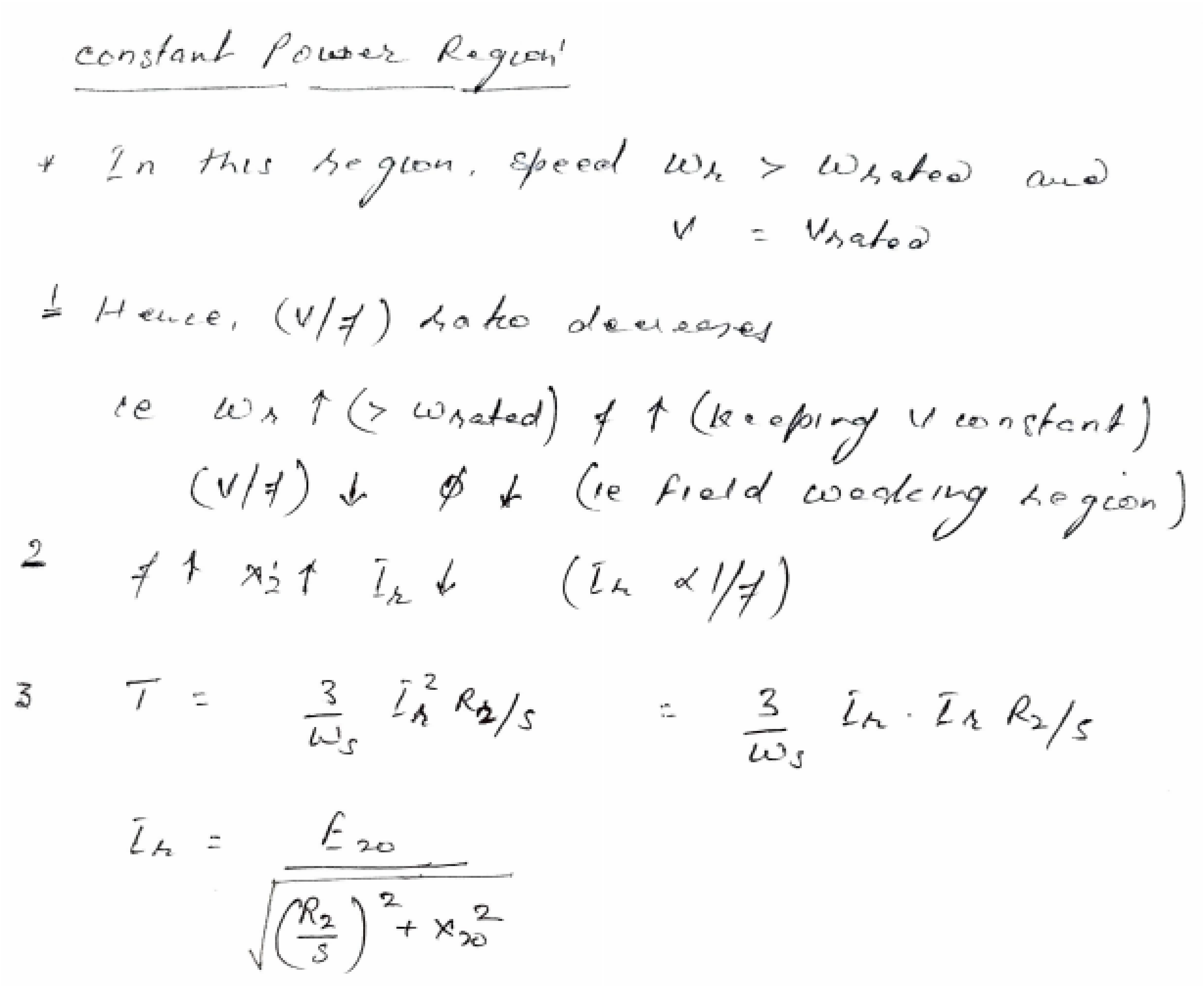
So $T \propto \omega_{sl}$

So. Same torque can be obtained for small value of slip frequency at different speed

$\Downarrow$

constant torque region

B) Constant power region



Constant Power Region

* In this region, speed $\omega_r > \omega_{rated}$ and
$V = V_{rated}$

1 Hence, (V/f) ratio decreases

ie $\omega_r \uparrow (> \omega_{rated})$ $f \uparrow$ (keeping V constant)

$(V/f) \downarrow$ $\phi \downarrow$ (ie field weakening region)

2 $f \uparrow$ $x_2' \uparrow$ $I_r \downarrow$ $(I_r \propto 1/f)$

3 $T = \frac{3}{\omega_s} I_r^2 R_2/s = \frac{3}{\omega_s} I_r \cdot I_r R_2/s$

$$I_r = \frac{E_{20}}{\sqrt{\left(\frac{R_2}{s}\right)^2 + X_{20}^2}}$$

$$T = \frac{3}{\omega_s} \frac{E_{20}}{\sqrt{\left(\frac{R_2}{s}\right)^2 + x_{20}^2}} \cdot I_r \frac{R_2}{s}$$

$$= 3\left(\frac{E_{20}}{\omega_s}\right) \frac{R_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + x_{20}^2}} \cdot \frac{I_r}{s}$$

$$= 3\overset{\phi}{\left(\frac{E_{20}}{\omega_s}\right)} I_r \frac{R_2}{\sqrt{R_2^2 + (\omega_{sl} L_{lr})^2}}$$

$$\simeq K \phi I_r$$

$\phi \propto 1/f$

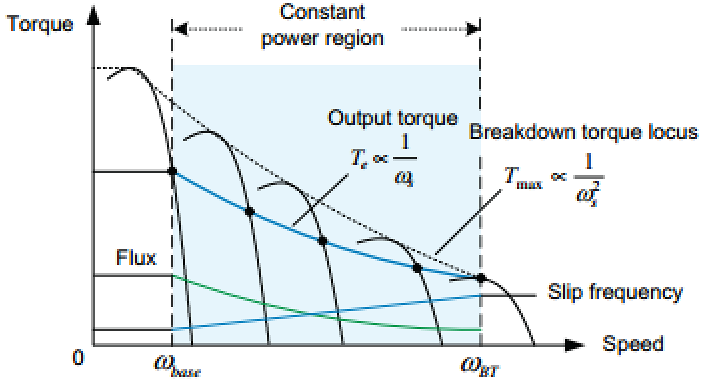
~~$I_r \propto f$~~

$I_r \propto 1/f$

Hence, $T \propto 1/\omega_s^2$

4 If slip frequency ω_{sl} increases as stator frequency ω_s increases. ⇒ Rotor current will constant because voltage drop in $\frac{R_2}{s}$ is compensated in x_{20}

Hence, $T \propto 1/\omega_s$



Linear Induction Motor (LIM)

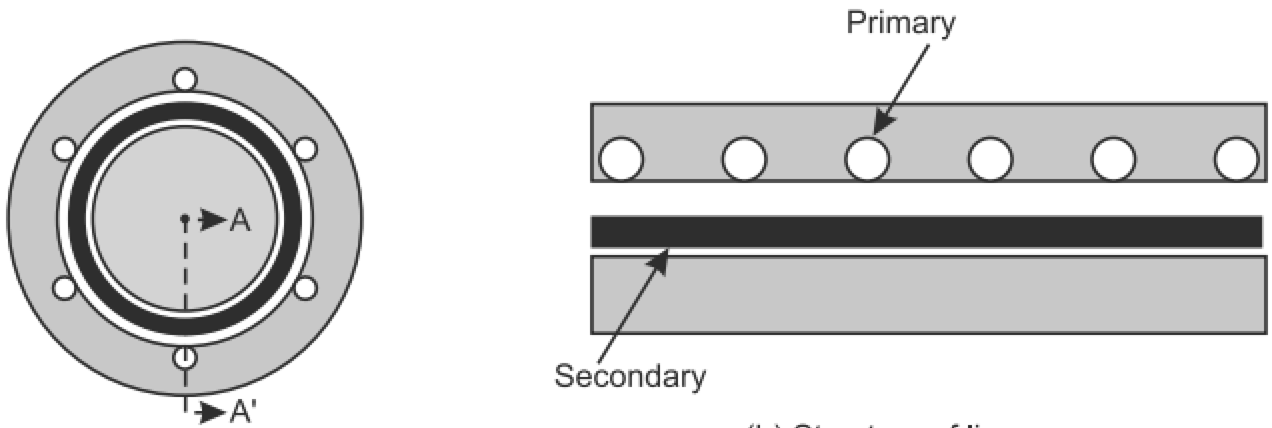
Linear induction motor (LIM)

is a motor which gives

a linear or translational motion

How and Why?

Construction of LIM



(a) Structure of conventional induction motor

(b) Structure of linear induction motor

Constructional features of Linear induction motor

Operation of LIM

Three-phase AC supply

↓

Resultant travelling flux is produced

↓

Flux linkage to rotor conductor (secondary)

↓

Current will flow in secondary

↓

Linear Force or thrust (F) will be produced

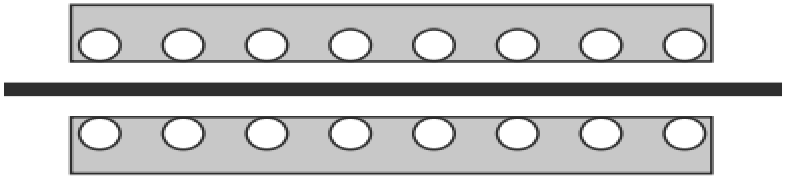
- For fixed secondary, primary will move in the direction of travelling wave

Types of LIM

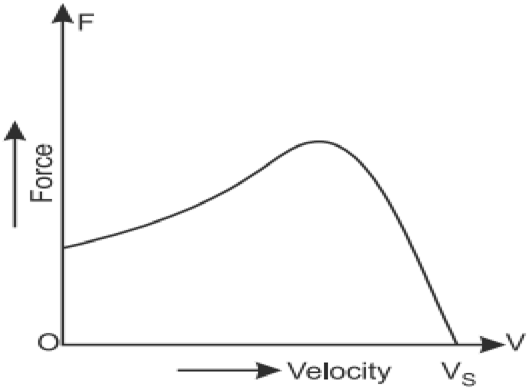
Single-sided linear induction motor (*SLIM*)

Double-sided linear induction motor (D*LIM*)

primary is placed on both the sides of the secondary



(a) Double sided linear induction motor



(b) Characteristics of linear induction motor

Performance

The linear synchronous speed of the travelling field,

$$v_s = 2f \times \textit{pole pitch} \text{ metre/second}$$

The linear speed of the secondary

$$v = v_s (1 - S)$$

Where S is the slip of the linear induction motor

$$S = \frac{v_s - v}{v_s} \text{ pu}$$

The thrust or linear force acting on the secondary or primary

$$F = \frac{\text{air gap power}}{\text{linear synchronous velocity i.e., } v_s}$$

Comparison of Conventional (rotary) and Linear Induction Motors

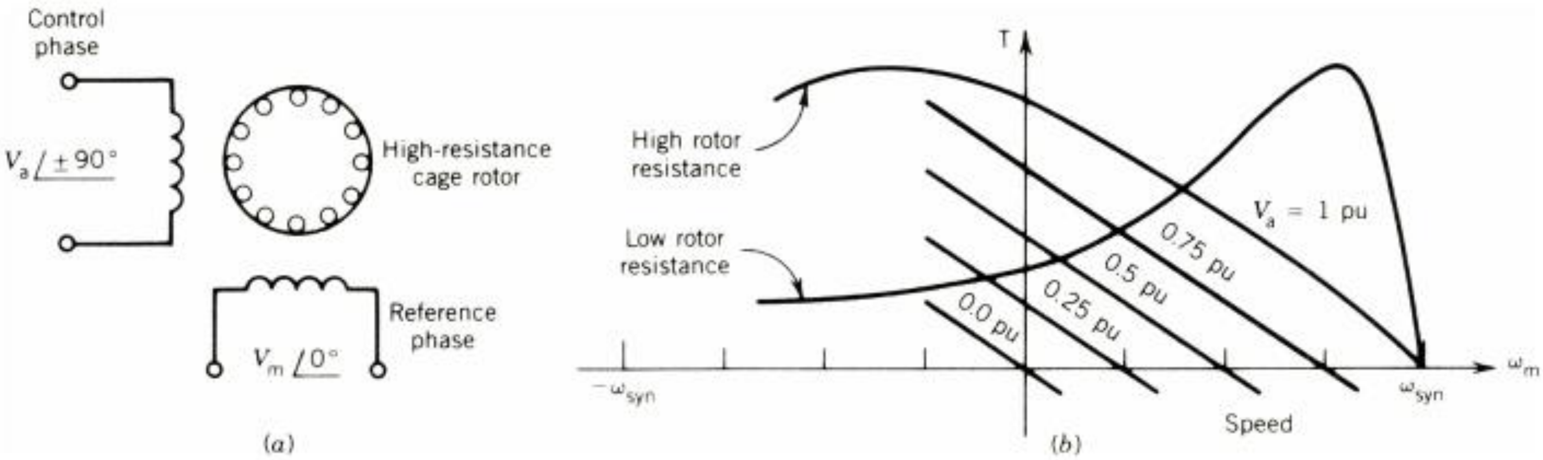
Se. No	*Particulars*	*Conventional (or rotary) induction motor*	*Linear induction motor*
1.	Type of movement	Angular rotation is obtained	Linear movement is obtained.
2.	Torque or Thrust	Torque develops in the rotor	Thrust (force) is exerted on the secondary (or primary)
3.	Parts	Main parts are called stator and rotor	Main parts are called primary and secondary
4.	Magnetising current	Air-gap is very small, magnetising current is small and pf is reasonably high.	Air-gap in more and magnetising current is more which reduces the pf
5.	Transients	Flux distribution is regular and transients are less	Due to shorter secondary, flux distribution is not regular, transients are more

Applications

The main application of this motor is in transportation where primary is mounted on the vehicle and secondary is laid along the track.

There are employed in cranes for material handily pumping of liquid metals, actuators for door movements, actuators for high voltage circuit breakers, etc.

Two Phase AC SERVOMOTORS

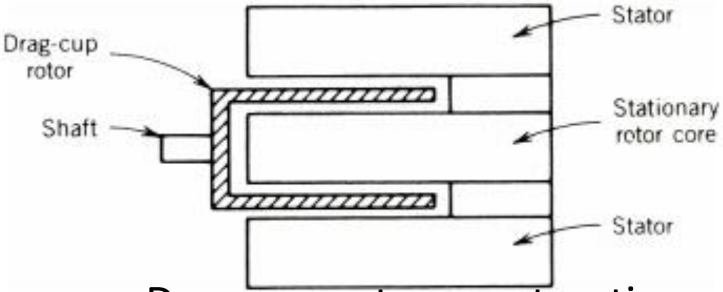


Two-phase ac servomotor. (*a*) Schematic diagram. (*b*) Torque–speed characteristics.

- To control the machine it is operated with fixed voltage for the reference phase and variable voltage for the control phase.

- The torque–speed characteristics are essentially linear (high rotor resistance assumed) for various control phase voltages.

- In low-power control applications (below a few watts), a special rotor construction is used to reduce the inertia of the rotor. A thin cup of nonmagnetic conducting material is used as the rotor (*drag-cup rotor*).



Drag-cup rotor construction

- A stationary iron core at the middle of the conducting cup completes the magnetic circuit

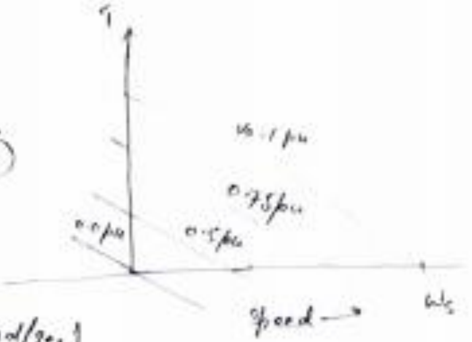
ANALYSIS: TRANSFER FUNCTION AND BLOCK DIAGRAM

Motor torque is given by

$$T = K_m V_a - F_m \omega_m \quad — ①$$

K_m: Motor torque constant (N-m/volt)

F_m: Motor viscous friction (N-m/rad/sec)



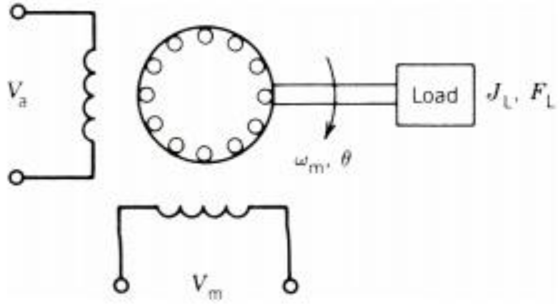
Equation of motion of servomotor

$$T = T_L$$

$$K_m V_a - F_m \omega_m = (\underset{\text{motor inertia}}{J_m} + \underset{\text{load inertia}}{J_L})\frac{d\omega_m}{dt} + F_L \omega_m \quad — ②$$

Also, $\frac{d\theta}{dt} = \omega_m$

$$\therefore K_m V_a - F_m \frac{d\theta}{dt} = (J_m + J_L)\frac{d^2\theta}{dt^2} + F_L \frac{d\theta}{dt} \quad — ③$$



Servo system using a two-phase motor.

Taking Laplace transformation of equation ②, we get

$$K_m V_a(s) - f_m \omega_m(s) = s(J_m + J_L)\,\omega_m(s) + f_L\,\omega_m(s)$$

or $$K_m V_a(s) = \cancel{(sJ + f_m)\,\omega_m(s)}$$

$$= (sJ + f)\,\omega_m(s) \quad ; \; J = J_m + J_L$$

$$f = f_m + f_L$$

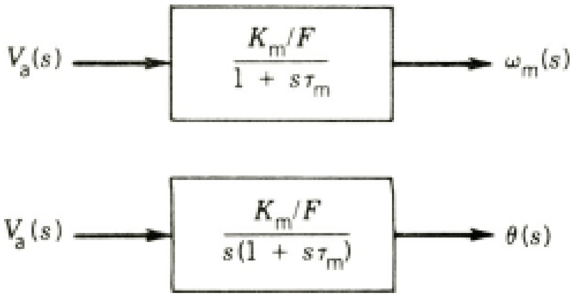
or $$\frac{\omega_m(s)}{V_a(s)} = \frac{K_m}{(sJ + f)}$$

or $$\boxed{\frac{\omega_m(s)}{V_a(s)} = \frac{K_m/F}{(1 + s\tau_m)}} \quad — ④$$

; $\tau_m = J/f$ (mechanical time constant of drive

After taking Laplace transformation and simplification of ③

$$\boxed{\frac{\theta(s)}{V_a(s)} = \frac{K_m/F}{s(1 + s\tau_m)}} \quad — ⑤$$



Transfer functions.

Time Response for a Step Change in Control Phase Voltage: Open-Loop Operation

Consider a step change in the control phase voltage Va

$$V_{\mathrm{a}}(s) = \frac{V}{s}$$

$$\omega_{\mathrm{m}}(s) = \frac{K_{\mathrm{m}}/F}{1 + s\tau_{\mathrm{m}}}\frac{V}{s}$$

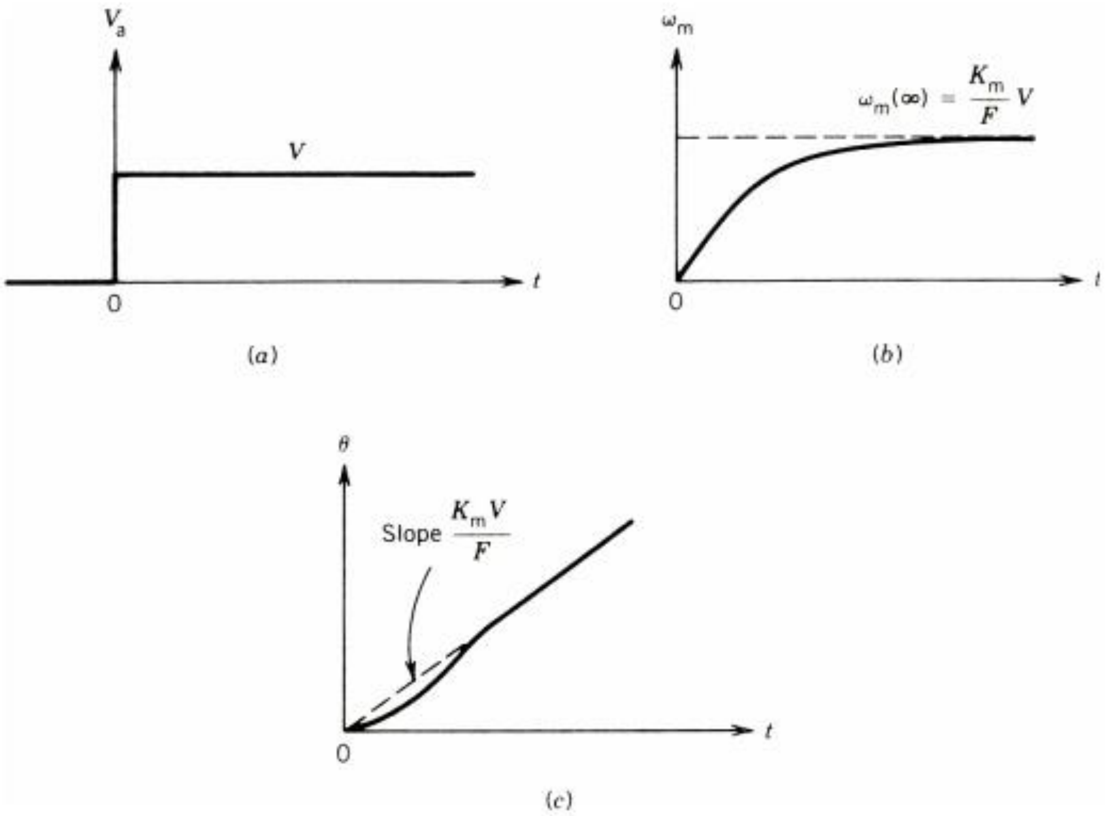
$$= \frac{K_{\mathrm{m}}V}{F}\left(\frac{1}{s} - \frac{1}{s + 1/\tau_{\mathrm{m}}}\right)$$

The corresponding time function is

$$\omega_{\mathrm{m}}(t) = \frac{K_{\mathrm{m}}V}{F}(1 - e^{-t/\tau_{\mathrm{m}}})$$

The steady-state speed is

$$\omega_{\mathrm{m}}(\infty) = \frac{K_{\mathrm{m}}V}{F}$$



Step response in a two-phase servo system.
(a) Step change in Va.
(b) Response in speed.
(c) Response in position.

Q1

A two-phase servomotor has rated voltage applied to its reference phase winding. The torque–speed characteristic of the motor with $V_a = 115\,\text{V}$, 60 Hz applied to its control phase winding is shown in Fig. E8.1. The moment of inertia of the motor and load is $10^{-5}\,\text{kg}\cdot\text{m}^2$, and the viscous friction of the load is negligible (Fig. 8.4).

(a) Obtain the transfer function between shaft position θ and control voltage V_a.

(b) Obtain an expression for the shaft position due to the application of a step voltage $V_a = 115\,\text{V}$ to the control phase winding.

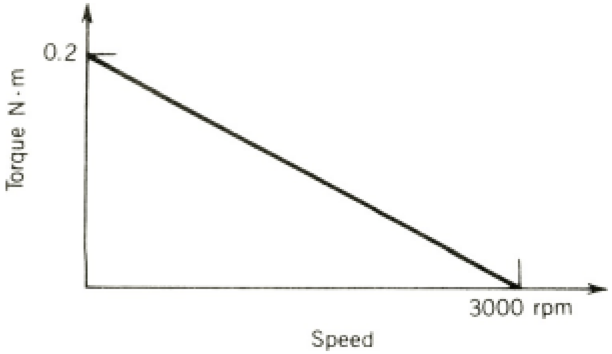


FIGURE E8.1

Solution

(a)

$$K_m = \left.\frac{T}{V_a}\right|_{\omega_m = \text{ constant}} = \left.\frac{0.2}{115}\right|_{\omega_m = 0} = 0.00174 \text{ N}\cdot\text{m/V}$$

$$F_m = \left.\frac{T}{\omega_m}\right|_{V_a = \text{ constant}} = \frac{0.2}{3000 \times 2\pi/60} = 0.0006366 \text{ N}\cdot\text{m/rad/sec}$$

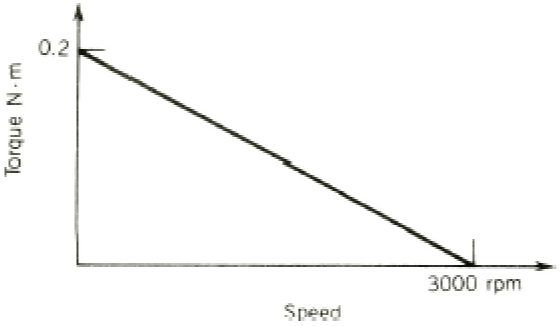


FIGURE E8.1

$$F = F_m + F_L = F_m + 0 = F_m$$

$$J = 10^{-5} \text{ Kg}\cdot\text{m}^2$$

$$\tau_m = \frac{J}{F} = \frac{10^{-5}}{0.0006366} = 15.71 \times 10^{-3} \text{ sec}$$

$$\frac{K_m}{F} = \frac{0.00174}{0.0006366} = 2.733$$

$$\frac{\theta(s)}{V_{\mathrm{a}}(s)} = \frac{2.733}{s(1+0.01571s)}$$

$$V_{\mathrm{a}}(s) = \frac{115}{s}$$

$$\frac{K_{\mathrm{m}}V}{F} = 2.733 \times 115 = 314.3$$

$$\frac{K_{\mathrm{m}}V}{F}\tau_{\mathrm{m}} = 314.3 \times 0.01571 = 4.94$$

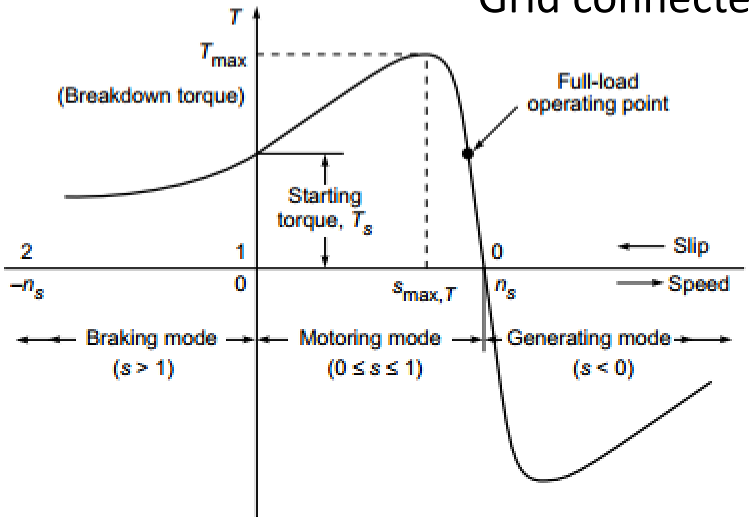
$$\theta(t) = 314.3t - 4.94 + 4.94\,e^{-t/0.01571}$$

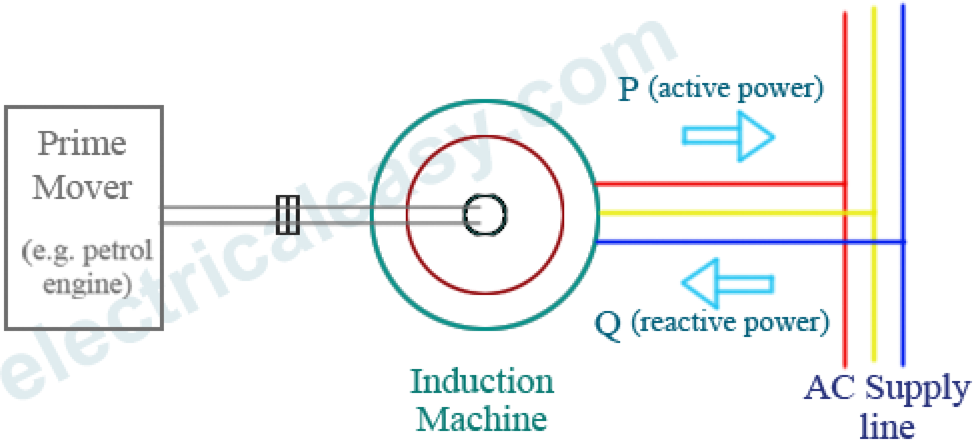
$$\simeq 314.3t \quad \blacksquare$$

Induction machine as generator

Grid connected

Self-exited





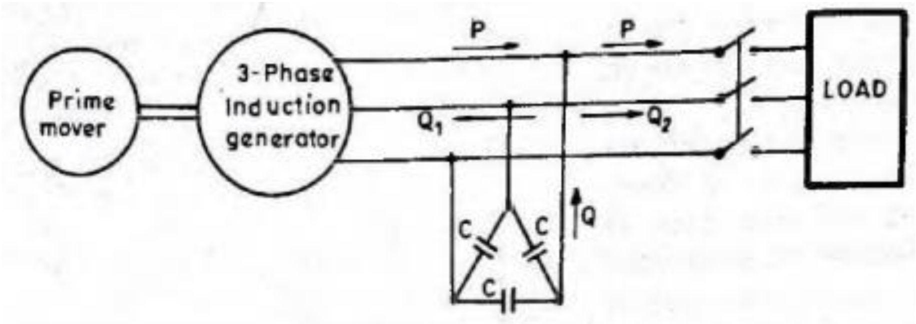
- Now, if the rotor is accelerated to the synchronous speed by means of a prime mover, the slip will be zero and hence the net torque will be zero. The rotor current will become zero when the rotor is running at synchronous speed.

- If the rotor is made to rotate at a speed more than the synchronous speed, the slip becomes negative. A rotor current is generated in the opposite direction, due to the rotor conductors cutting stator magnetic field.

- This generated rotor current produces a rotating magnetic field in the rotor which pushes (forces in opposite way) onto the stator field. This causes a stator voltage which pushes current flowing out of the stator winding against the applied voltage. Thus, the machine is now **working as an induction generator (asynchronous generator)**.

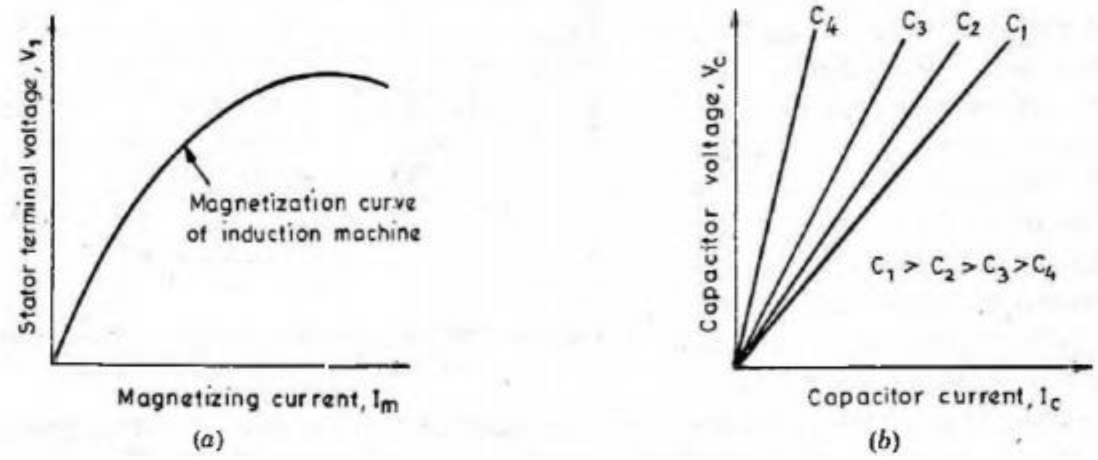
In polyphase induction motors, rotating magnetic field is set up by the magnetizing current drawn by the stator from the supply mains. When the speed of the machine is made more than synchronous speed, even then this magnetizing current must be delivered to the induction generator by the supply mains so that rotating flux is established. In other words, induction generator is not a *self-excited machine* and must therefore continue to get its magnetizing current and reactive power from the supply mains to which it was connected to run as a polyphase induction motor. Thus, 3-phase induction generator cannot work in isolation ; it must work in parallel with the bus-bar or other synchronous generator/s, which can supply the magnetizing current and reactive power needed by 3-phase induction generator.

Self exited induction generator



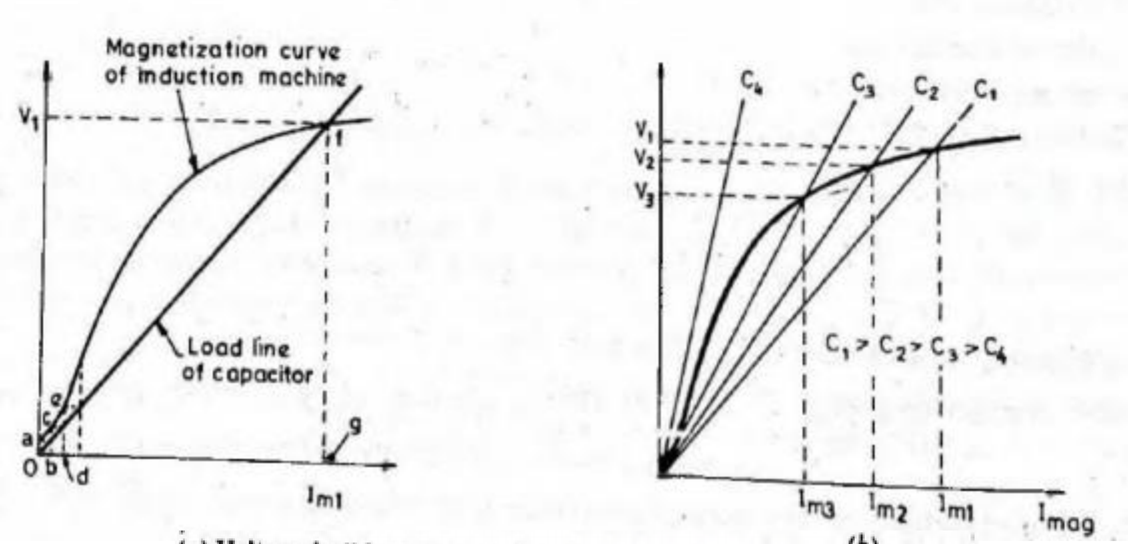
- Capacitive reactive power, Q = Ql, reactive power needed by 3-phase induction generator + Q2, reactive power needed by the load.

- The reactive current of a capacitor depends upon the voltage across its terminals, *i.e.*, capacitor voltage $V_c = I \cdot X_c$ where $X_c = \frac{1}{\omega C}$



Characteristics of (*a*) induction generator and (*b*) capacitor bank.

Voltage Build-up in self-exited induction generator



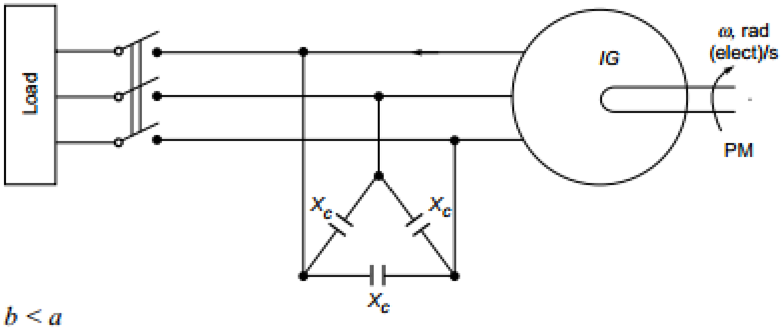
(*a*) Voltage build-up process in a 3-phase induction generator and (*b*) No-load voltages V_1, V_2, V_3 for C_1, C_2, C_3.

Self exited induction generator

- The delta- connected capacitors across the generator terminals provide the magnetizing current necessary to excite the isolated generator

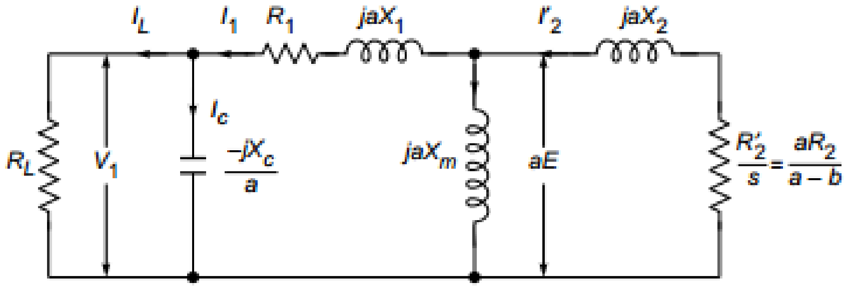
- Operating frequency depends primarily upon rotor speed but is affected by the load, while the voltage is mainly decided by capacitor reactance (*XC*) at the operating frequency.

Let ω_0 = rated frequency
ω_s = operating frequency (stator)
ω_r = stator frequency corresponding to rotor speed
$a = \omega_s/\omega_0$ and $b = \omega_r/\omega_0$



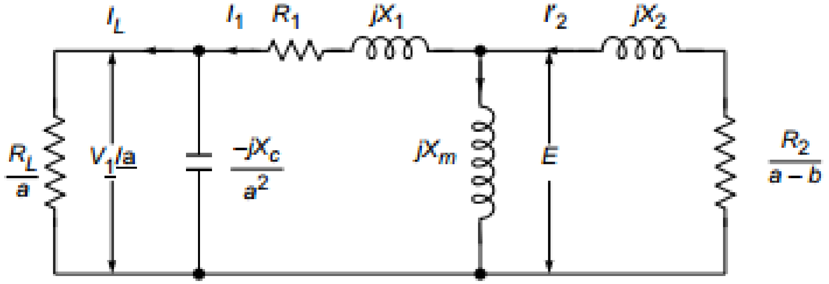
The machine slip (which should be negative) can be expressed as

$$s = (\omega_s - \omega_r)/a\omega_0$$
$$= (a\omega_0 - b\omega_0)/a\omega_0 = (a - b)/a;\ b < a$$



I_L
I_1
R_1
jaX_1
I'_2
jaX_2
I_c
R_L
V_1
$\frac{-jX_c}{a}$
jaX_m
aE
$\frac{R'_2}{s} = \frac{aR_2}{a-b}$

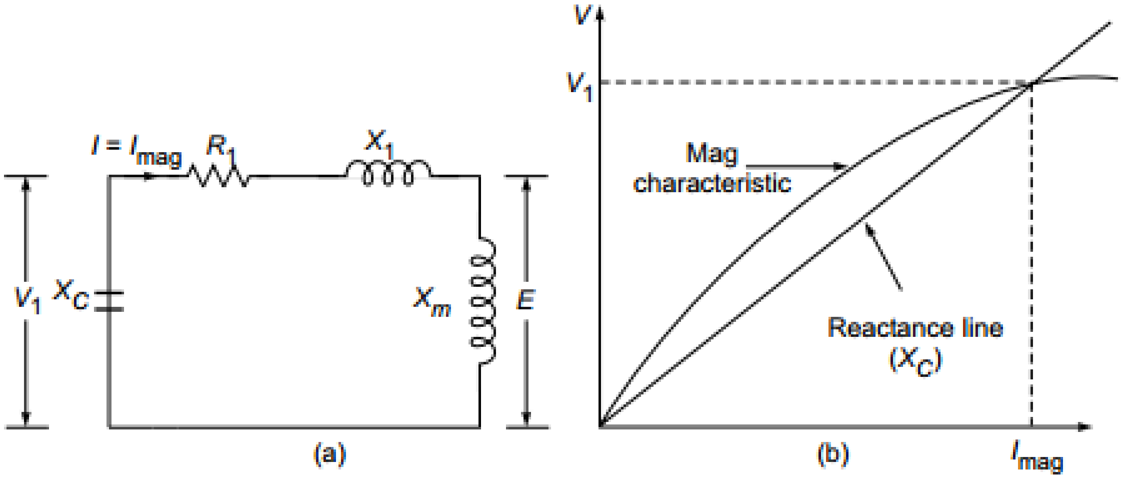
(a)



I_L
I_1
R_1
jX_1
I'_2
jX_2
$\frac{R_L}{a}$
V_1/a
$\frac{-jX_c}{a^2}$
jX_m
E
$\frac{R_2}{a-b}$

(b)

Voltage Buildup in SEIG



The current fed into the motor is $I_1 = I_{mag}$, the magnetizing current. If the voltage drop in stator impedance, R_1, X_1, is ignored, we have

$$V_1 = E \text{ (induced emf)}$$
$$= E\,(I_{mag}), \text{ the magnetization characteristic}$$

Also $$V_1 = X_C I_{mag}; \text{ the reactance line}$$

Unit 2

Stepper motor

- *Def. A motor in which the rotor turns in discrete movements is called a* ***stepper motor.***
- Typical step sizes are 2°, 2.5°, 5°, 7.5°, and 15° for each electrical pulse.
- The angle through which the motor shaft rotates for each command pulse is called *the step angle* β.
- It is observed, the smaller the step angle, the greater the number of steps per revolution and higher the resolution or accuracy of positioning obtained.

The value of step angle can be expressed either in terms of the rotor and stator poles (teeth) Nr and Ns, respectively, or in terms of the number of stator phases (m) and the number of rotor teeth.

$$\beta = \frac{(N_s - N_r)}{N_s \,.\, N_r} \times 360^\circ$$

$$\beta = \frac{360^\circ}{m.N_r} = \frac{360^\circ}{\text{No. of stator phases} \times \text{No. of rotor teeth}}$$

- The resolution is given by the number of steps needed to complete one revolution of the rotor shaft. The higher the resolution, the greater the accuracy of positioning of objects by the motor.

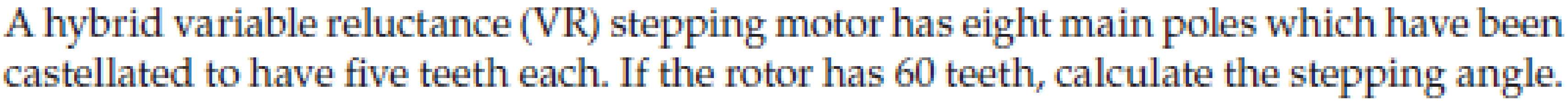
Resolution = No. of steps/revolution = 360°/β

- When the pulse rate is high the shift rotation seems continuous and operation at a high rate is called slewing.

- If f is the stepping frequency (or pulse rate) in pulses per second (pps) and β is the step angle, then motor shaft speed is given by

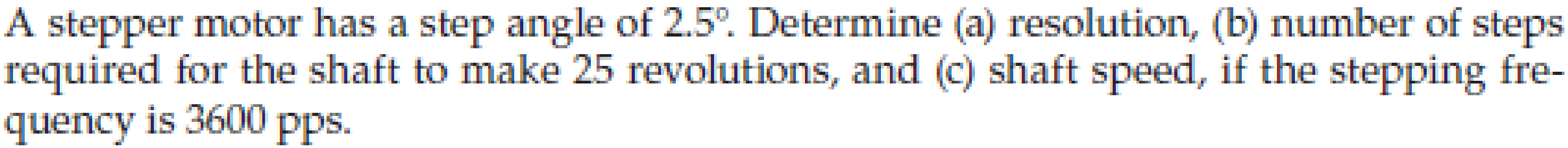
$$n = \beta \times f / 360 \text{ rps} = \text{pulse frequency resolution}$$

Q1



A hybrid variable reluctance (VR) stepping motor has eight main poles which have been castellated to have five teeth each. If the rotor has 60 teeth, calculate the stepping angle.

Q2



A stepper motor has a step angle of 2.5°. Determine (a) resolution, (b) number of steps required for the shaft to make 25 revolutions, and (c) shaft speed, if the stepping frequency is 3600 pps.

Solution

$$Ns = 8 \times 5 = 40; \text{ and } Nr = 60$$

$$\beta = \frac{(N_s - N_r)}{N_s . N_r} \times 360^\circ$$

$$\beta = \frac{(N_r - N_s)}{N_s . N_r} \times 360^\circ = \frac{(60 - 40)}{40.60} \times 360^\circ = 3^\circ$$

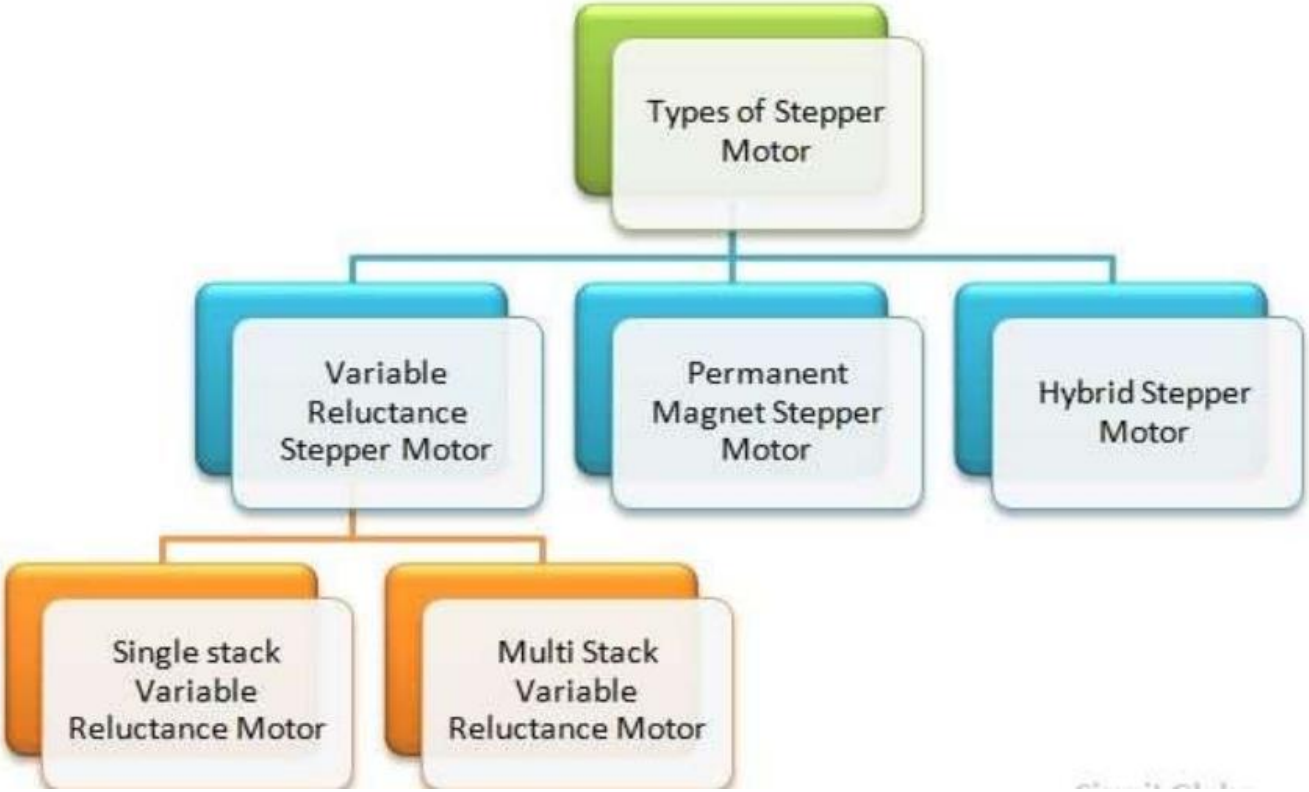
Solution

a. resolution = $360^\circ/\beta = 360^\circ/2.5^\circ = 144$ steps/revolution

b. steps/revolution = 144. Hence, steps required for making 25 revolutions = $144 \times 25 = 3600$

c. $n = \beta \times f/360^\circ = 2.5 \times (3600/360^\circ) = 25$ rps.

Classification



Types of Stepper Motor

- Variable Reluctance Stepper Motor
  - Single stack Variable Reluctance Motor
  - Multi Stack Variable Reluctance Motor
- Permanent Magnet Stepper Motor
- Hybrid Stepper Motor

Advantages

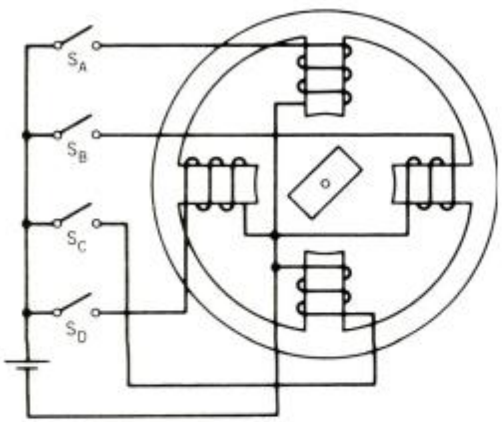
- Low cost for control achieved
- High torque at startup and low speeds
- Ruggedness
- Simplicity of construction
- Can operate in an open loop control system
- Low maintenance
- Less likely to stall or slip
- Will work in any environment
- Can be used in robotics in a wide scale.
- High reliability

Applications

- The stepper motor is used for precise positioning with a motor like hard disk drives, robotics, telescopes and some toys.
- Industrial Machines-Stepper motors are used in automotive gauge and machine tooling automated production equipments.
- Security-New surveillance products for the security industry
- Medical-Stepper motors are used inside medical scanner , sampler and also found inside digital dental photography.

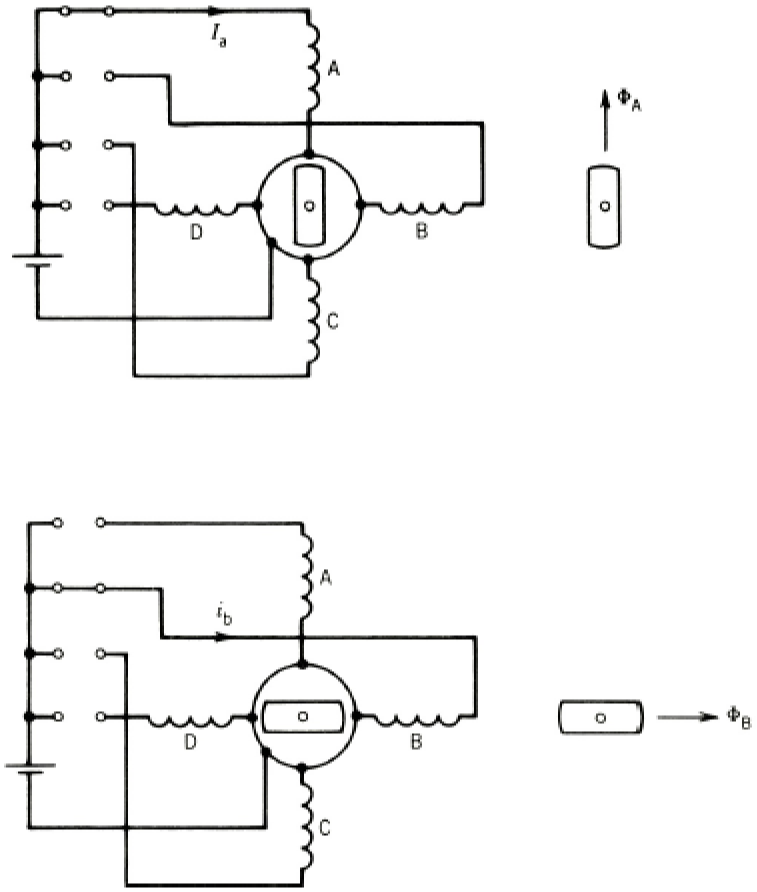
VARIABLE-RELUCTANCE STEPPER MOTOR

A. Single-Stack Stepper Motor



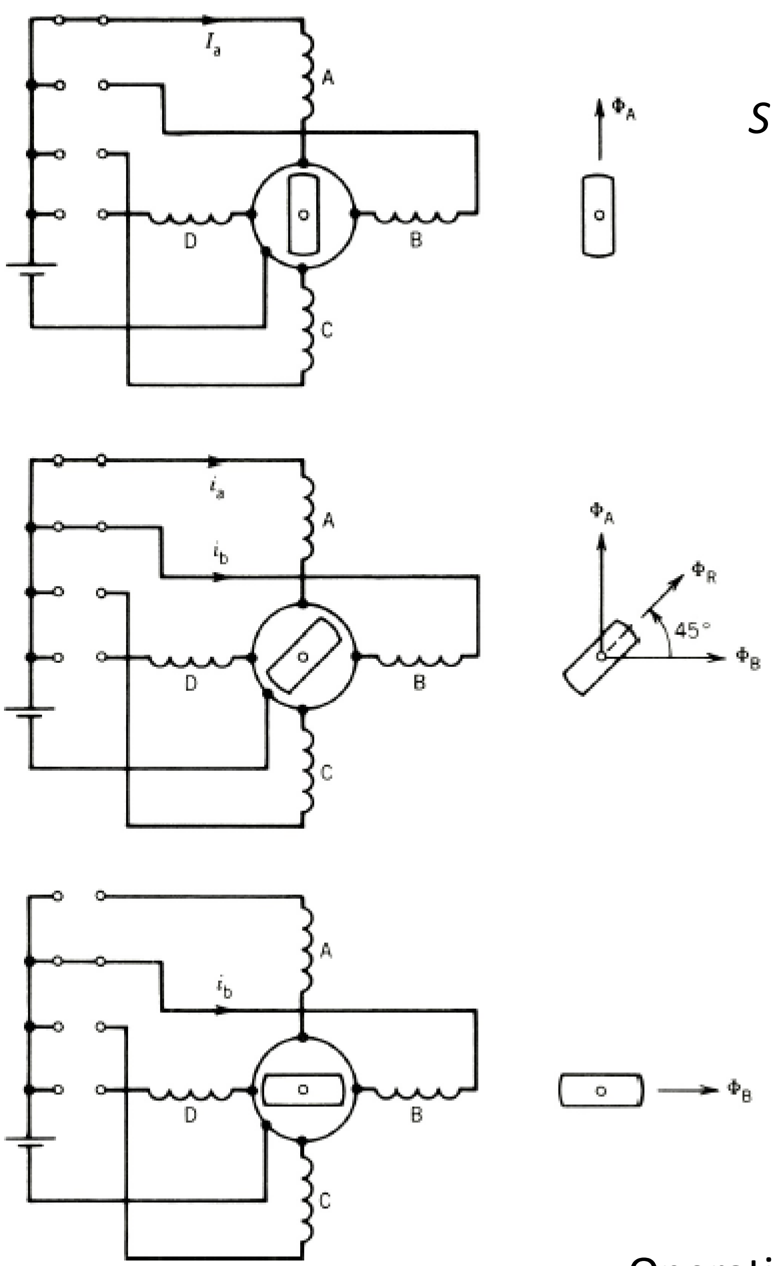
Basic circuit for a four-phase, 4/2 pole stepper motor.

- Reluctance torque is generated because of the tendency of the ferromagnetic rotor to align itself along the direction of the resultant magnetic field.
- Sequence: A, B, C, D etc. for 90 degree rotation in clockwise direction.



Operating modes of stepper motor for 90 step.

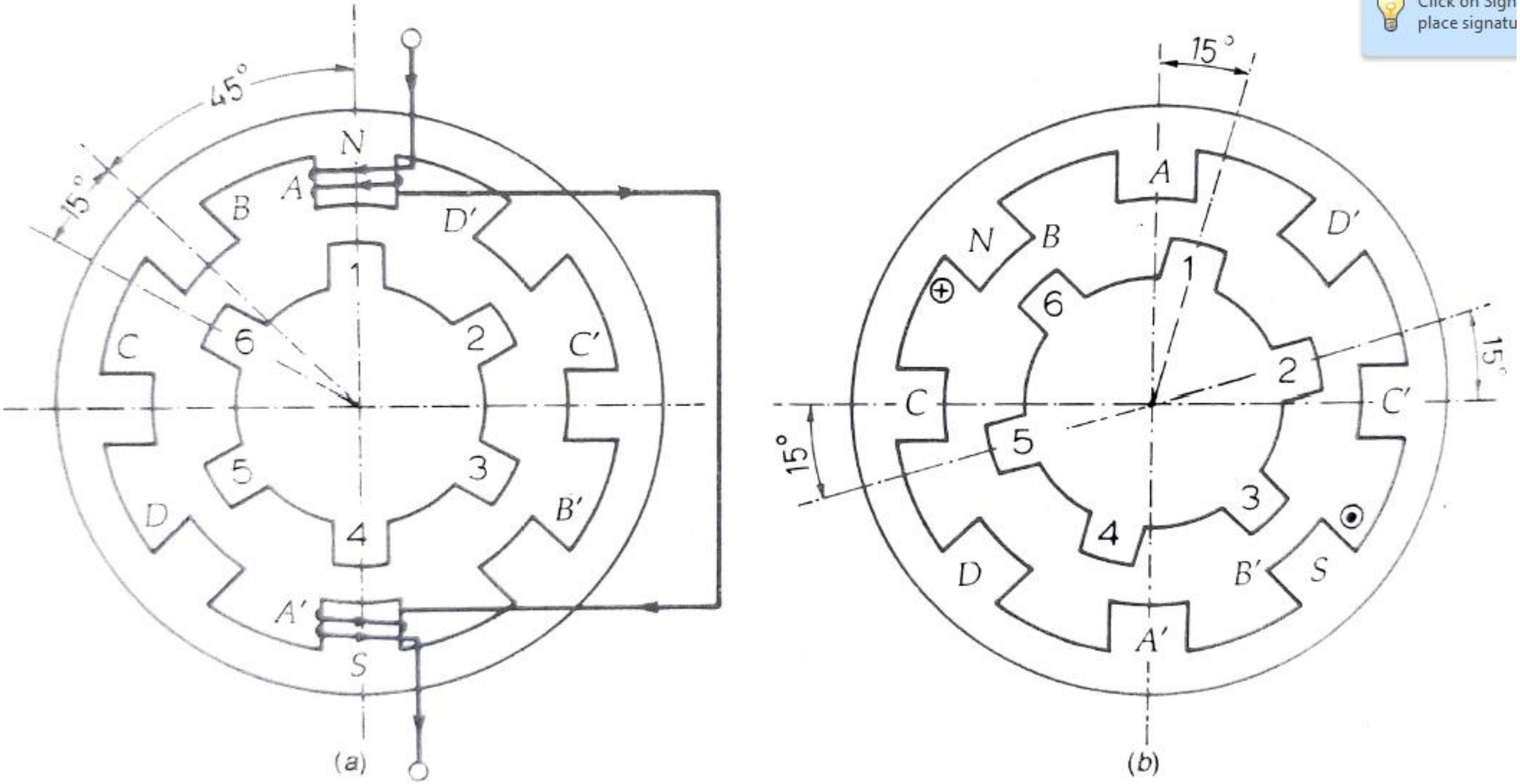
Sequence: A, A + B, B, B + C, and so forth



Operating modes of stepper motor for 45 step.

- Microstepping: The method of gradually shifting of excitation from one phase to another (e.g. From A to B with an intermediate step A+B) is known as microstepping.

- Lower values of step angle can be obtained by using a stepping motor with more number of poles on stator and teeth on rotor.

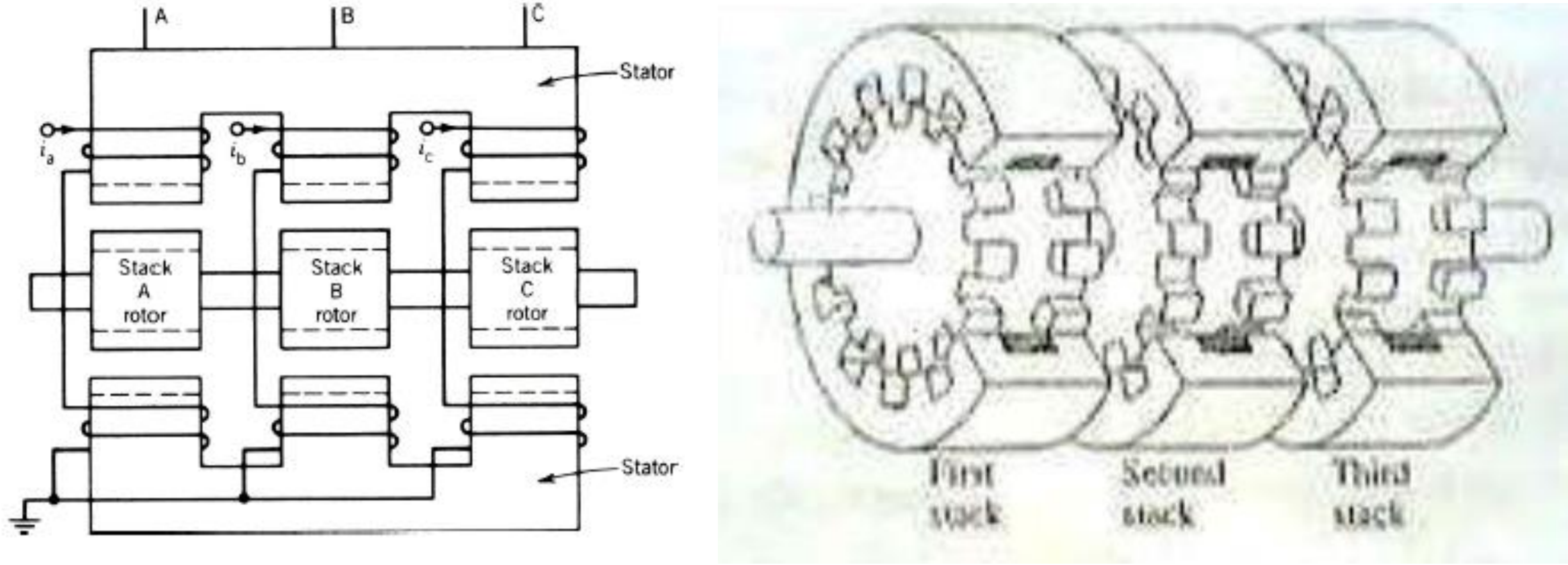


Four-phase, 8/6-pole variable reluctance stepper motor.

Multistack Stepper Motor

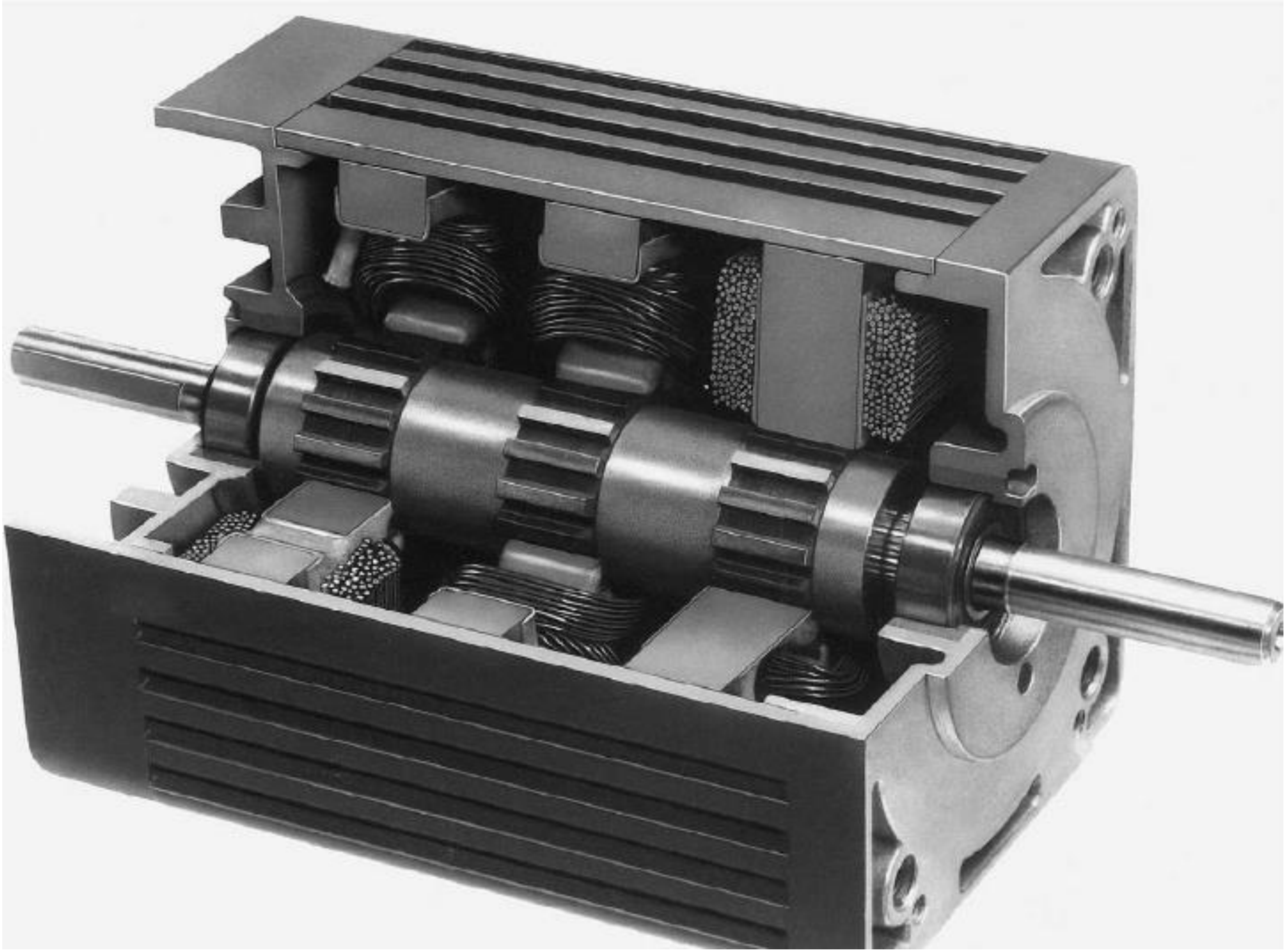
- A Multi Stack or m stack variable reluctance stepper motor is made up of m identical single stack variable reluctance motor. The rotor is mounted on the single shaft. The stator and rotor of the **Multi Stack Variable motor** have the same number of poles and hence, the same pole pitch.

- All the stator poles are aligned in a Multi-Stack motor. But the rotor poles are displaced by 1/m of the pole pitch angle from each other. The stator windings of each stack forms one phase as the stator pole windings are excited simultaneously. Thus, the number of phases and the number of stacks are same.



Cross section of a three-stack, variable-reluctance stepper motor parallel to the shaft.

- There are 12 stator and rotor poles in each stack. The pole pitch for the 12 pole rotor is 30, and the step angle or the rotor pole teeth are displaced by 10 degrees from each other.



Let Nr be the number of rotor teeth and m be the number of stacks or phases.

Hence, Tooth pitch is represented by the equation shown below.

$$T_p = \frac{360°}{N_r} \dots\dots..(1)$$

Therefore,

$$\text{Step angle} = \frac{360°}{m\,N_r} \dots\dots\dots(2)$$

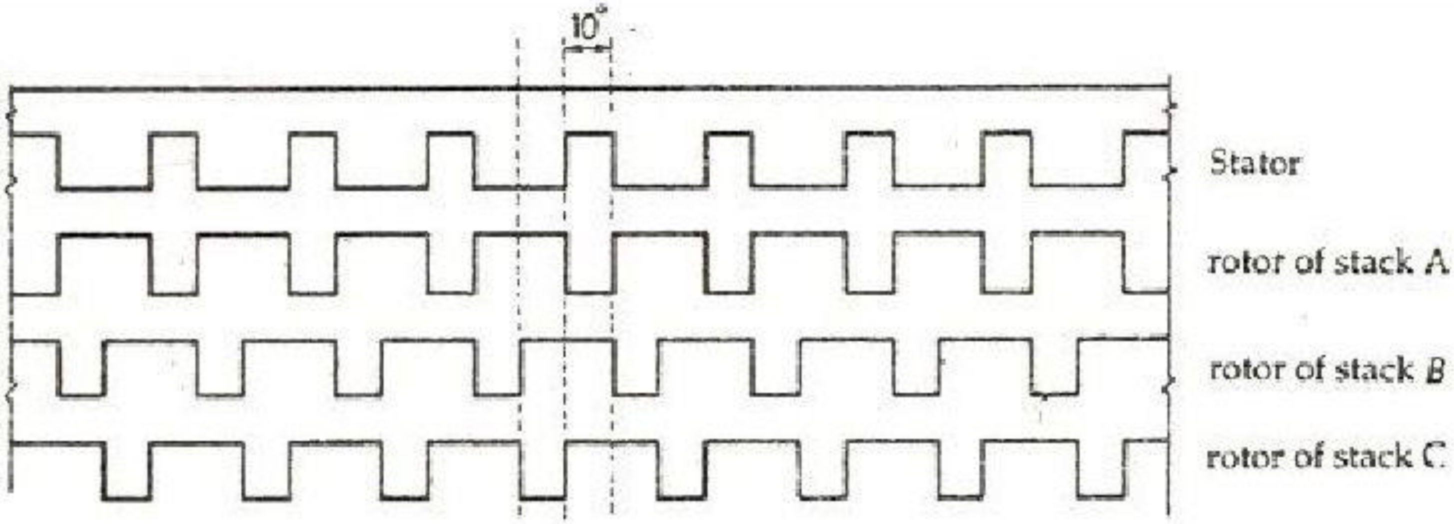
As there are 12 poles in the stator and rotor, thus the value of Nr = 12. Now, putting the value of Nr in the equation (1) we get

$$T_p = \frac{360°}{12} = 30° \dots\dots..(3)$$

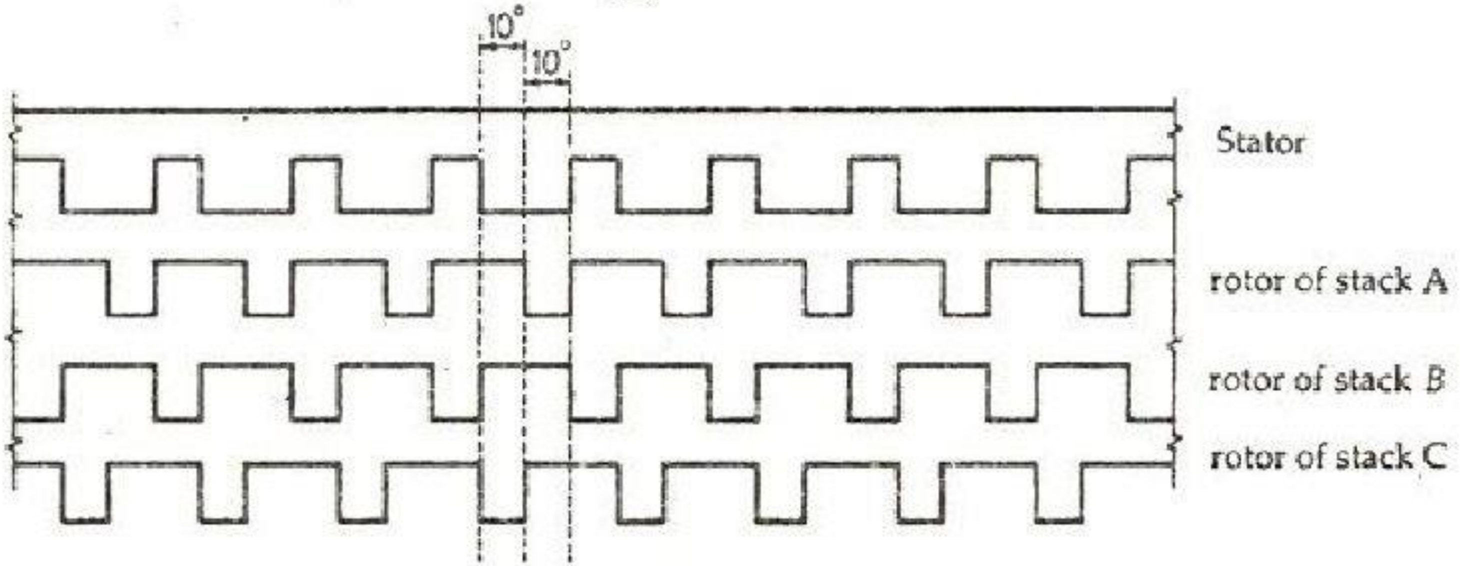
The value of m= 3. Therefore, the step angle will be calculated by putting the value of m in the equation (2).

$$\text{Step angle} = \frac{360°}{3 \times 12} = 10° \;\ldots\ldots\ldots (4)$$

- When the phase winding A is excited the rotor teeth of stack A are aligned with the stator teeth as shown in the figure below.



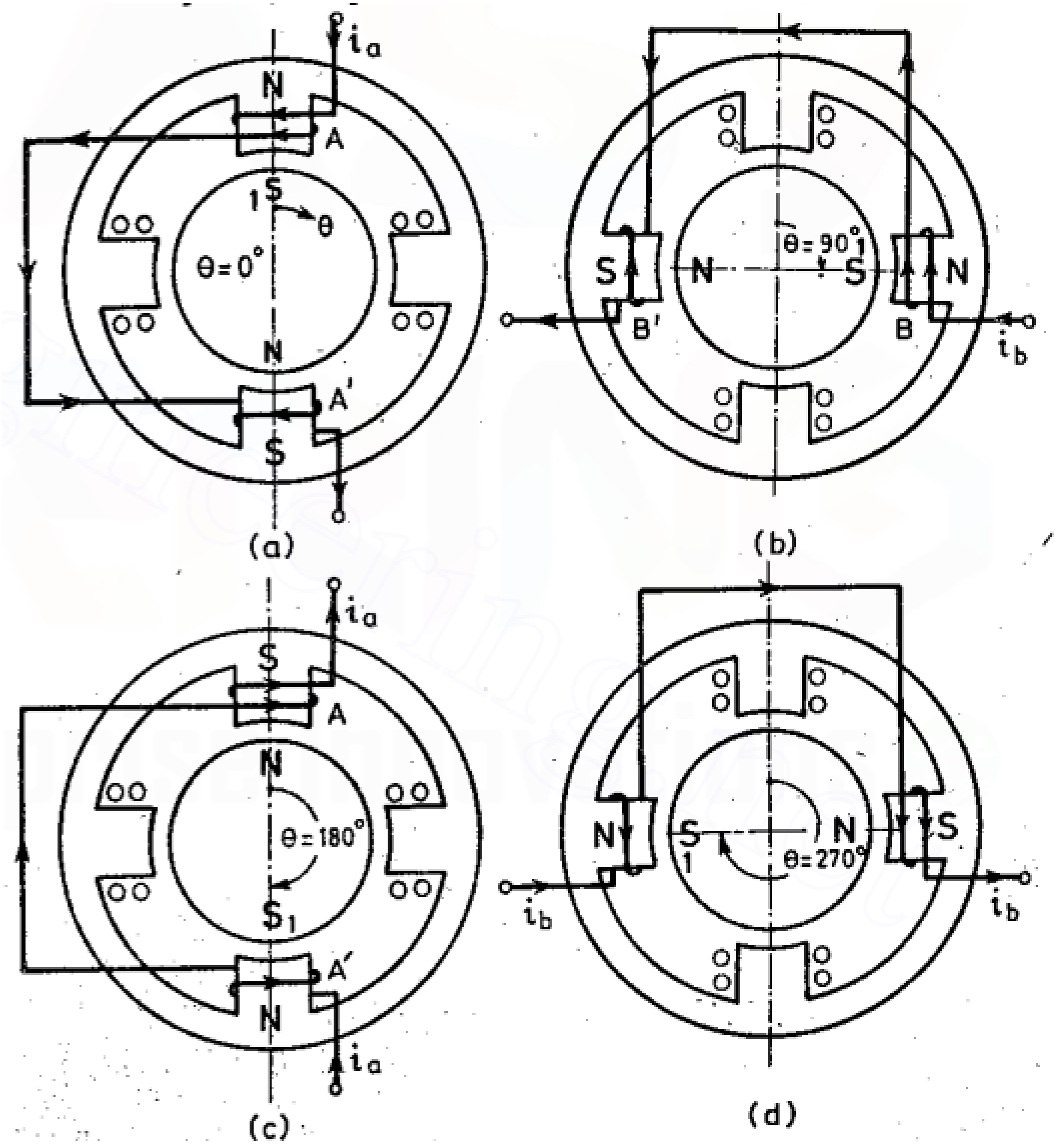
- When phase A is de-energized, and phase B is excited, rotor teeth of the stack B are aligned with the stator teeth. The rotor movement is about 10 degrees in the anticlockwise direction.



- Similarly, now phase B is de-energized, and phase C is excited. The rotor moves another step of 1/3 of the pole pitch in the anticlockwise direction.

- However, during this whole process (A – B – C – A) the rotor has moved one rotor tooth pitch.

Permanent magnet stepper motor



Two-phase 4/2-pole PM stepper motor.

(*a*) In Fig. 9.5 (*a*), two coils *AA′* connected in series constitute phase *A* winding. When this winding is excited with current i_a as shown, the stator produced poles attract the rotor PM poles so that their magnetic axes coincide. Let the exciting of phase *A* winding in Fig. 9.5 (*a*) be denoted by + *A*.

(*b*) In Fig. 9.5(*b*), the current i_a in phase *A* winding is reduced to zero whereas phase *B* winding is energized with current i_b. Stator produced poles now attract the rotor poles causing a CW step rotation through $\theta = 90^\circ$ as shown. Let the exciting of phase *B* winding in Fig. 9.5 (*b*) be designated +*B*.

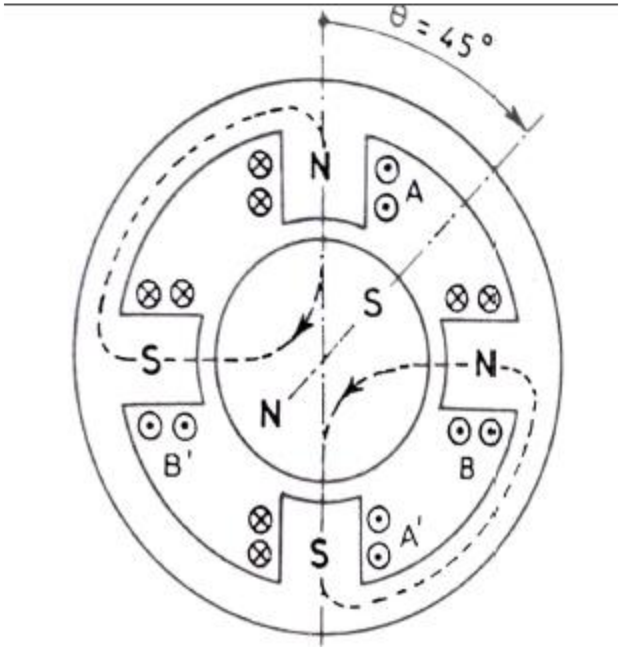
(*c*) In Fig. 9.5 (*c*), the phase winding *A* is again excited but with current opposite to that in Fig. 9.5 (*a*). The rotor poles now move through a further step of 90°CW so that $\theta = 180^\circ$. This step of exciting the phase *A* winding must be designated – A.

(*d*) In Fig. 9.5 (*d*), the phase winding *B* is made to carry exciting current i_b opposite to that in Fig. 9.5 (*b*). The rotor again executes a further step of 90° CW so that $\theta = 270^\circ$. This method of exciting phase winding *B* must be designated – B.

It is seen from above that by the application of each current pulse to the stator winding in proper sequence, the rotor can be made to execute discrete angular steps of 90°. The sequence of exciting the stator phase windings is $+A, +B, -A, -B, +A$, ... for CW rotor movement. For CCW rotor rotation, the sequence of switching the phase winding must be reversed to $+A, -B, -A, +B, +A, -B$, ...

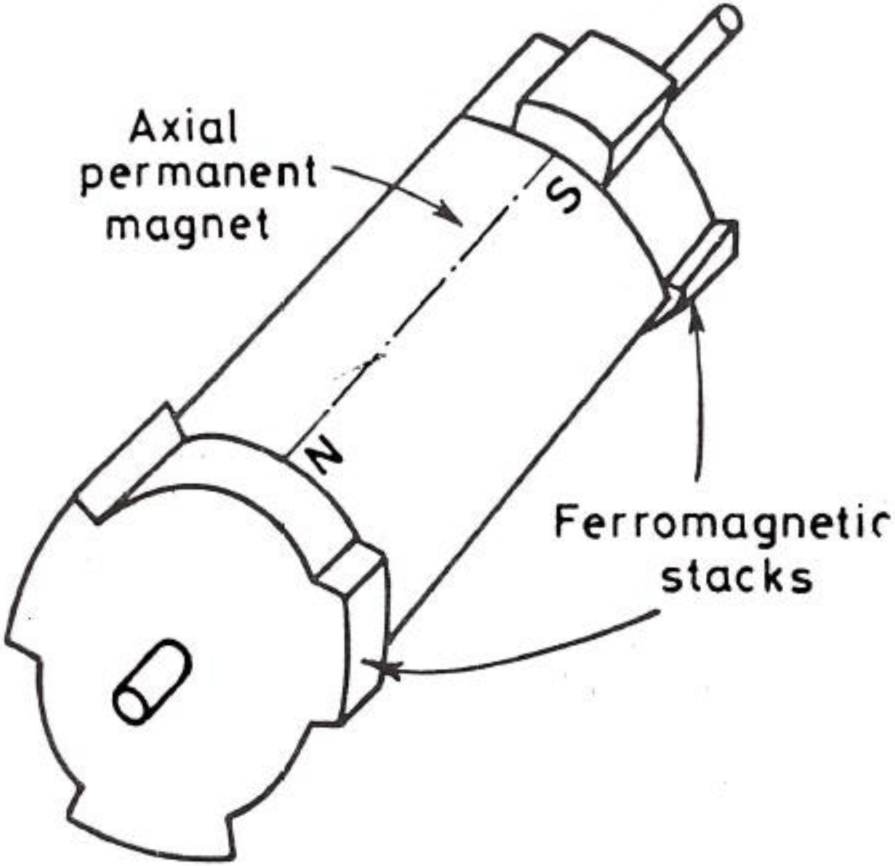
Half Step mode excitation

$+A, (+A+B), +B, (+B-A), -A, (-A-B), -B, (-B+A), +A, \ldots$

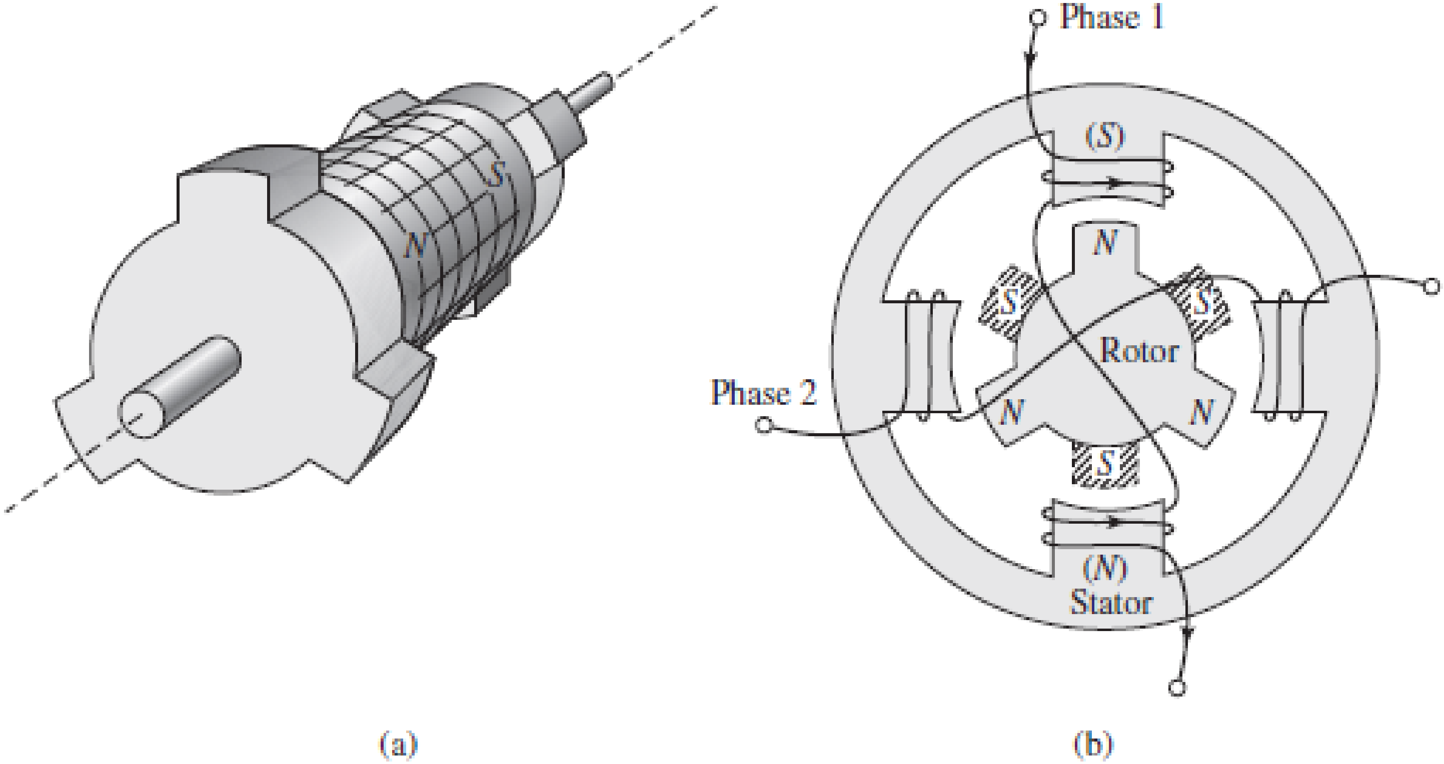


Rotor rotation through 45° with both windings excited together in PMSM

Hybrid Stepper Motor



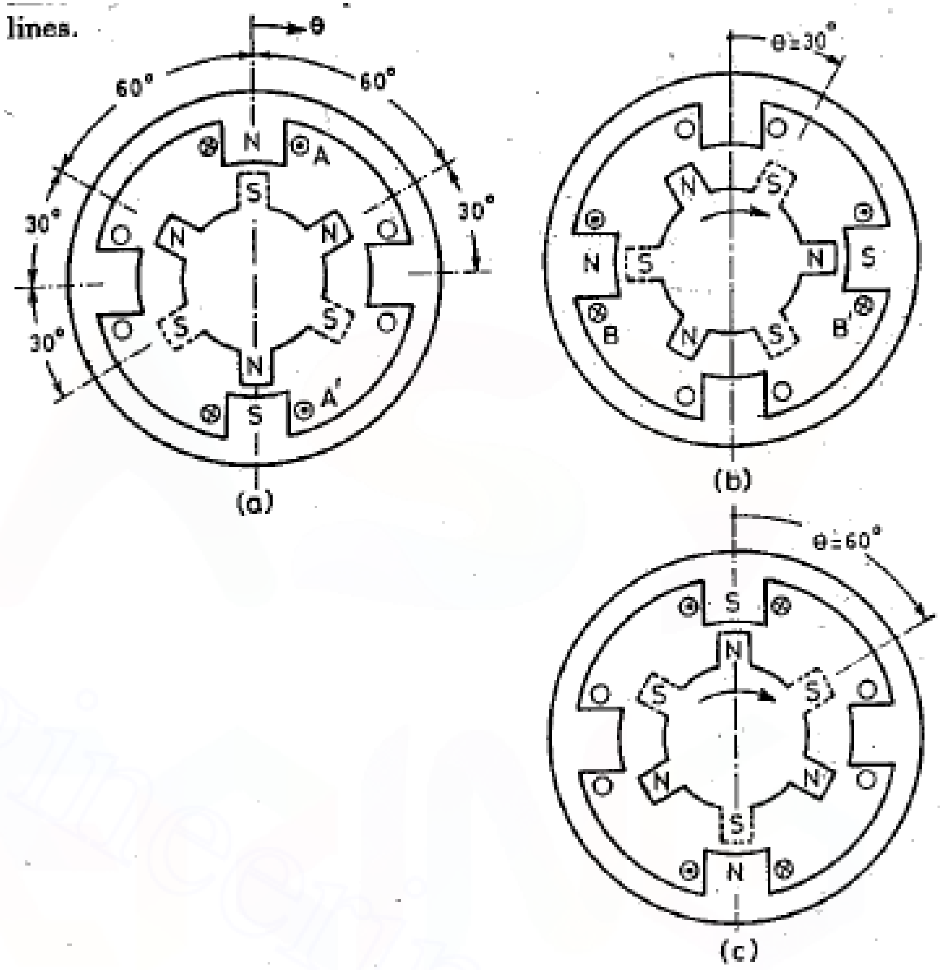
Schematic view of hybrid stepping motor.



(a) (b)

Figure 8.24 Schematic view of a hybrid stepping motor. (a) Two-stack rotor showing the axially directed permanent magnet and the pole pieces displaced by one-half the pole pitch. (b) End view from the rotor north poles and showing the rotor south poles at the far end (shown crosshatched). Phase 1 of the stator is energized to align the rotor as shown.

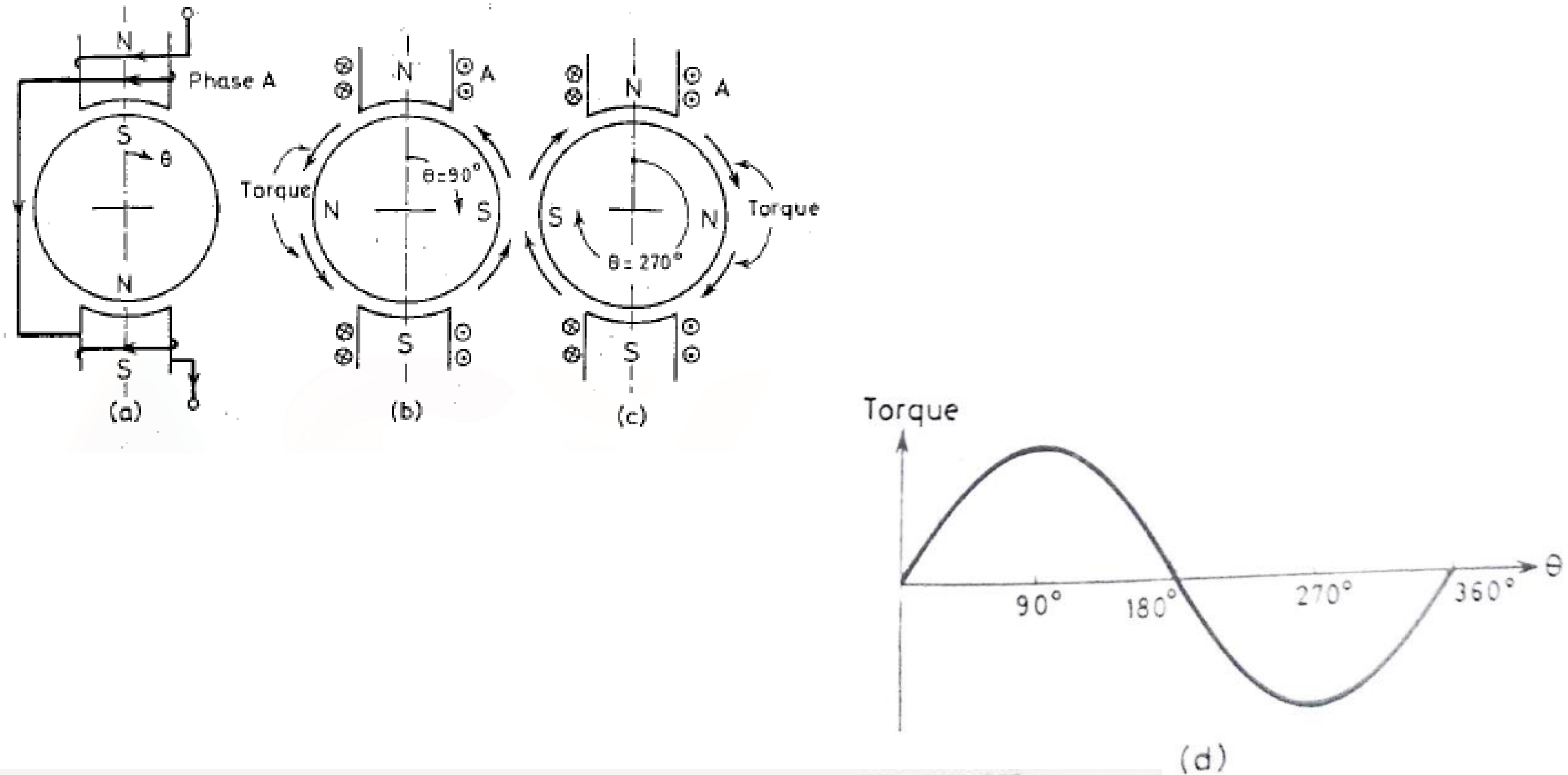
lines.



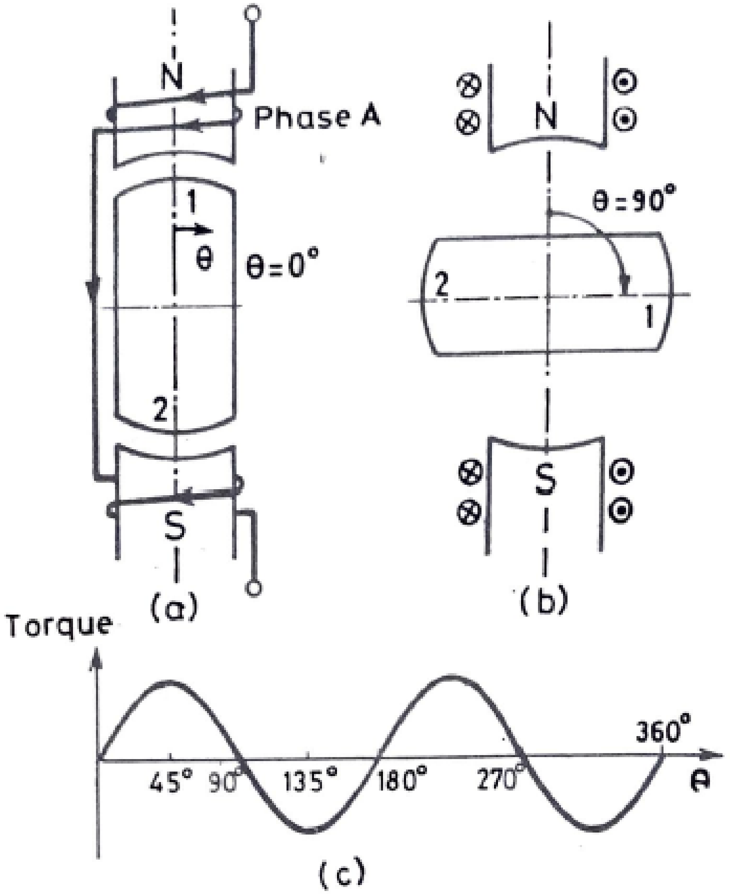
. Two-phase, 4/6-pole hybrid stepping motor (*a*) stator phase winding A excited (*b*) phase winding B excited and (*c*) phase winding A excited in the reversed direction.

Characteristics of Stepper Motor

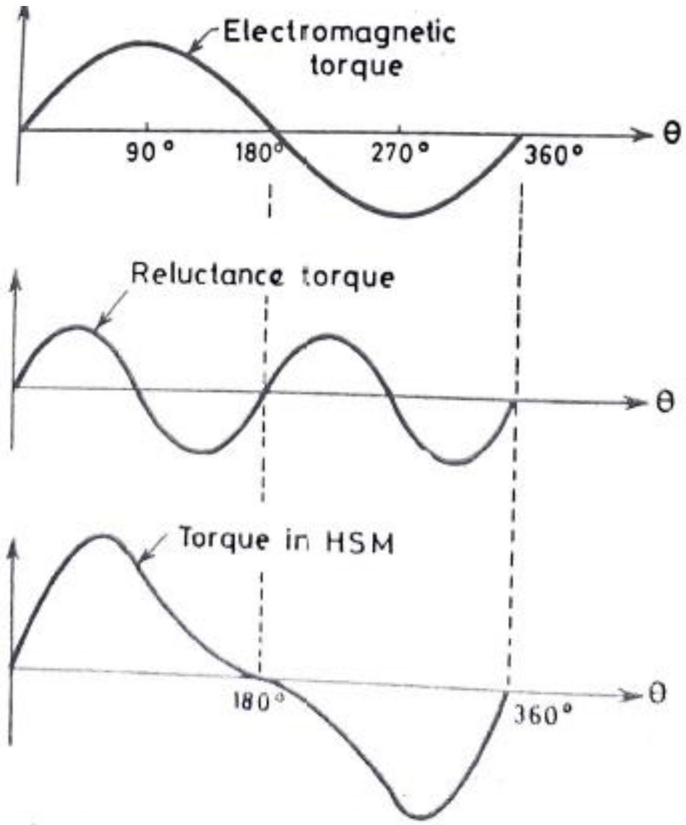
A) Torque-Displacement Characteristic



Pertaining to PM stepping motor (*a*) no torque (*b*) CCW torque (*c*) CW torque (*d*) torque-displacement characteristic.



. Pertaining to a VR stepping motor (*a*) no torque (*b*) no torque and (*c*) torque-displacement characteristic.



Pertaining to torque in a hybrid stepping motor

Some terms applicable to stepping motors are as under :

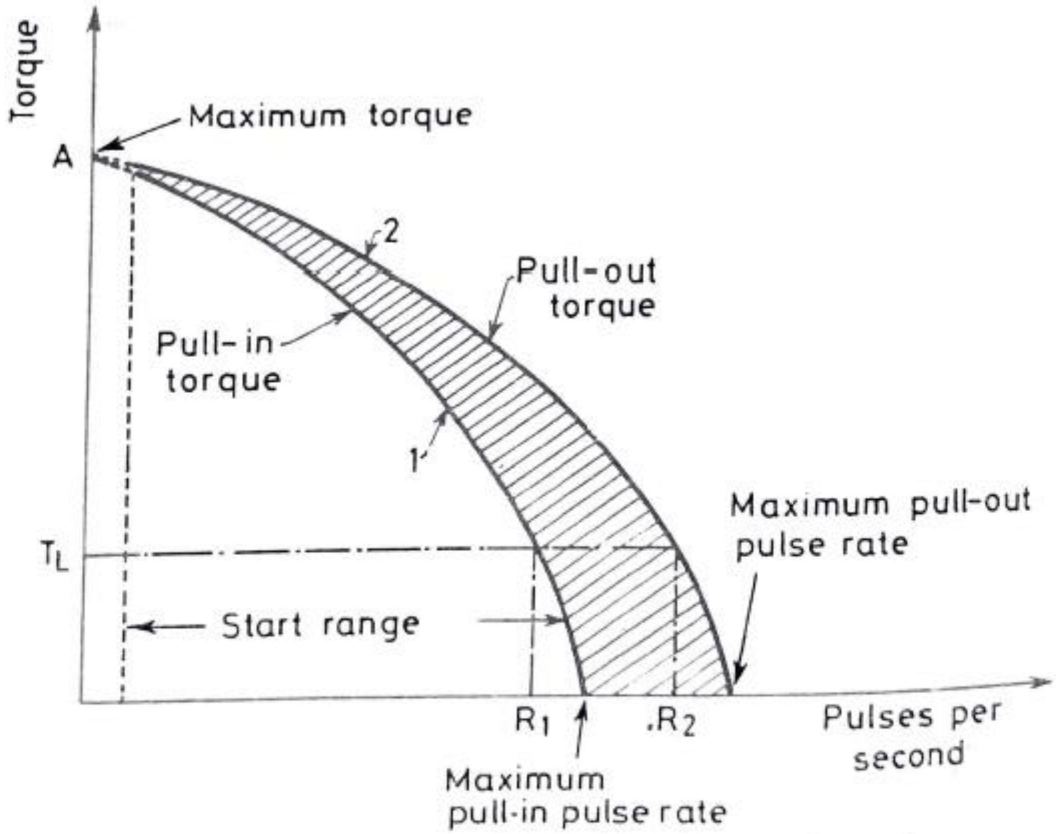
(*i*) *Step angle* is the angle through which shaft rotates in response to one input pulse.

(*ii*) *Single-step resolution* is inversely proportional to step angle. Smaller the step angle, greater the number of steps per revolution and therefore higher the single-step resolution.

(*iii*) At standstill, the excited motor opposes the rotor rotation due to load torque. Holding torque is a term introduced for the measure of this opposing torque. Thus, *holding torque* is defined as the maximum load torque that can be applied to the shaft of an *excited motor* without causing continuous rotation.

(*iv*) In case the motor is unexcited, the permanent magnet and hybrid stepping motors are able to develop a torque restricting the rotor rotation. The term detent torque takes this into consideration. Thus, *detent torque* is defined as the maximum load torque that can be applied to the shaft of an *unexcited motor* without causing continuous rotation.

B) Torque pulse rate characteristic



- With increase in stepping rate, the rotor gets less time in driving the load from one position to the next, the driving torque is therefore decreased

- **Pull-in torque:** It shows the maximum stepping rate for the various values of the load torque at which the motor can start, synchronize, stop or reverse.

- **Pull-out torque characteristics**: It shows the maximum stepping rate of the motor where it can run for the various values of load torque. But it cannot start, stop or reverse at this rate.

- The area between curves 1 and 2 represents the various torque values, the range of stepping rate, which the motors follow without losing the synchronism when it has already been started and synchronized. This is known as **Slew Range**.

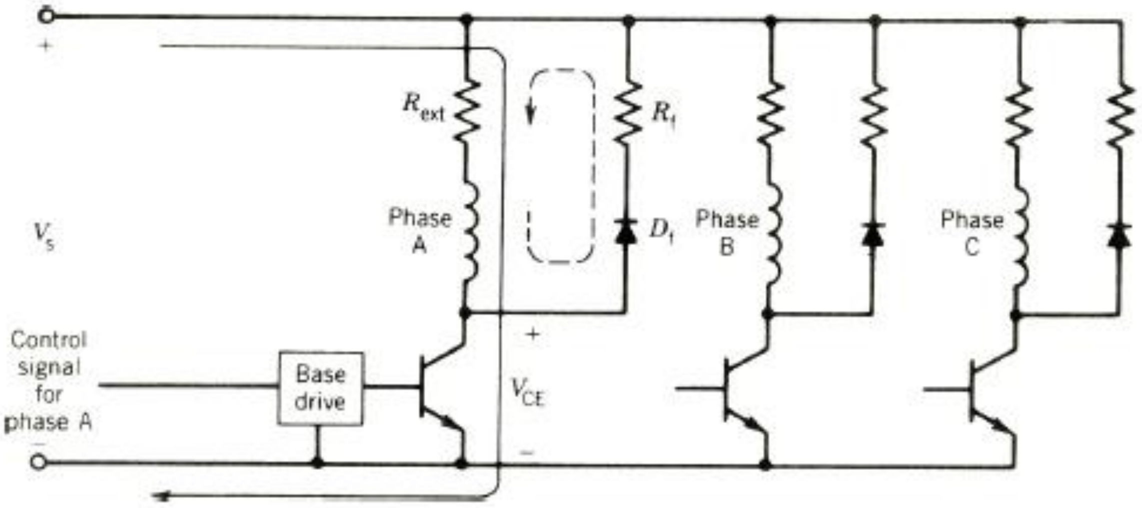
DRIVE CIRCUITS

For optimum torque output from a stepping motor, the drive circuits should possess the following features :

(*i*) During turn-on, the current in the phase winding should rise from zero to the desired level in a short duration.

(*ii*) During turn-off, the current in the phase winding should be forced to decay to zero in the shortest possible time.

A) Unipolar Drive Circuit

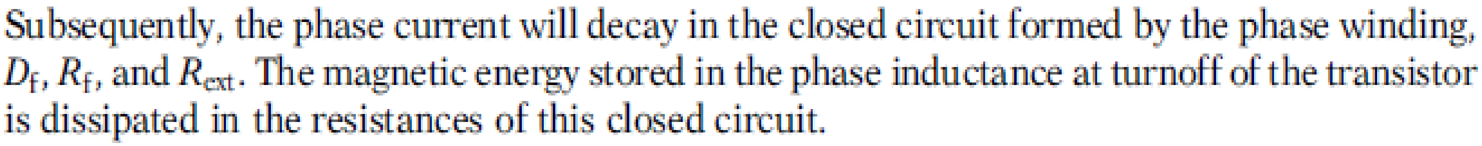


- A phase winding is excited by applying a control signal to the base of the transistor (a sufficiently high base current is passed through the base of the transistor)
- The dc supply voltage Vs is chosen so that it produces the rated current I in the winding.

$$V_s = I(R_w + R_{ext})$$

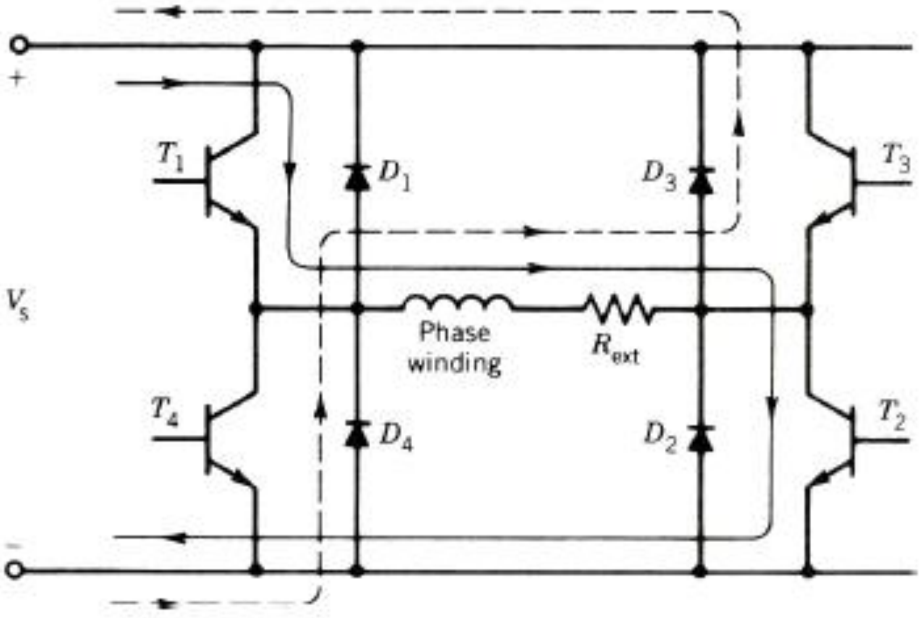
- The maximum voltage across the transistor occurs at the instant of switch off, and is

$$V_{CE(max)} = V_s + IR_f$$



Subsequently, the phase current will decay in the closed circuit formed by the phase winding, D_f, R_f, and R_{ext}. The magnetic energy stored in the phase inductance at turnoff of the transistor is dissipated in the resistances of this closed circuit.

B) Bipolar Drive Circuit

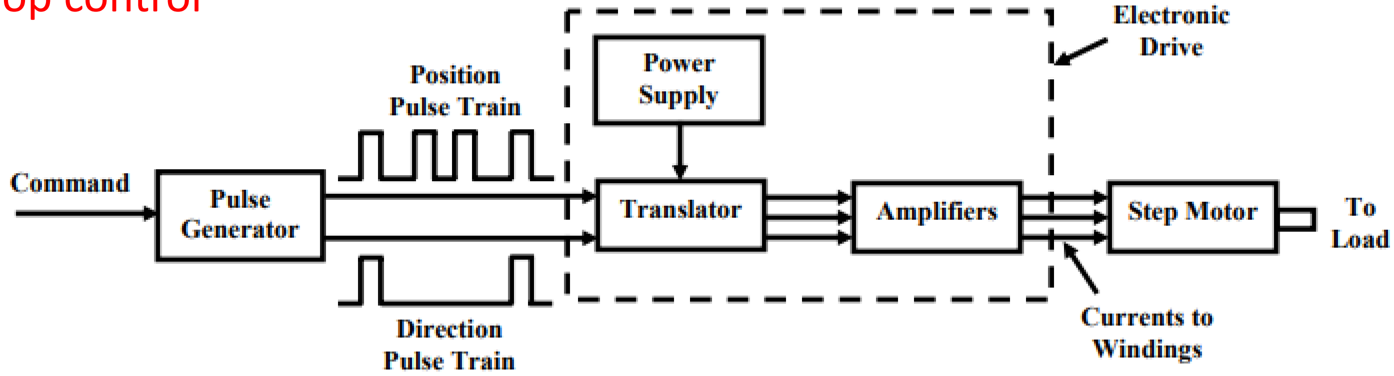


One phase of a bipolar drive circuit

- Bipolar drive circuit suitable for a permanent magnet or hybrid-type stepper motor.
- The transistors are switched in pairs according to the current polarity required for the phase winding
- The four diodes D1 to D4 connected in antiparallel with the switching transistors provide the paths for the freewheeling currents
- Note that when current flows through D3 and D4 to the dc supply, some of the energy stored in the phase winding inductance at turnoff (of the transistors) is returned to the supply. This **improves the overall system efficiency**, and is a significant advantage of the bipolar drive circuit over the unipolar drive circuit.
- Bipolar drive circuits require more switching devices and are therefore more expensive than unipolar drive circuits.
- Note that the freewheeling currents in the bipolar drive circuit decay more rapidly than in the unipolar drive circuit, because the dc supply opposes them. Consequently, no additional freewheeling resistance is necessary in the bipolar drive circuit.

Control of Step Motors

A) Open-loop control



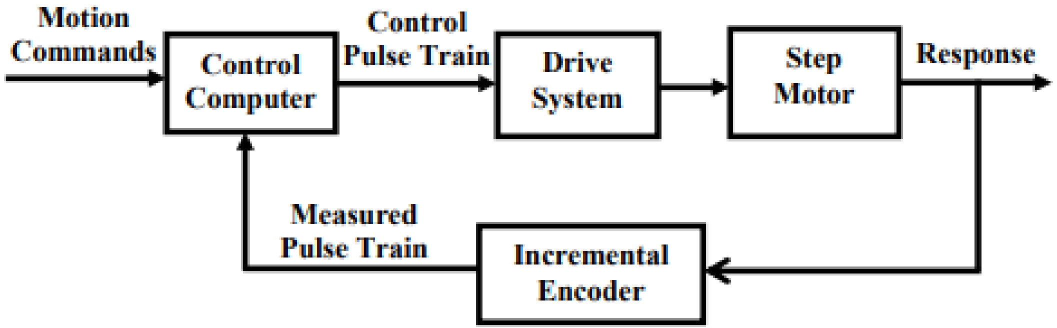
Open-loop control of a step motor

- The command to the pulse generator sets the number of steps for rotation and direction of rotation
- The Translator is a simple logical device and distributes the position pulse train to the different phases
- The amplifier block amplifies this signal and drives current in the corresponding winding.

- The direction of rotation can also be reversed by sending a direction pulse train to the translator. After receiving a directional pulse the step motor reverses the direction of rotation.

- The **major disadvantage** of the open loop scheme is that in case of a missed pulse, there is no way to detect it and correct the switching sequence. A missed pulse may be due to malfunctioning of the driver circuit or the pulse generator. This may give rise to erratic behaviour of the rotor.

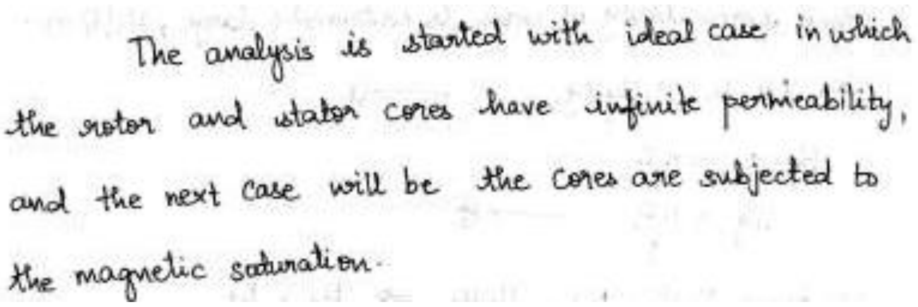
B) Closed-loop control



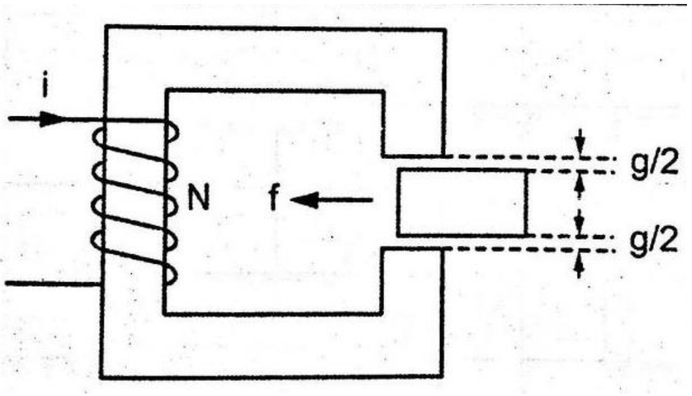
Feedback control of a step motor

The closed loop arrangement has the advantage over open loop control, since it does not allow any pulse to be missed and a pulse is send to the driving circuit after making sure that the motor has rotated in the proper direction by the earlier pulse sent. In order to implement this, we need a **feedback mechanism that will detect the rotation in every step and send the information back to the controller**. The incremental encoder here is a digital transducer used for measuring the angular displacement.

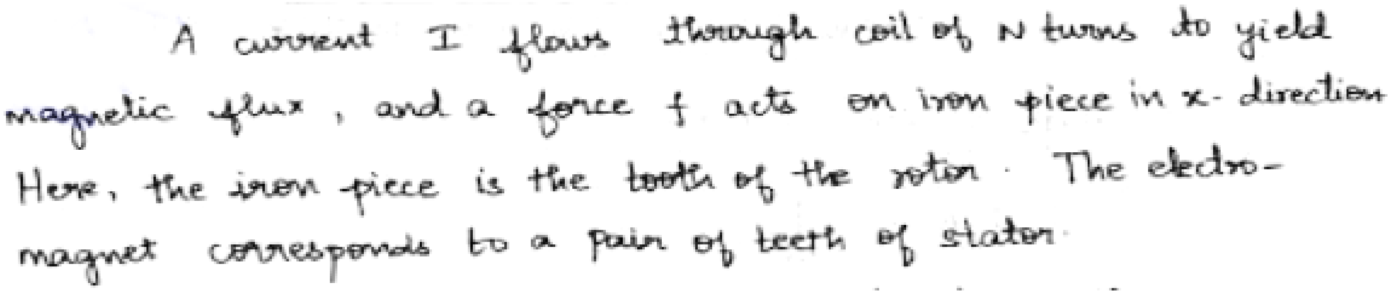
Torque/Force equation of Stepper Motor



The analysis is started with ideal case in which the rotor and stator cores have infinite permeability, and the next case will be the cores are subjected to the magnetic saturation.



Case I: Core permeability is infinite



A current I flows through coil of N turns to yield magnetic flux, and a force f acts on iron piece in x- direction Here, the iron piece is the tooth of the rotor. The electro-magnet corresponds to a pair of teeth of stator.

Let us consider, B_g - Magnetic flux density in the air gaps.

According to ampere's law,

$$\oint H \cdot dl = nI \quad \text{———} \; ①$$

$$\oint H \cdot dl = H_g\left(g/2\right) + H_g\left(g/2\right) + H_i\left(l\right) \quad \text{———} \; ②$$

$$= H_g(g) + H_i\left(l\right)$$

where, H_g - Magnetic field intensity in airgap.

H_i - Magnetic field intensity in the cores.

l - total magnetic path in the cores.

when permeability of cores is extremely large, $H_i(l) = 0$.

$$\text{So, } \oint H \cdot dl = H_g(g) \quad \text{———} \; ③$$

$$H_g \cdot g = nI$$

$$H_g = \frac{nI}{g} \quad \text{———} \; ④$$

We know that, $B_g = \mu_0 H_g \Rightarrow H_g = \frac{B_g}{\mu_0}$

Substitute H_g in ④,

$$B_g = \frac{\mu_0 n I}{g} \quad \text{——} ⑤$$

where, μ_0 - permeability in the gap length.

Let ω - transverse length of iron piece.

x - distance by which ~~rotor~~ tooth & iron piece overlap

Overlapped area $= x \times \omega$.

$$B_g = \phi/A = \frac{\phi}{x\omega}$$

$$\phi = \frac{\mu_0 n I x \omega}{g} \quad \text{——} ⑥$$

Flux linkages, $\psi = n\phi$ — ⑦

Substitute ⑥ in ⑦,

$$\psi = \frac{x\omega\mu_0 n^2 I}{g} \qquad \text{— ⑧}$$

Let us assume that there is an incremental displacement Δx at time Δt,

$$\Delta\psi = \frac{\omega\mu_0 n^2 I\Delta x}{g} \qquad \text{— ⑨}$$

$$\text{Emf, } e = -\frac{\Delta\psi}{\Delta t} = -\frac{\omega\mu_0 n^2 I}{g}\cdot\frac{\Delta x}{\Delta t} \qquad \text{— ⑩}$$

$$\text{Workdone, } \Delta P_i = I|e|\Delta t$$

$$= \frac{\omega\mu_0 n^2 I^2}{g}\,\frac{\Delta x}{\cancel{\Delta t}}\cdot\cancel{\Delta t}$$

$$\Delta P_i = \frac{\omega\mu_0 n^2 I^2}{g}\Delta x \qquad \text{— ⑪}$$

Coil resistance is zero.

$$\Delta P_i = \frac{B_g^2 \cdot g \omega \Delta x}{\mu_0} \quad \text{——} \quad ⑫$$

Work done by the source is converted partly to mechanical work and the rest in increasing the magnetic field energy in the gaps.

$$\Delta W_m = 1/2 \, \Delta P_i$$

$$= 1/2 \frac{B_g^2}{\mu_0} \cdot g \cdot \omega \Delta x \quad \text{——} \quad ⑬$$

ΔP_i is converted into magnetic field energy and other half of ΔP_i is converted into the mechanical work. Since the mechanical work is the force 'f' multiplied by the displacement Δx.

Comparing ⑬ & ⑭,

$$f = \frac{1}{2} \frac{B_g^2}{\mu_0} g w \qquad \text{—} \; \textcircled{15}$$

Substitute ⑮ to B_g,

$$f = \frac{1}{2} \frac{\mu_0^2 n^2 I^2}{g^2 \mu_0} g w$$

$$f = \frac{1}{2} \frac{w \mu_0 n^2 I^2}{g} \qquad \text{—} \; \textcircled{16}$$

Magnetic energy $W_m = f x$

$$W_m = \frac{1}{2} \frac{B_g^2}{\mu_0} g x w$$

$$W_m = \frac{1}{2} \frac{\mu_0 n^2 I^2}{g} x w \qquad \text{—} \; \textcircled{17}$$

Compare ⑮ & ⑰,

$$f = \left[\frac{\partial W_m}{\partial x}\right]_{I = \text{constant}}$$ (in rigorous form).

$$f = \left[\frac{d W_m}{d x}\right]_{I = \text{constant}}$$

$$f = -\left[\frac{\partial W_m}{\partial x}\right]_{\phi = \text{constant}}$$ (coil resistance is not zero)

Case (ii) - Constant permeabilities :-

In the infinitely permeable cores, the magnetic field appears only in the gaps. When cores are of finite permeability, the magnetic energy appears also in the cores and spaces other than the gaps.

$$\psi = LI \quad \text{——} \; ⑱$$

where ψ = flux linkages

L = coil inductance

The magetic energy,

$$W_m = \frac{1}{2} LI^2$$

Emf, $e = -\dfrac{\Delta \psi}{\Delta t} = \dfrac{-\Delta (LI)}{\Delta t}$

$$e = -I \frac{\Delta L}{\Delta t} \quad \text{——} \; ⑲$$

Work ΔP_i can be expressed as,

$$\Delta P_i = I|e|\Delta t$$

$$= I\left|-I\frac{\Delta L}{\Delta t}\right|\Delta t$$

$$\Delta P_i = I^2\Delta L \qquad \text{—— ⑳}$$

Increase in the magnetic energy ΔW_m,

$$\Delta W_m = \frac{1}{2}\Delta P_i$$

$$\Delta W_m = \frac{1}{2} I^2\Delta L \qquad \text{—— ㉑}$$

Mechanical work, $\Delta P_o = f\Delta x$ —— ㉒

comparing ㉑ & ㉒,

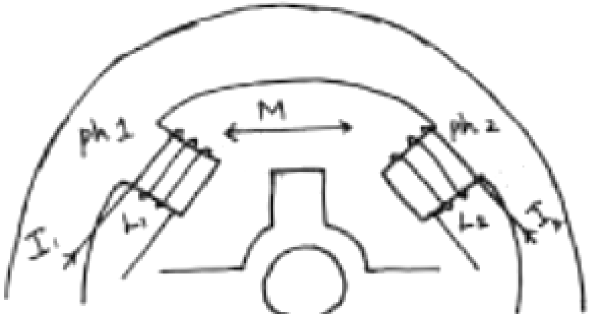
$$f\Delta x = \frac{1}{2} I^2\Delta L$$

$$f = \frac{1}{2} I^2 \frac{\Delta L}{\Delta x} \qquad \text{—— ㉓}$$

In the above procedure, it was assumed that the coil resistance was zero and the power supply was a current source.

Effects of mutual induction:-

In a drive scheme, other than one phase on drive, it is desirable that the mutual inductance is minimum. When mutual inductance is not negligible, the torque in terms of linear theory is derived by the following procedure.



mutual Induction,

$$e = -I\frac{\Delta L}{\Delta t}$$

$$e_1 = -I_1\frac{\Delta L_1}{\Delta t} - I_2\frac{\Delta M}{\Delta t}$$

$$e_2 = -I_2\frac{\Delta L_2}{\Delta t} - I_1\frac{\Delta M}{\Delta t}$$

where, e_1 = induced voltage of phase 1.

e_2 = induced voltage of phase 2

f_1 = Inductance of phase 1.

f_2 = Inductance of phase 2

M = mutual inductance between the two phases

The work done by the two power supplies during Δt,

$$\Delta P_i = -(e_1 I_1 + e_2 I_2)\,\Delta t$$

$$= -\left[-I_1^2\frac{\Delta L_1}{\Delta t} - I_1 I_2\frac{\Delta M}{\Delta t} - I_2^2\frac{\Delta L_2}{\Delta t} - I_1 I_2\frac{\Delta M}{\Delta t}\right]$$

$$\Delta P_i = I_1^2\Delta L_1 + I_2^2\Delta L_2 + 2I_1 I_2\Delta M$$ — ㉔

The increment of magnetic energy,

$$\Delta W_m = \frac{1}{2} \Delta P_i$$

$$= \frac{1}{2} \left[I_1^2 \Delta L_1 + I_2^2 \Delta L_2 \right] + I_1 I_2 \Delta M \quad \text{— ㉕}$$

Mechanical output,

$$\Delta P_o = T \Delta \theta \quad \text{— ㉖}$$

$$T \Delta \theta = \frac{1}{2} \left(I_1^2 \Delta L_1 + I_2^2 \Delta L_2 \right) + I_1 I_2 \Delta M$$

Torque is expressed as,

$$\boxed{T = \frac{1}{2} I_1^2 \frac{\partial L_1}{\partial \theta} + \frac{1}{2} I_2^2 \frac{\partial L_2}{\partial \theta} + I_1 I_2 \frac{\partial M}{\partial \theta}}$$

Unit 3

Switched Reluctance Motors

VRMs are now finding increasing use in adjustable-speed drives because of the following features :

1. This motor is simple in construction with no winding on rotor and simple concentrated coils on stator.

2. It can run successfully at high speeds (about 2×10^5 rpm) because of no winding on rotor and rugged rotor construction.

3. Stator windings can be cooled easily and efficiently.

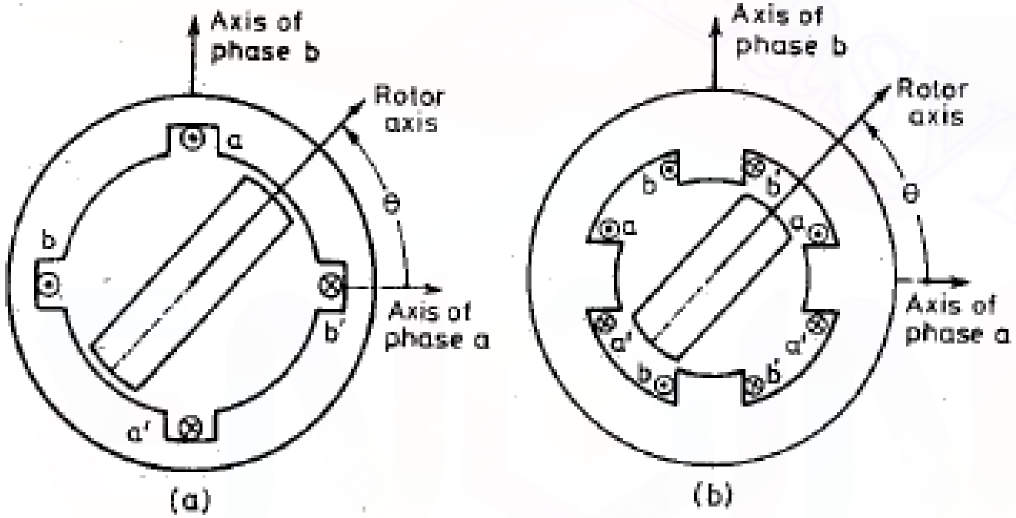
4. Because of effective stator cooling, the motor dimensions decrease for a given machine rating

5. As VRM can be operated from unidirectional drive circuits, cost of micro-and power-electronics is reduced.

6. VRM operates successfully, though at reduced output, even if one or more phases are out of circuit due to some fault.

7. As these motors can be manufactured with a large number of stator and rotor teeth, they give large torque per unit volume.

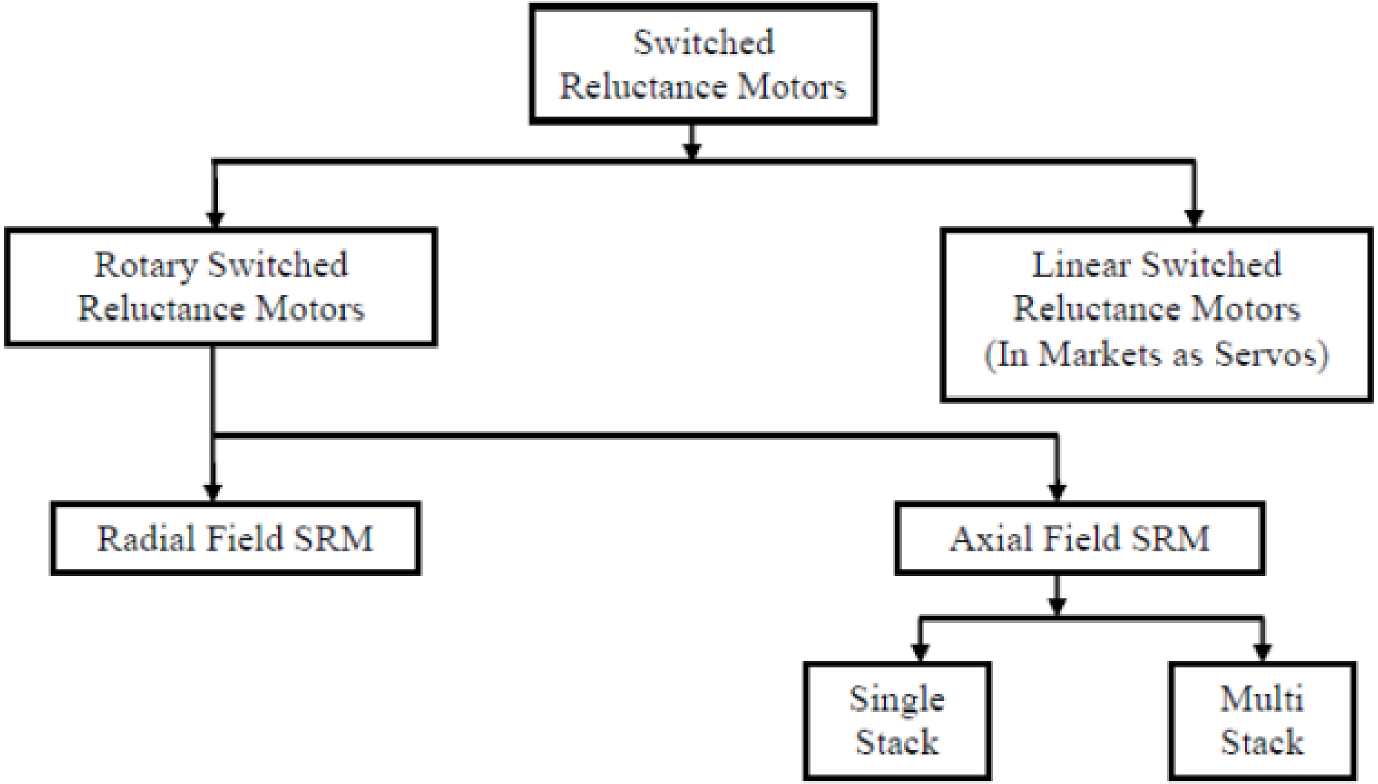
Constructional Features



Elementary two-phase VRMs (*a*) single salient and (*b*) doubly salient.

singly-salient VRM is shown to consist of a non-salient stator and a two-pole salient rotor. The rotor has no winding but the cylindrical stator has two-phase winding as shown. In an actual machine, the number of phase windings may be more than two.

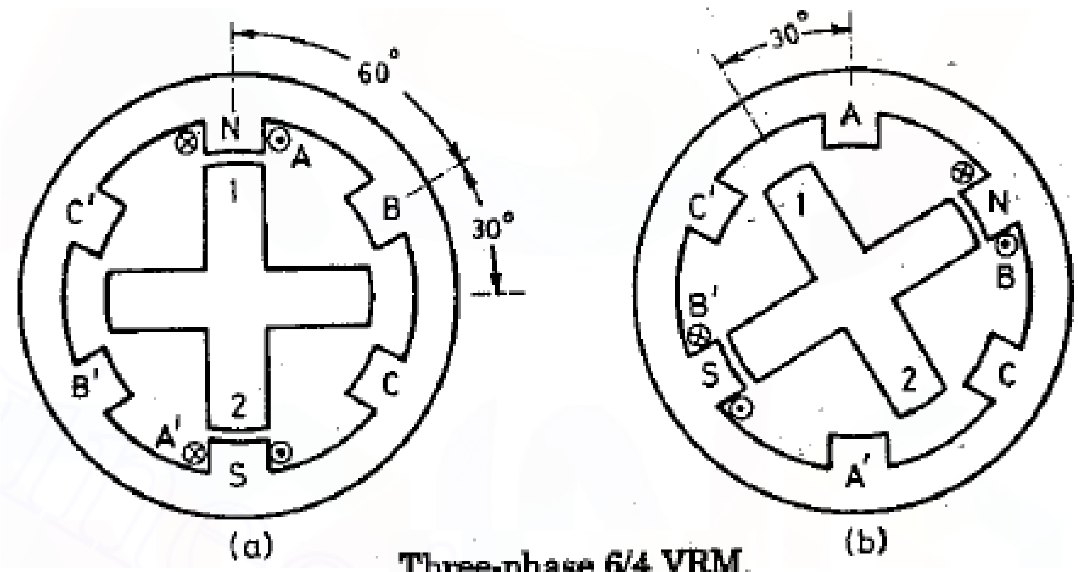
TYPES OF SRM


Switched Reluctance Motors
Rotary Switched Reluctance Motors
Linear Switched Reluctance Motors (In Markets as Servos)
Radial Field SRM
Axial Field SRM
Single Stack
Multi Stack

S. No	SRM	Stepper motor
1	SRM is designed for continuous rotation	Stepper motor is designed to rotate in step by step rotation.
2	SRM requires a rotor-position sensor	It does not require rotor-position sensor.

S. No	Switched Reluctance Motor (SRM)	Variable Reluctance stepper motor
1	It's normally operated with shaft position feedback to synchronize with the rotor position sensor.	It's usually fed by a square wave of phase current without rotor position feedback.
2	SRM rotates continuously	It rotates in steps
3	The SRM usually operates at high speeds	The stepper motor is usually designed as a torque motor with a limited speed range.
4	No half step operation and micro stepping are possible.	It is capable of half step operation and micro stepping.

Principle of operation



Three-phase 6/4 VRM.

For CCW rotation sequence of excitation is ABC.

Torque Developed Expression

→ Torque is Produced due to Variable reluctance Principle.

The flux linkage (λ) due to excitation of winding :

$$\lambda = Li \longrightarrow ①$$

According to Faraday's law of electromagnetic induction, emf (e) due to change in flux linkage,

$$e = (-)\frac{d\lambda}{dt} \longrightarrow ②$$

Substituting λ from eqn ① in ②,

$$② \Rightarrow \quad e = (-)\frac{\partial(Li)}{\partial t} = -L\frac{\partial i}{\partial t} - i\frac{\partial L}{\partial t}$$

Multiply (x) & Divide (÷) by $\partial\theta$ on the second term,

$$e = (-)L\frac{\partial i}{\partial t} - i\frac{\partial L}{\partial \theta} \times \left(\frac{\partial\theta}{\partial t}\right)$$

$$= -L\frac{\partial i}{\partial t} - i\frac{\partial L}{\partial \theta}(\omega)$$

$$\left[\because \theta = \omega t, \quad \therefore \omega = \frac{\partial \theta}{\partial t}\right]$$

$$e = (-)\left[L\frac{\partial i}{\partial t} + i\frac{\partial L}{\partial \theta}(\omega)\right] \longrightarrow \textcircled{3}$$

Considering only magnitude,

$$\therefore \; e = L\frac{\partial i}{\partial t} + i\frac{\partial L}{\partial \theta}(\omega) \longrightarrow \textcircled{4}$$

Energy stored in the magnetic field,

$$W_e = \frac{1}{2}Li^2 \longrightarrow \textcircled{5}$$

Mechanical Power developed = Power input to motor (-) Power due to variation in stored energy

Power input to the motor $= ei \rightarrow ⑥$

$$= \left[L \frac{\partial i}{\partial t} + i(\omega) \frac{\partial L}{\partial \theta} \right] i$$

$$= Li \frac{\partial i}{\partial t} + i^2 \omega \frac{\partial L}{\partial \theta} \rightarrow ⑦$$

Power due to Variation in Stored energy } $= \dfrac{dW_e}{dt}$

$$= \frac{d}{dt} \left[\frac{1}{2} L i^2 \right]$$

$$= \frac{1}{2} \left[2Li \frac{\partial i}{\partial t} + i^2 \frac{\partial L}{\partial t} \right]$$

$$= \frac{1}{2} \left[2Li \frac{\partial i}{\partial t} \right] + \frac{1}{2} \left[i^2 \frac{\partial L}{\partial t} \right]$$

$$= Li \frac{\partial i}{\partial t} + \frac{i^2}{2} \frac{\partial L}{\partial t}$$

$$= Li \frac{\partial i}{\partial t} + \frac{i^2}{2} \frac{\partial L}{\partial \theta}\left(\frac{\partial \theta}{\partial t}\right)$$

$$= Li \frac{\partial i}{\partial t} + \frac{i^2}{2}(\omega)\frac{\partial L}{\partial \theta} \longrightarrow ⑧$$

Subtracting ⑧ from ⑦,

~~⑧~~ ⑦ - ⑧ ⇒

Mechanical Power developed $= P_m = \cancel{Li \frac{\partial i}{\partial t}} + i^2 \omega \frac{\partial L}{\partial \theta}$

$$- \cancel{Li \frac{\partial i}{\partial t}} - \frac{i^2}{2}(\omega)\frac{\partial L}{\partial \theta}$$

$$\therefore P_m = \frac{1}{2}\left[i^2 \omega \frac{\partial L}{\partial \theta}\right] \longrightarrow ⑨$$

In general,

$$P_m = \frac{2\pi N T}{60} = \left[\frac{2\pi N}{60}\right] T = \omega T$$

$$\therefore \; P_m = \omega T \longrightarrow ⑩$$

Where,

$\omega \rightarrow$ angular velocity,

$T \rightarrow$ Electromagnetic torque developed.

From ⑩ ⇒

$$T = \frac{P_m}{\omega} \longrightarrow ⑪$$

Subtituting ⑨ in ⑪,

$$T = \frac{\frac{1}{2}\left(i^2 \cancel{\omega} \frac{\partial L}{\partial \theta}\right)}{\cancel{\omega}}$$

$$\therefore \; \boxed{T = \frac{1}{2}\left[i^2 \frac{\partial L}{\partial \theta}\right]} \longrightarrow ⑫$$

Torque corresponds to motoring,

when $\frac{\partial L}{\partial \theta}$ is +ve.

Torque corresponds to generating

when $\frac{\partial L}{\partial \theta}$ is -ve.

As, $\underline{\underline{T \propto i^2}}$, it is independent of direction of current.

* If there is magnetic saturation equation ⑫ is invalid & the torque should be derived as the derivative of co-energy or field stored energy.

Torque Production

Under the assumption of magnetic linearity, the reluctance torque* developed in VRM is given by

$$T_e = \frac{1}{2} i^2 \frac{dL}{d\theta}$$

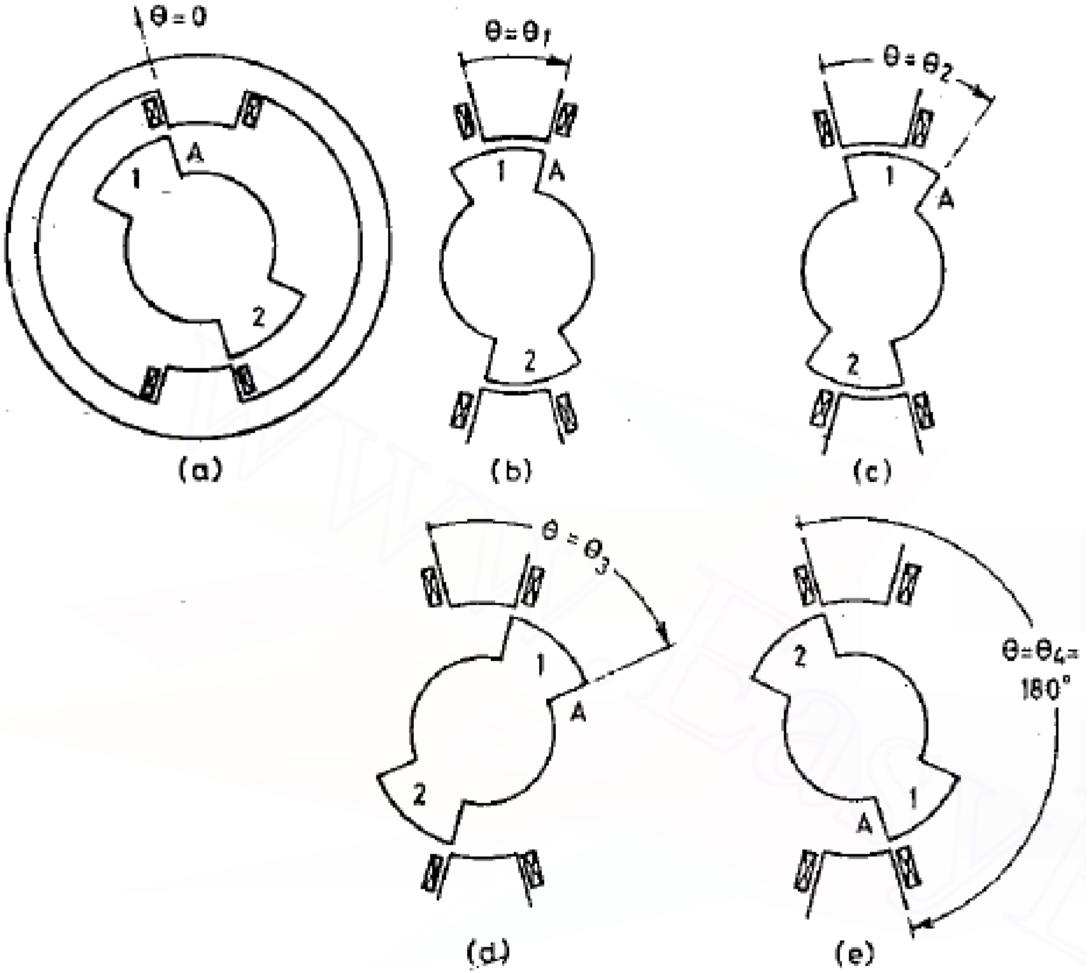
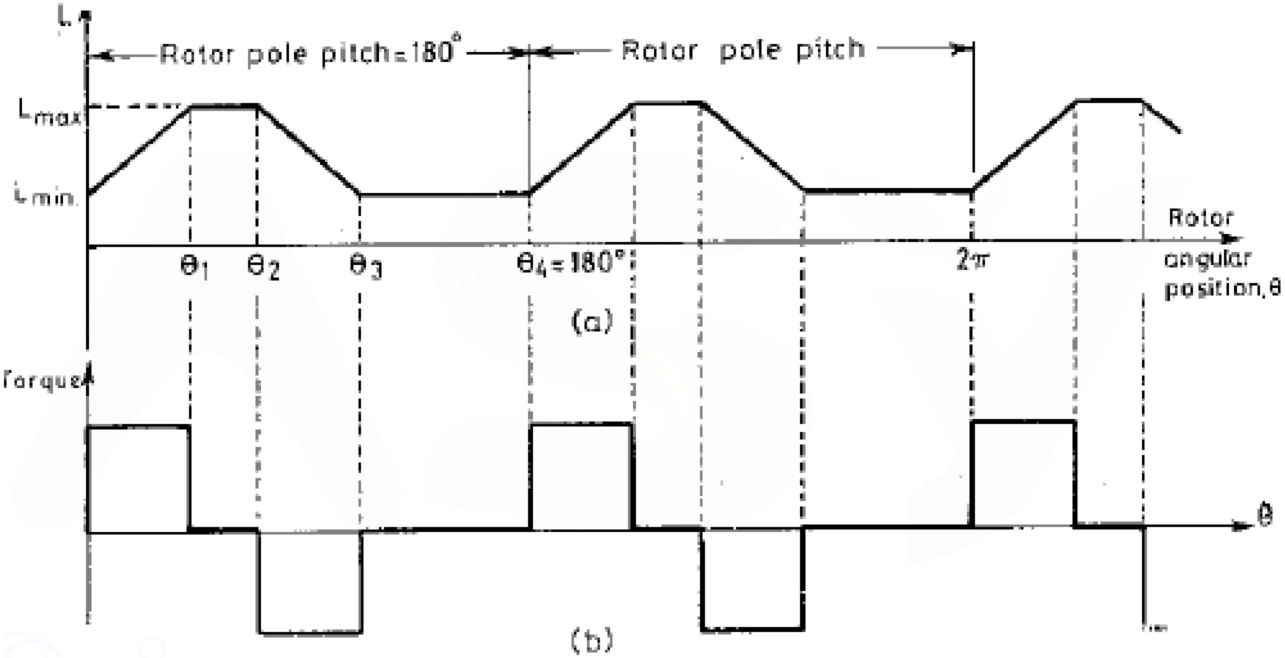


Fig. ~~9.23~~. A two-pole stator with two-pole rotor shown in different orientations for a VRM.



(*a*) Inductance variation of stator phase winding with respect to θ and (*b*) torque-angle characteristics.

Block diagram of SRM:

- DC power is given to power semiconductor switching circuitry.
- Output of switching circuit is given to phase windings of SRM.
- Shaft carries Rotor position sensor which gives information about the position of rotor.
- Controller gets the information from rotor and reference speed signal and turns ON and OFF the concerned power device in switching circuit.
- Current signal is also feedback to the controller to limit the current.

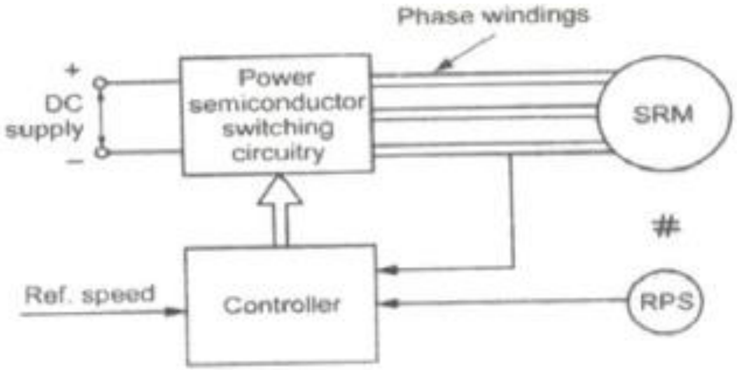
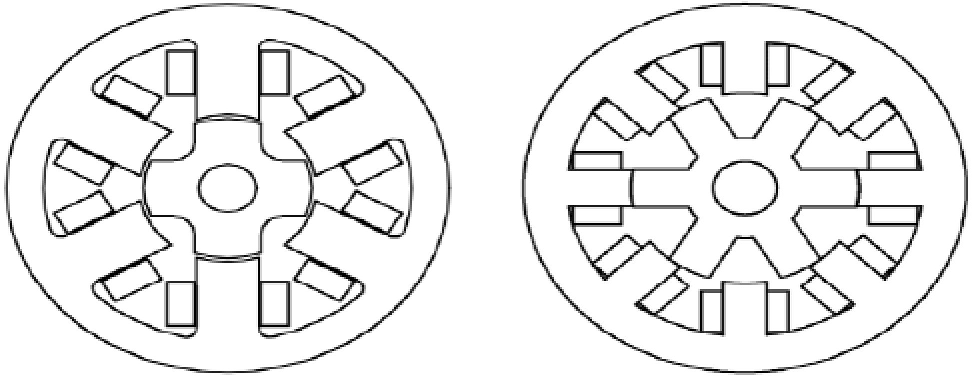


Fig. Block diagram of SRM

1. Rotary SRM

➢ ***Radial field SRM:***

  - Magnetic field perpendicular to axial length of machine (or) magnetic field along the radius of stator or rotor.



6/4 Pole 8/6 Pole

(a) Long flux path:

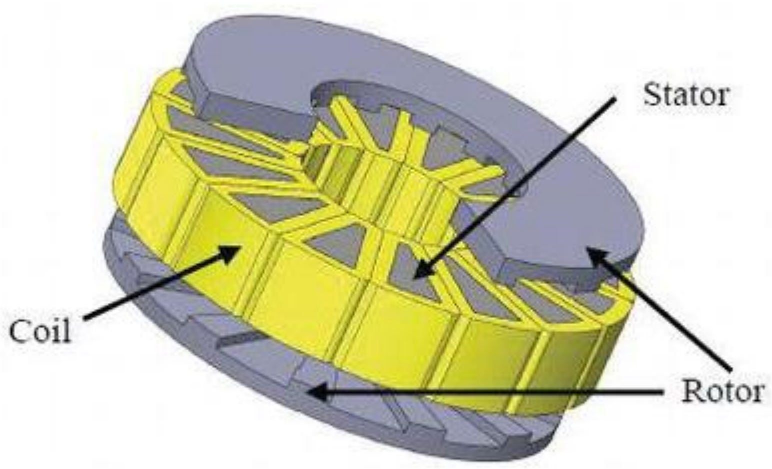
  - Diametrically opposite windings are in series to form a phase winding.

(b) Short flux path:

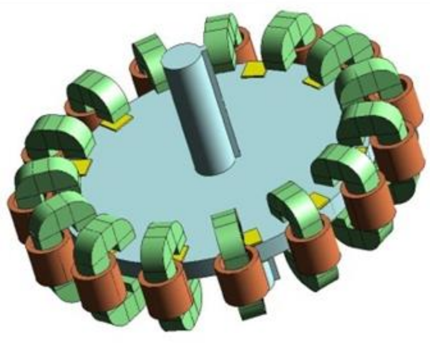
  - Adjacent pole windings are in series to form a phase windings.

Axial-flux switched reluctance motors

In these motors, the flux path is aligned with the motor axis, and they are used for cases where the motor length is of high importance and the motors with a small length and high torque are preferred for applications such as air conditioning fans and electric vehicles

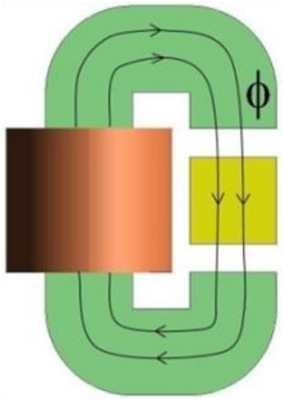


Axial field SRM: The magnetic field path is along the axial direction.



Whole motor

Rotor

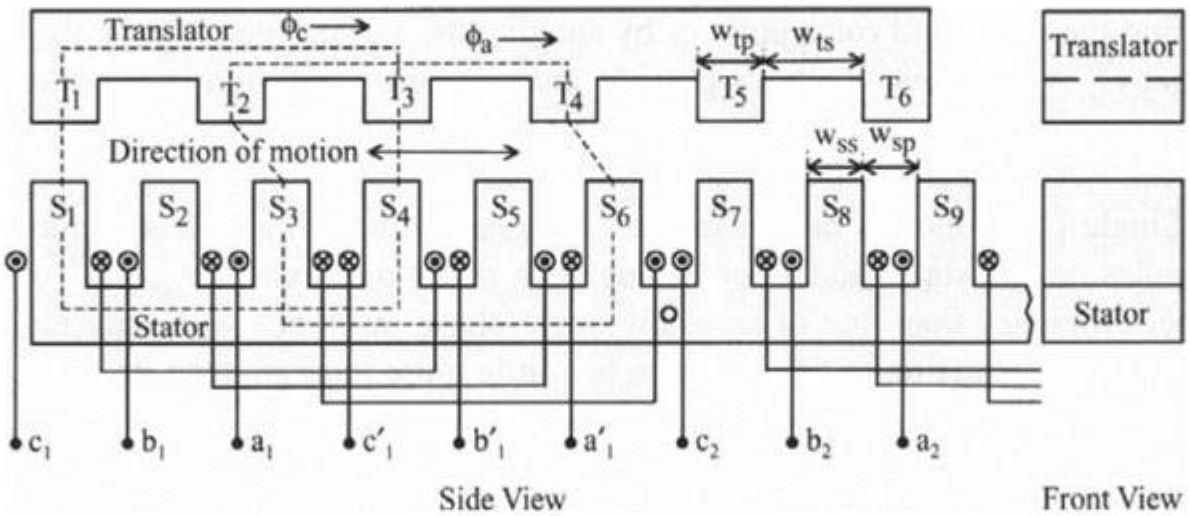


The short magnetic flux path

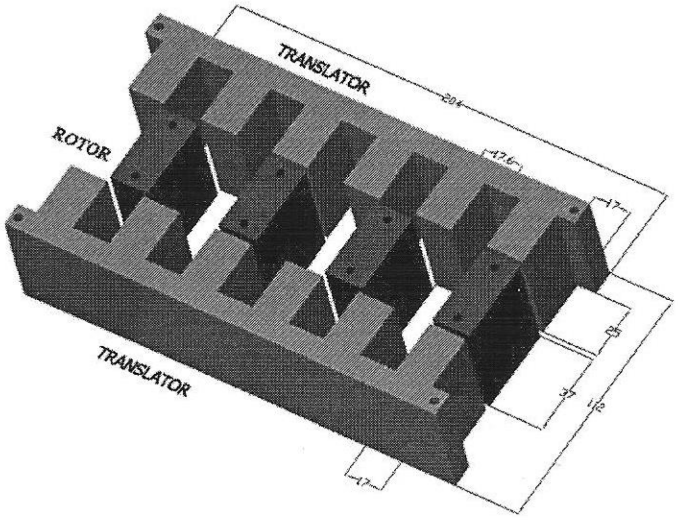
Linear-switched reluctance motors

- Linear-switched reluctance motors (LSRMs) are similar to conventional SRMs in their structure except that their rotor and stator are cut open taking a linear form

- One of the applications of these motors is in electric trains and subways



One side LSRM



Two sided LSRM with winding on the translator

Operating modes

Single-Pulse mode

PWM (or chopping) mode.

A) Single pulse mode $\xrightarrow{\text{also known as}}$ High speed mode

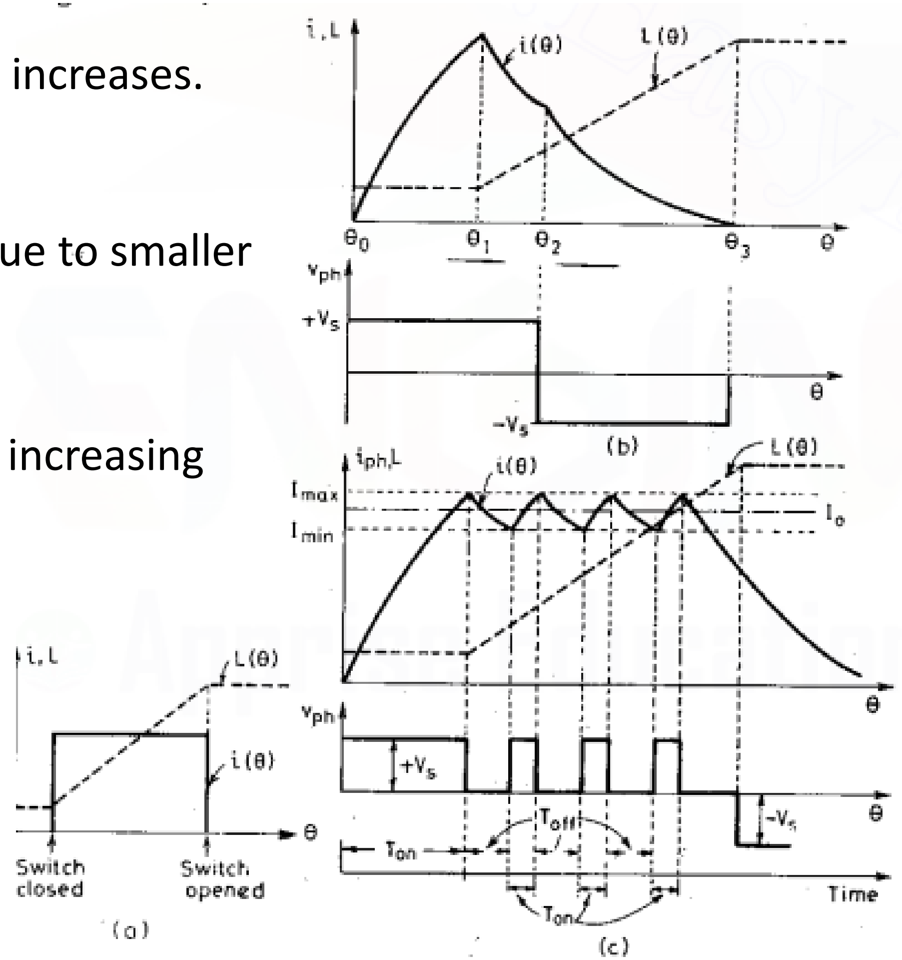
Current rise is within limits during the small time interval of each phase excitation.
It depends winding inductance and motional counter emf generated in stator winding

1. Input voltage Vs is applied before L increases.

2. Current is buildup to larger value due to smaller Inductance L, up to θ1.

3. After θ1, current decreases due to increasing L and back emf.

4. At θ2, negative voltage is applied And current falls to 0 at θ3.



Operation of VRM (*a*) ideal current waveform. Typical waveforms for phase current and phase voltage for (*b*) single-phase mode and (*c*) chopping mode.

Single-Pulse Mode

Ideal case

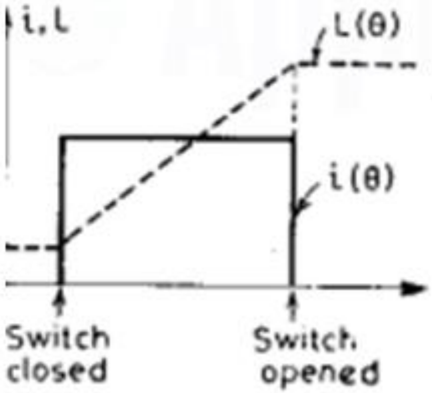
1 Switch ON (closed)

↓

current increase at once to a constant value

↓

Inductance increases to yield +ve Te.



(a)

2 Actual case (single pulse mode)

1 Input voltage V_s is switched ON and current $i(\theta)$ ↑ to higher value in short duration.

Reason: → $L(\theta)$ during the short duration is almost constant at minimum value.

→ Motional EMF is also very low. (E_m)

2 At $\theta = \theta_1$ (Torque producing zone starts)

$L(\theta)$ ↑ and E_m ↑ $\xrightarrow{\text{resulting}}$ $i(\theta)$ ↓ up to θ_2.

3 At $\theta = \theta_2$, $-V_s$ is applied

↓ resulting

$i(\theta)$ decay to zero at $\theta = \theta_3$

↓

$L(\theta)$ ↑ to max. value at θ_3

4 Positive torque is produced due to +ve $dL/d\theta$

5 Terms: $(\theta_2 - \theta_0)$ → Transistor/thyristor conduction angle

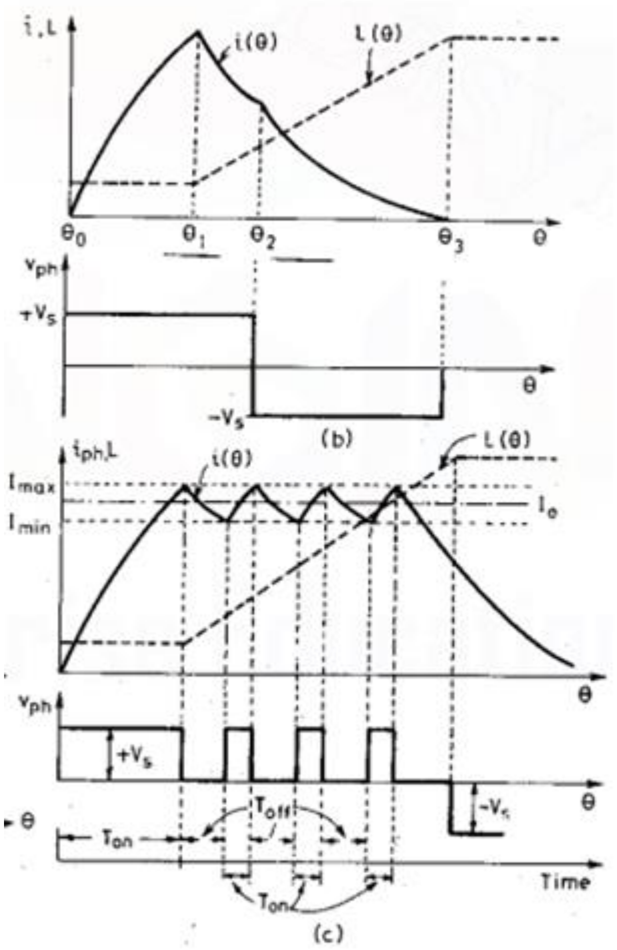
$(\theta_3 - \theta_2)$ → Diode conduction angle of inverter.

$(\theta_3 - \theta_0)$ → conduction angle (θ_c)

$(\theta_1 - \theta_0)$ → Angle of Advance

θ_0 → Switch-on angle

θ_2 → Switch-off angle



(b)

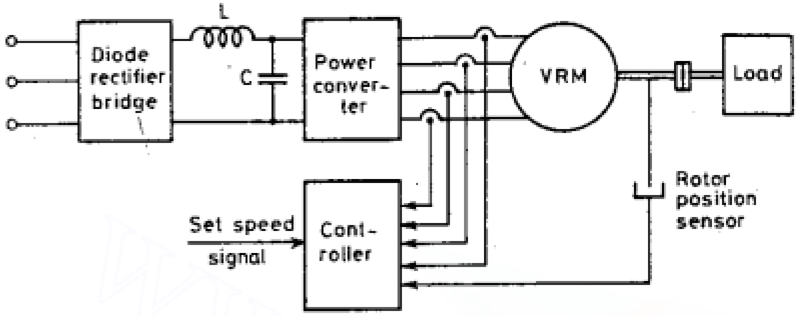
(c)

B) PWM (or chopping) pulse mode

also known as → Low speed mode

1. Initially, current in each phase is buildup during long period to higher value.

2. Current rise is kept within acceptable rating of motor and inverter components. Hence, current limiting device is used.



Block diagram for 4-phase VRM drive system.

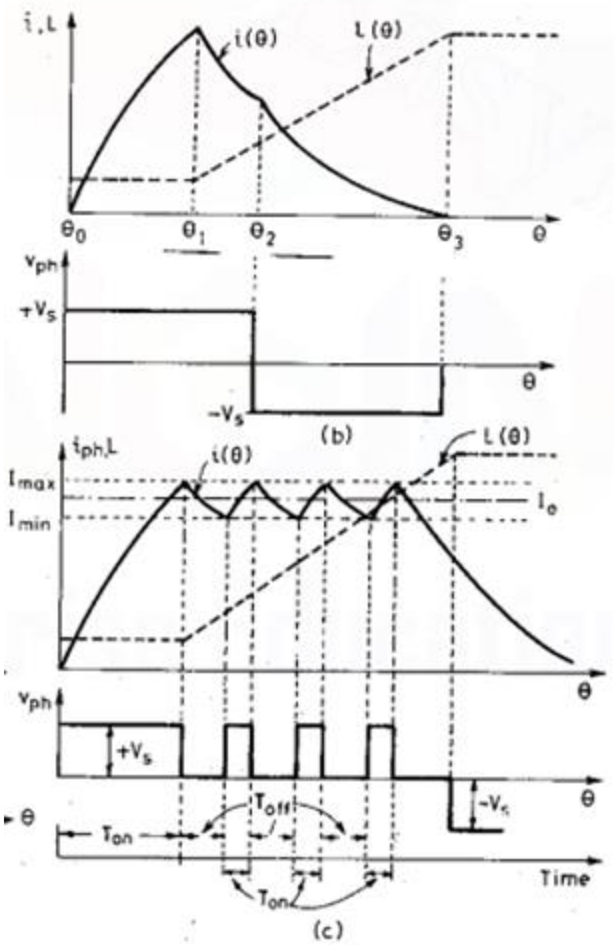
PWM (chopping) mode:

1 Phase winding is excited for long time ~~to~~ may attain prohibitively high current

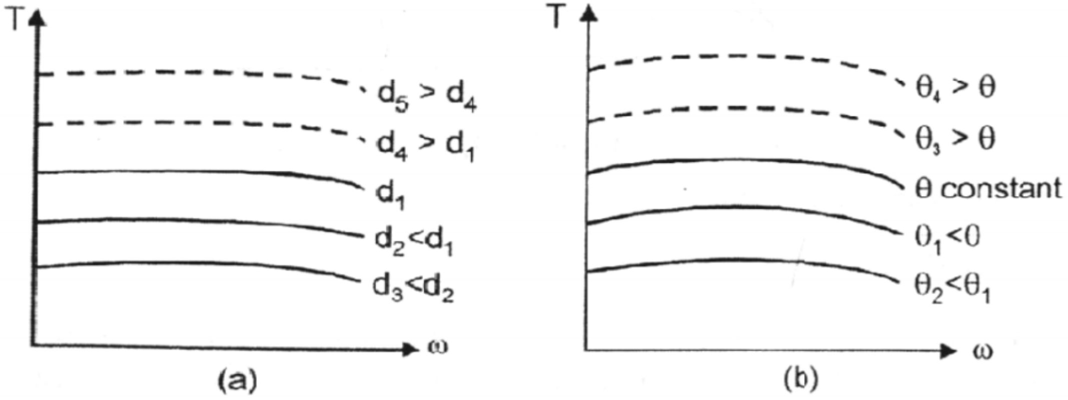
2 current limiting device is used to limit the current (not more than the rating of motor and inverter)

3 So, converter components are alternately switch ON and OFF to limit the current in limits : I_{min} to I_{max}. It is achieved in closed loop drive system.

4 upper and lower current limits may be shifted to change mean current i_o and hence torque and speed is controlled.



TORQUE-SPEED CHARACTERISTICS



θ = constant

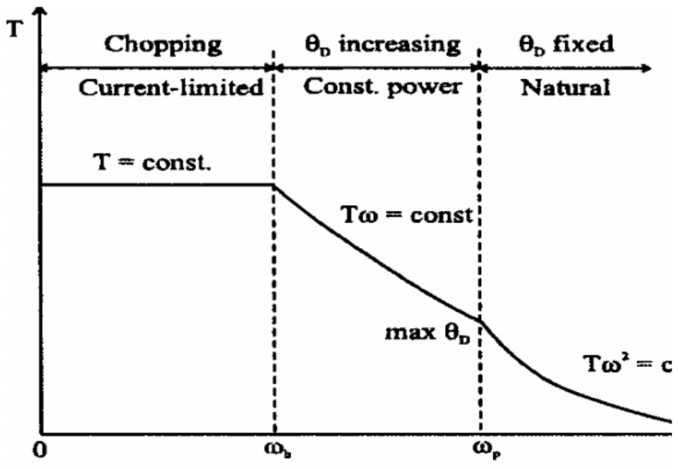
Duty cycle = constant

Torque developed (i.e.) average torque developed but SRM depends upon the current wave form of SRM phase winding. Current waveform depends upon the conduction period and chopping details. It also depends upon the speed.

- The first region is known as the constant torque region, where the supply of voltage and commutation angles is fixed and is located below the rated speed, ωb

- The second region identified as the constant power region, starts from the rated speed ωb. In this region applied voltage remains at its rated value and as the rated speed is increased, the maximum torque is sustained by advancing the turn on angle with a fixed commutation angle. In this region, the torque falls inversely to the motor speed, 1/ω.

- Finally, there is a practical limitation to advancing the turn on angle, which, lets say, begins at some speed called ωp. At this point, the third region starts and the torque decreases as 1/ ω2. This mode is known as the natural mode and is usually neglected for most motor technology.



. General torque-speed characteristics and regions of SRM

Torque Speed Capability Curve

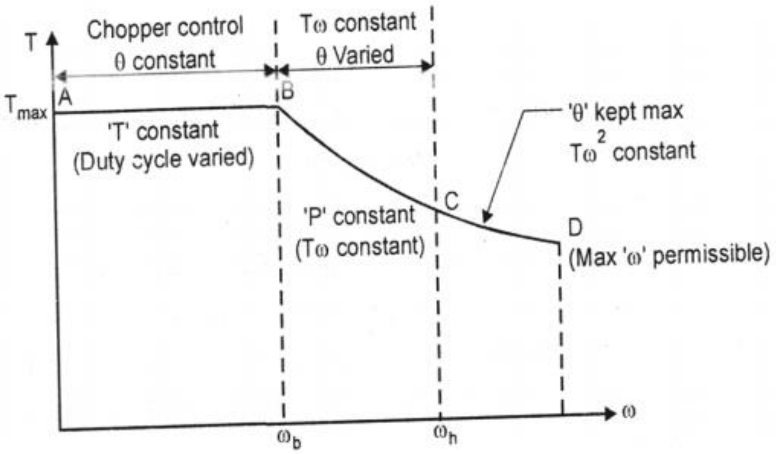


Fig. Torque-speed capability characteristics

- Maximum torque developed and maximum power transferred are restricted by mechanical subsystem design parameters.
- For a given conduction angle, T can be varied by the duty cycle of chopper.

In region AB:

- ✓ Maximum torque region.
- ✓ θ = constant, duty cycle = varied, T = constant
- ✓ At B –> conduction angle = θ, point with maximum speed and constant torque, $P = T\omega$.

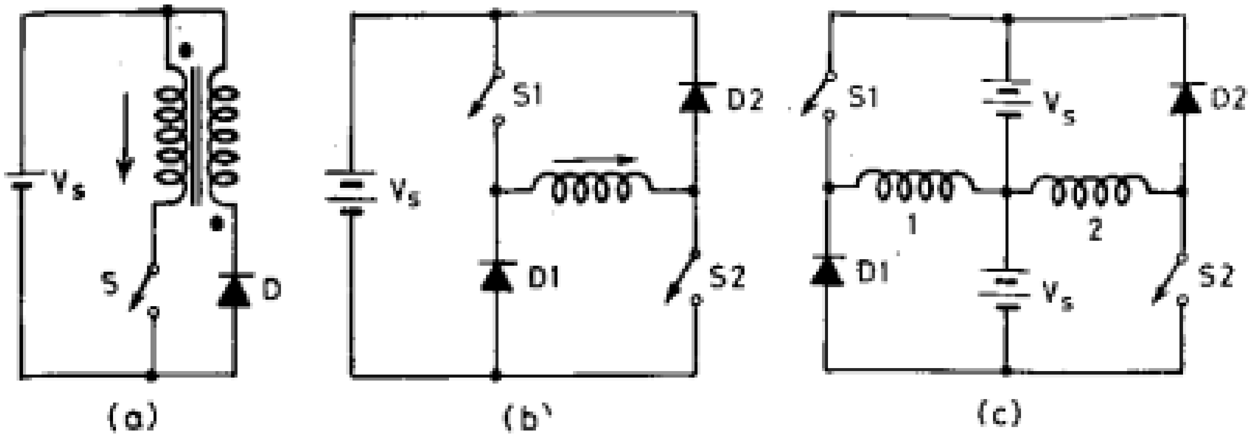
In region BC:

- ✓ Max permissible torque at each speed without exceeding max power transfer.
- ✓ $P = T\omega$ = constant, vary θ to its maximum value.
- ✓ At C -> max power, max θ, duty cycle = 1.

In region CD:

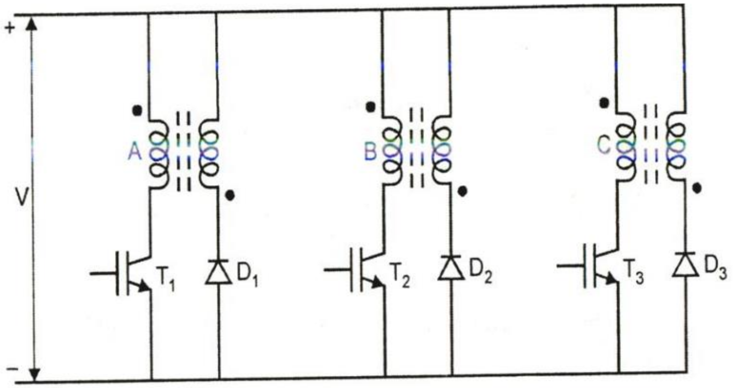
- ✓ $T\omega^2$ = constant, θ = max, duty cycle = max.
- ✓ At D –> ω = max
- ✓ Region ABCD -> operating region of SRM

Inverter drive circuits



Inverter drive circuit configurations for VRMs.

Phase winding using bifilar wires



- Each stator pole carries a coil using bifilar wires.
- When T_1 is turned ON, corresponding phase windings A gets energized.
- When T_1 is turned OFF, stored energy feed back to the dc source through A and diode D_1.
- Operation is similar for phase winding B and C also.

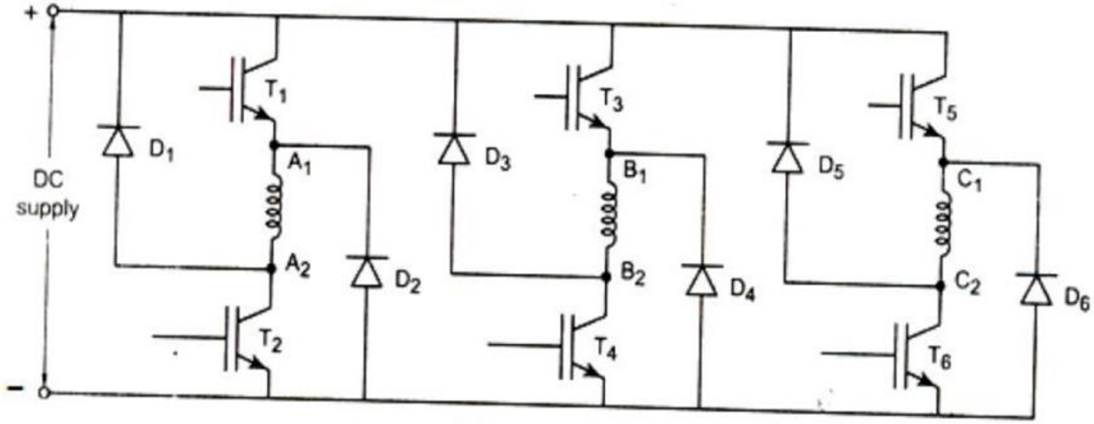
Advantages:

- ✓ Cost is low.
- ✓ Fast demagnetization.

Disadvantages:

- ✓ Presence of voltage spikes in bifilar windings.
- ✓ Poor utilization of copper.
- ✓ Copper losses are high.

Two Power Semiconductor Switching Devices and two diodes per phase



- Two power switches and two diodes per phase.
- To energize the phase windings A, the devices T_1 and T_2 are turned ON.
- Current flows through: +, T_1, A_1, A_2, T_2, -.
- To disconnect the phase, devices T_1 and T_2 are turned OFF.
- The stored energy in the phase winding A tends to maintain the current in the same direction.
- Stored energy is fed back to mains by: A_1, A_2, D_1, +, -, D_2.
- Other phases are excited in the similar manner.
- Upper devices T_1, T_3 and T_5 are turned ON and OFF from the signals obtained from the rotor position sensor.
- Lower devices T_2, T_4 and T_6 are controlled by the signals obtained from chopping frequency signal of power semiconductor devices.

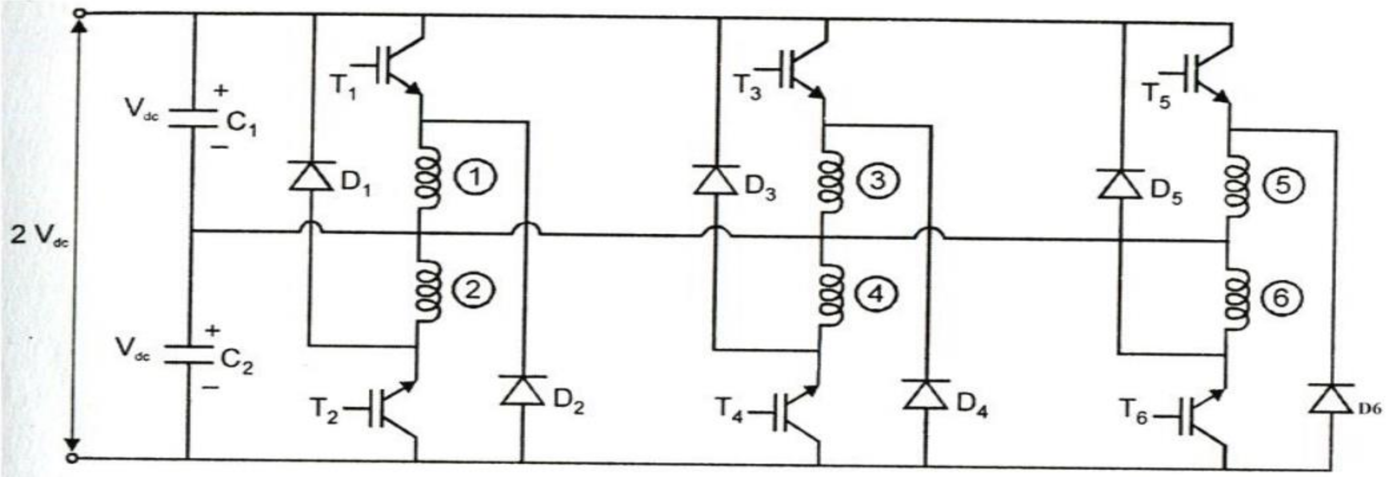
Advantages:

- ✓ Reduces switching losses of converter circuit.
- ✓ Control of each phase is completely independent of the other phases.

Disadvantages:

- ✓ Converter circuit is expensive.

Split – link circuit used with even phase number



- Used with GTO, thyristor or IGBT's.
- Main power supply of 2 V_{dc} is split into two halves using split capacitors.
- During the conduction period, energy supplied by one half of the power supply.
- During turn OFF period, the phases demagnetize into other half of the power supply.
- When T_1 is turned ON, phase 1 gets energized by capacitor C_1.
- When T_1 is turned OFF, the stored energy in the phase winding 1 is feedback to capacitor C_2, through the diode D_2.

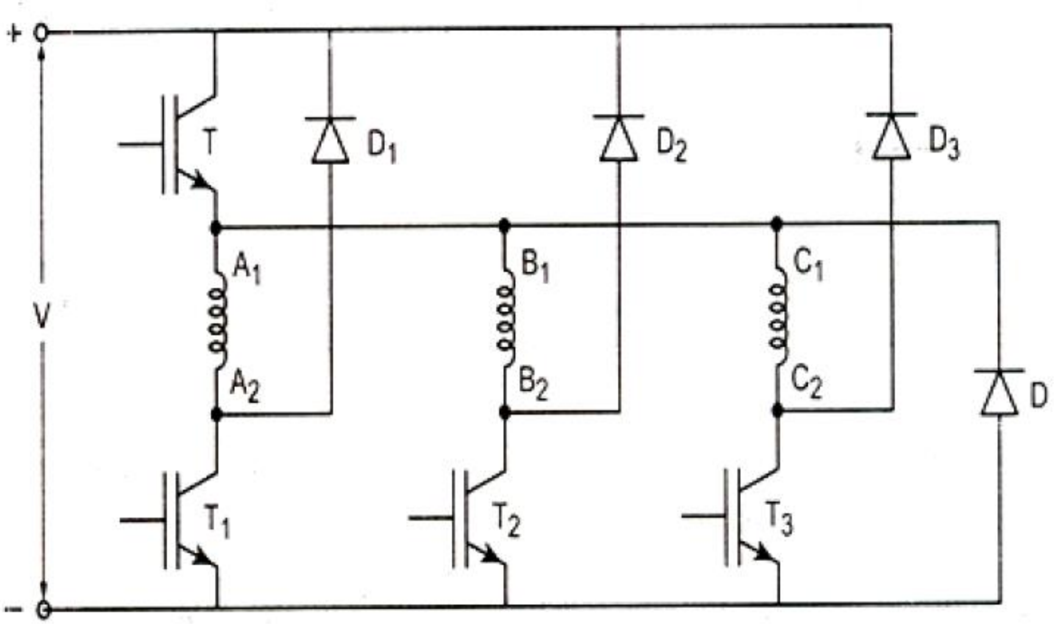
Advantages:

- ✓ Optimum choice for highly efficient drive.
- ✓ Capable of compete with bifilar wire circuit.

Disadvantages:

- ✓ If fault occurs in one phase means, circuit imbalance occurs.
- ✓ For low voltage applications, it is poor choice.

(n+1) power switching devices and (n+1) diodes



- Uses (n+1) semiconductor switches for n phase motor.
- To energize the phase windings A, the devices T and T_1 are turned ON.
- Current flows through: +, T, A_1, A_2, T_1, - .
- To disconnect the phase, device T_1 is turned OFF.
- The stored energy in the phase winding A tends to maintain the current in the same direction.
- Stored energy is fed back to mains by: A_1, A_2, D_1, +, -, D.
- Other phases are excited in the similar manner.

Merits

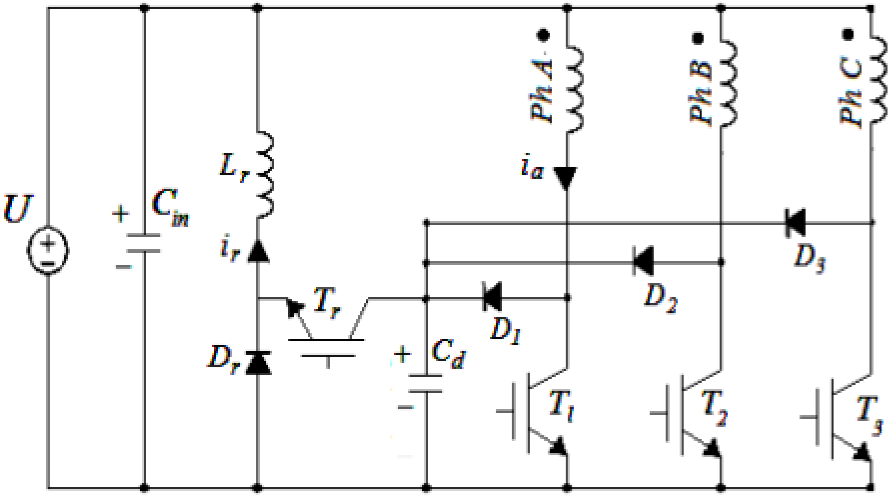
- The converter uses low number of switching devices, which reduces the cost of the converter.
- The converter is able to freewheel during the chopping, thus reducing the switching frequency and losses.
- Voltage rating of all the switching devices and the diodes are V_{dc}, which is relatively low.
- The energy for the off going phase is transferred back into the source, which results in useful utilization of the energy and also improves the efficiency.

Demerits

- Disability to magnetize a phase while the off going phase is still demagnetizing which results in higher torque ripple during commutation.
- At higher speeds of the off going phase cannot be de-energized fast enough because the common switch "T" keeps turnings on intermediately, disabling forced demagnetization.
- The common switch conducts for all the phases and thus has higher switching stress.

C-Dump Circuit

In that converter, assume that *T1* is turned on to energize the phase *a*. When the phase current *ia* exceeds the reference value, *T1* is turned off, this enables the diode *D1* to be forward biased, and the current path is closed through the dump capacitor *Cd* which increases the voltage across it in order to achieve fast demagnetization as asymmetric converter. Then the excess energy from the dump capacitor is transferred into the dc source via *Lr* by turning on the dump switch *Tr*. The voltage of the capacitor is regulated to be maintained at twice the supply voltage (*2U*) in order to apply (−*U*) across the outgoing phase for yielding faster demagnetization. The dump switch *Tr* is operated at a higher frequency than the phase switches



Advantages:

- ✓ It uses lower number of switching devices.
- ✓ Faster demagnetization.

Disadvantages:

- ✓ Complicated circuit.
- ✓ Use of inductors and capacitors in the circuit.

Control Circuits for SRM

(Current control techniques)

- Transducer (tachogenerator) is connected to the rotor.
- Output signal from the transducer is given as feedback signal to the transistor T_2.
- This signal is fed at the input of the operational amplifier.
- The operational amplifier compares signal with the reference current and then amplified signal is given to the transistor T_1.
- This signal in combination with collector current flow through the emitter of the transistor T_1 through the phase winding.
- The current limiting resistor R_{CL} limits the current according to the design requirement.
- When the reference current increase, the torque developed also increases.
- At low currents, T α I^2 and this relationship becomes more linear at higher values of current.
- At high current, torque/ampere reduces due to saturation.

- When torque varies monotonically with speed, the speed adjustment is possible even without feedback.
- But to have accurate speed control, speed adjustment is needed.

- To obtain the speed feedback signals shaft position sensor, optical encoders are used.
- The 'hysteresis type' current regulator uses hall-effect sensors with built in current sensing.
- This type of control produces a constant – torque type of characteristics.

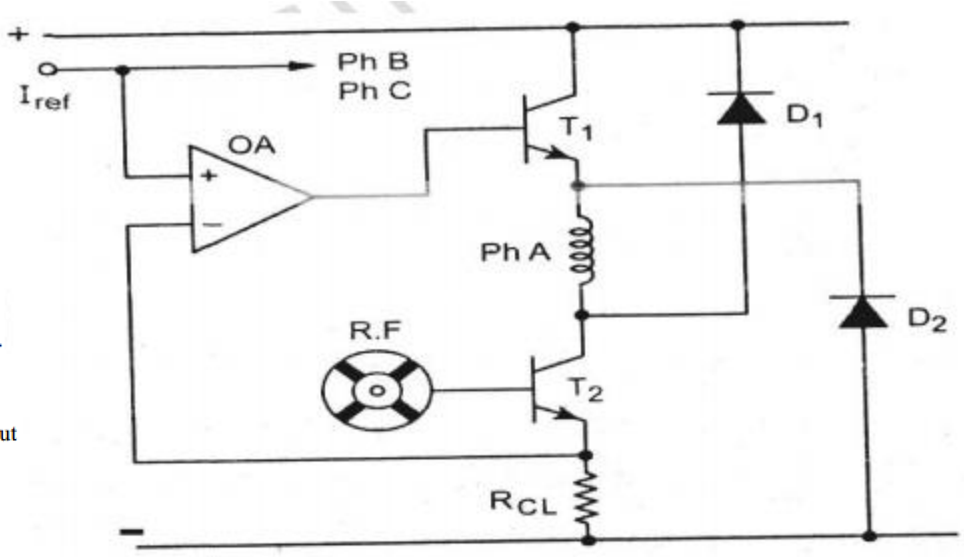
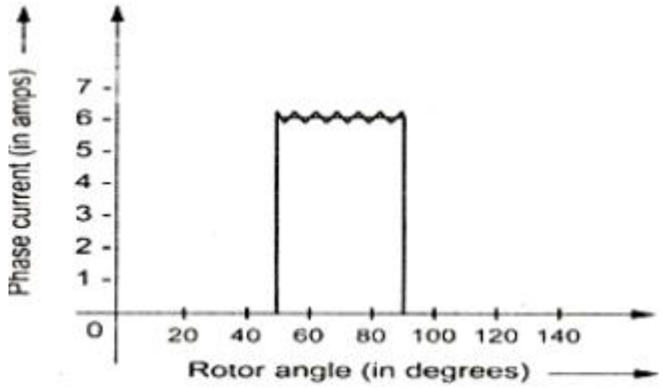


Fig. Hysteresis type current regulator



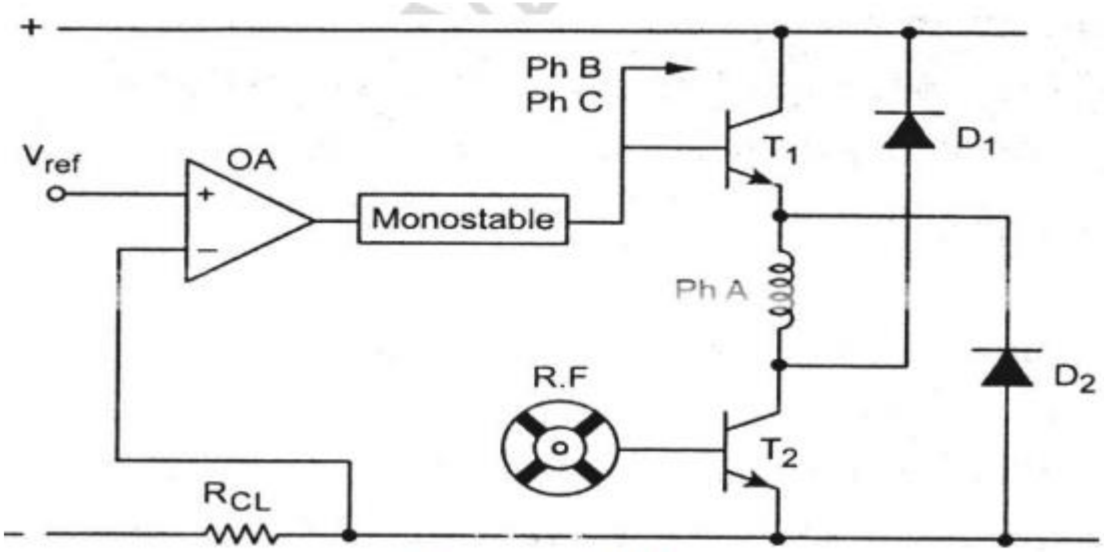
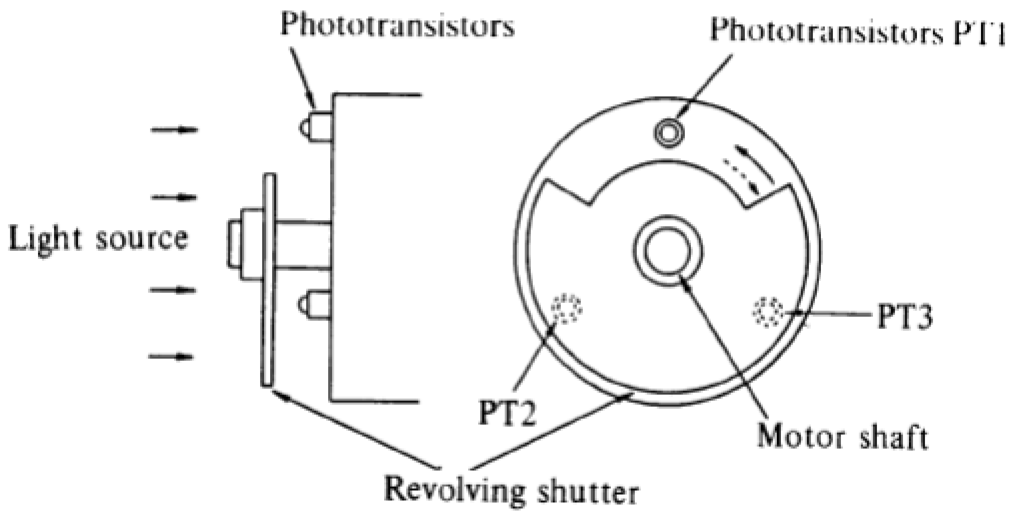


Fig: Voltage-PWM control

- Speed of the motor is converted into electrical signal (current), through the transducer (tachogenerator), which is fed to the transistor T_2.
- The resultant current from the emitter of the transistor T_2 flows through the current limiting resistor (R_{CL}) to the negative of the supply.
- The voltage at phase-A changes, because of the feedback signal.
- This feedback voltage is given as an input to the operational amplifier, which compares this input signal with the reference voltage.
- The difference of these two signals is amplified and fed to the monostable circuit.
- Monostable circuit modulates the pulse width of the incoming signal based on the requirement and the modulated signal is given at the base of T_1.
- This signal combines with collector current of T_1 and flows through phase A.
- Thus the current is regulated or controlled using pulse width modulation and rotor feedback.

Rotor Position Sensors

Phototransistor Sensors



- Consist of revolving shutter with 120 degree gap.
- When the gap is aligned with the phototransistor PT1, the phototransistor will generate a current due to the light, while phototransistor PT2 and PT3 have only a very small leakage currents because the light is blocked by the revolving shutter.

ROTARY ENCODER

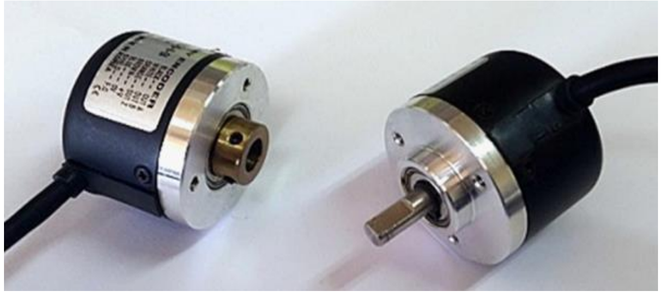
Absolute encoder, which can provide the absolute value of the rotation angle (complicated and expensive).

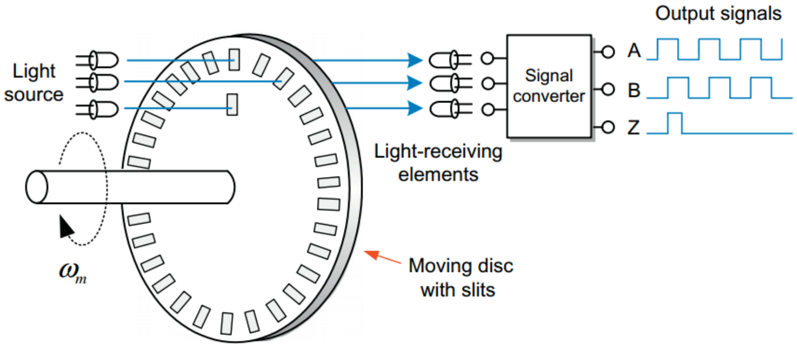
Incremental encoder, provides only the incremental value of the rotation (most widely used).

- There are two sensor types used in encoders to generate digital signals such as magnetic and optical. The latter is more commonly used.
- The resolution of an incremental encoder is frequently described in terms of pulse per revolution (PPR), which is the total number of output pulses per complete revolution of the encoder shaft.

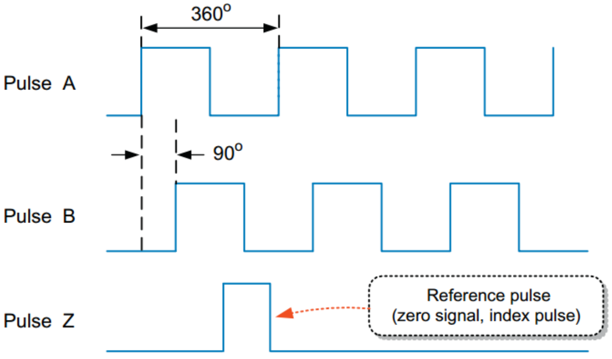
Two configurations

Shaft type that connects the shaft of a rotor with a coupling.
Hollow type, in which the shaft of a rotor is inserted into.

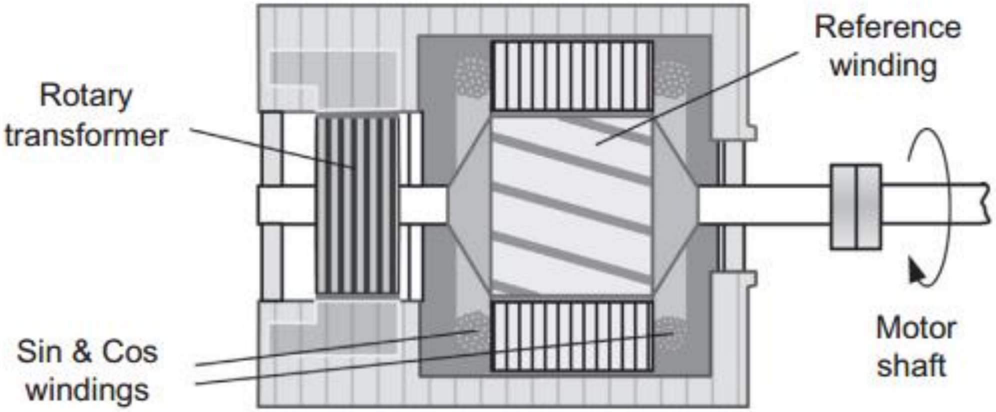




- It consists of a moving disc mounted to the rotating shaft, light sources (LEDs), and light receivers (phototransistors). The moving disc has the same number of slits as PPR. The light of LEDs passing through the slits on the disc is transmitted to phototransistors, and in turn, is converted to square wave shaped electric signals

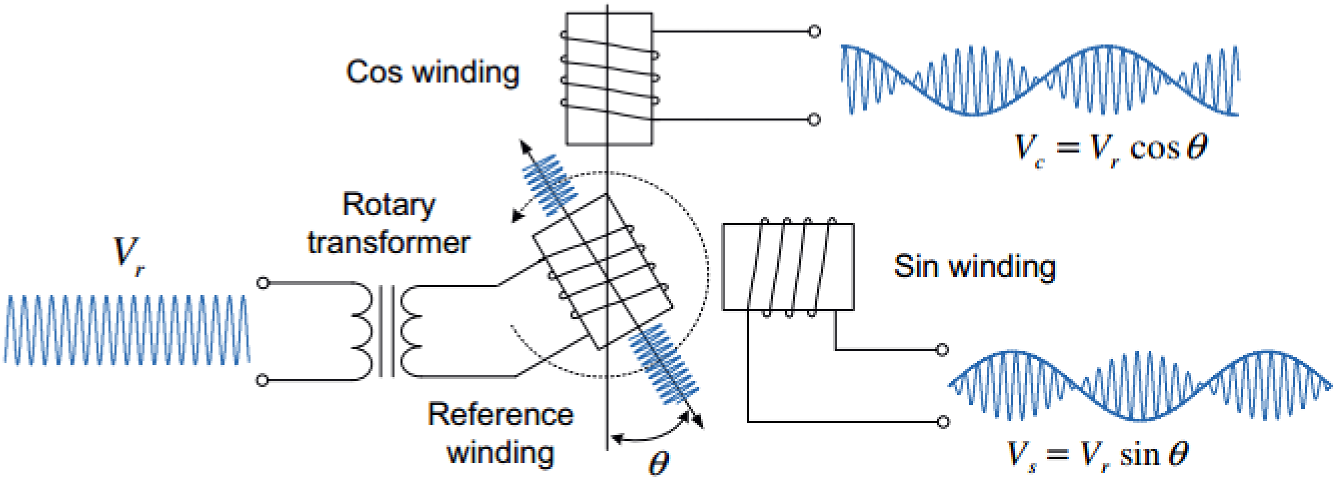


- Commonly, the encoder has three outputs called A, B, and Z. The total number of A and B pulses per revolution is equal to PPR, with which the angular position and speed can be calculated. The A and B pulses are 90 out of phase, which allows the identification of the direction of rotation. For example, when rotating in the forward direction, pulse A is ahead of pulse B. There is another pulse Z known as the index or reference pulse besides pulses A and B. Pulse Z is generated once per revolution and can be used to set the reference position.



Resolver.

Source: *http://www.amci.com/tutorials/tutorials-what-is-resolver.asp*



- The primary winding of a resolver, called reference winding, is located in the rotor and is excited through a rotary transformer. The two secondary windings, called SIN and COS Windings, are located in the stator and mechanically displaced by 90 degree from each other. In the resolver operation, a shaft angle θ can be measured from signals induced in the secondary windings after injecting AC voltage signal into the primary winding.
- When the primary winding is excited by an AC voltage Vr through a rotary transformer, voltages in the secondary windings are induced differently depending on the angle θ of the rotor shaft. The induced voltages vary as sine or cosine of the rotor angle, respectively. From an arctangent function of these signals, we can know the absolute angle θ of the rotor connected to the shaft as

$$\theta = \tan^{-1}\left(\frac{V_c}{V_S}\right) = \tan^{-1}\left(\frac{V_r \sin\theta}{V_r \cos\theta}\right)$$

Sensorless operation

Problem with sensor

- Position sensors are discrete in nature.
- Adds complexity to the system.
- More costly.
- Installation of sensor is time taking and tedious job.
- Tends to reduce the reliability of the drive system.

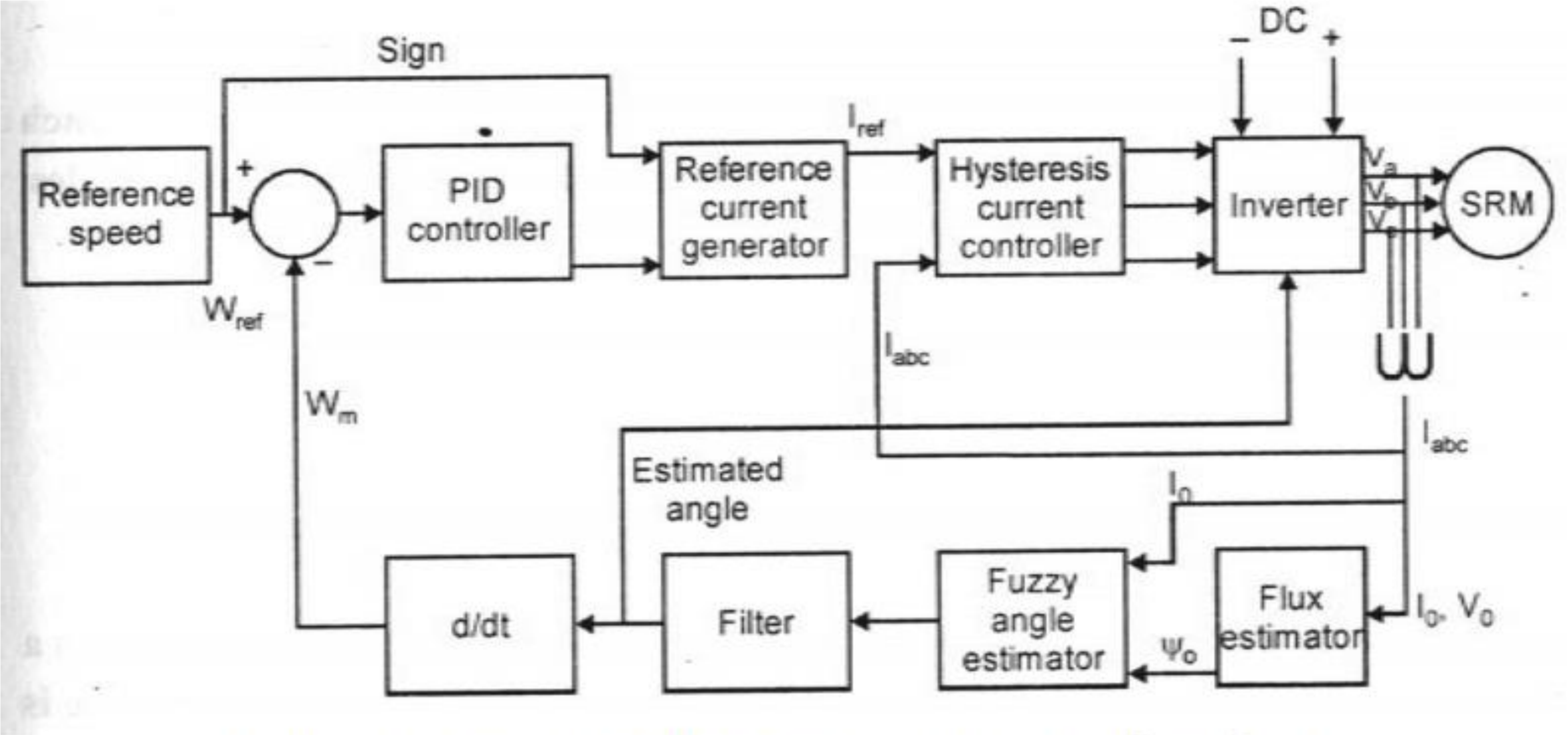
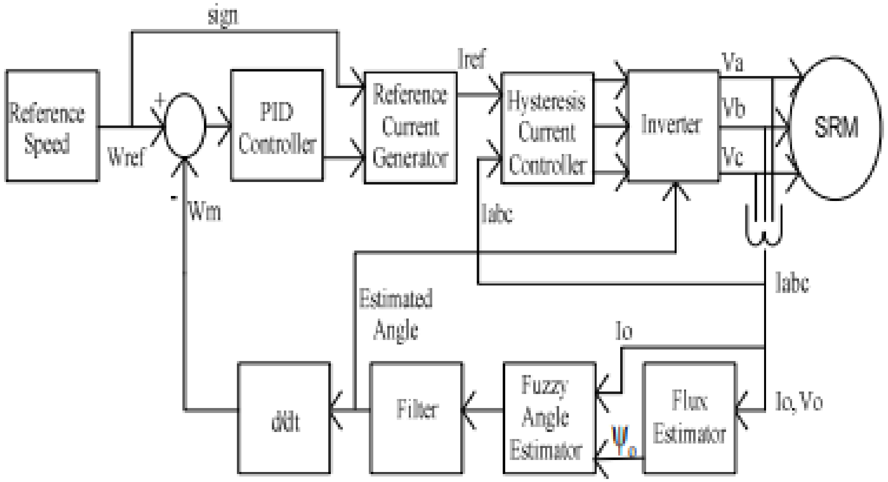


Fig. Sensorless Control of SRM with Fuzzy Rotor Position Estimator

The fundamental principle of operation of a SRM is based on the variation in flux linkage with the change in the angular position of the rotor. The proposed sensorless scheme relies on the fuzzy based rotor position estimator model of the SRM drive. The dynamics of the SRM drive can be represented by a set of non-linear first-order differential equations. The block diagram of the proposed sensorless scheme is shown in Fig. 1. It consists of various sub-systems necessary for PID speed controlled SRM drive with fuzzy logic use as a rotor position estimator. The flux estimator produced flux linkage by using phase voltage and current as inputs. The experimental data of flux linkage and phase current are used as inputs to fuzzy estimator and map them in fuzzy rule base for estimating the angle as an output. The suitable type of low pass filter has been used to produce refined estimated angle for inverter operation and simultaneously used to obtain estimated speed for comparison. This fuzzy based sensorless scheme is being simulated in MATLAB/Simulink environment. The simulated results have been compared with experimental results to verify the ability of this scheme for practicability.



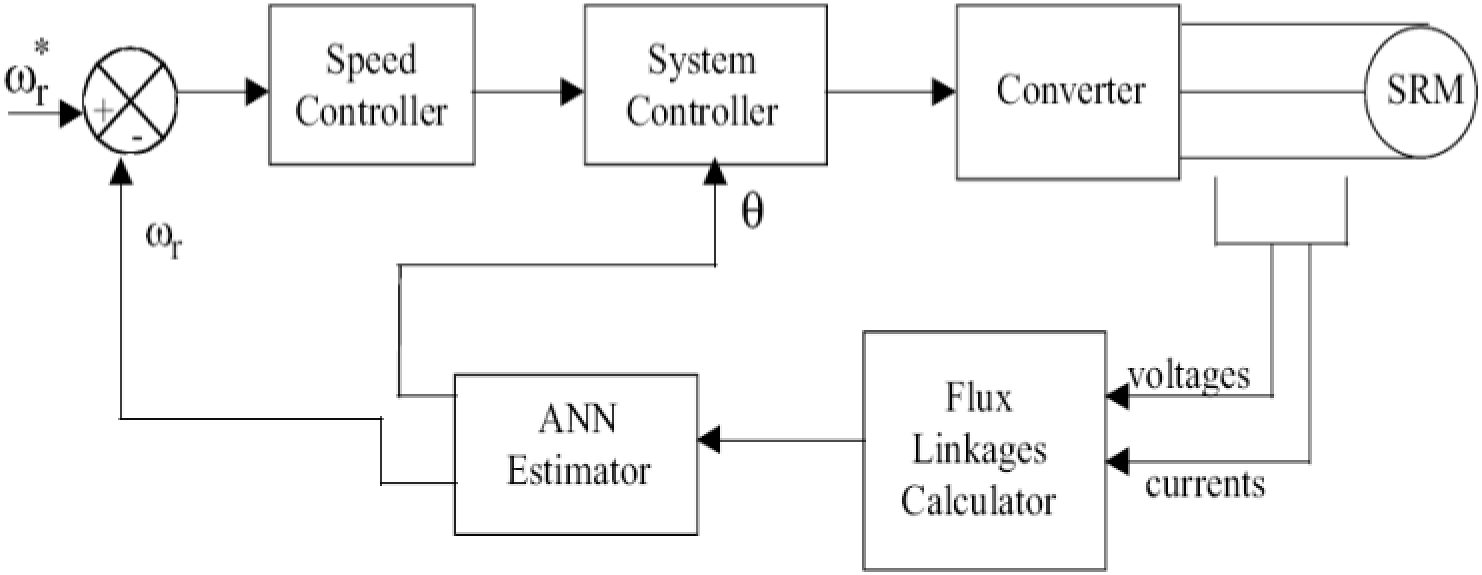
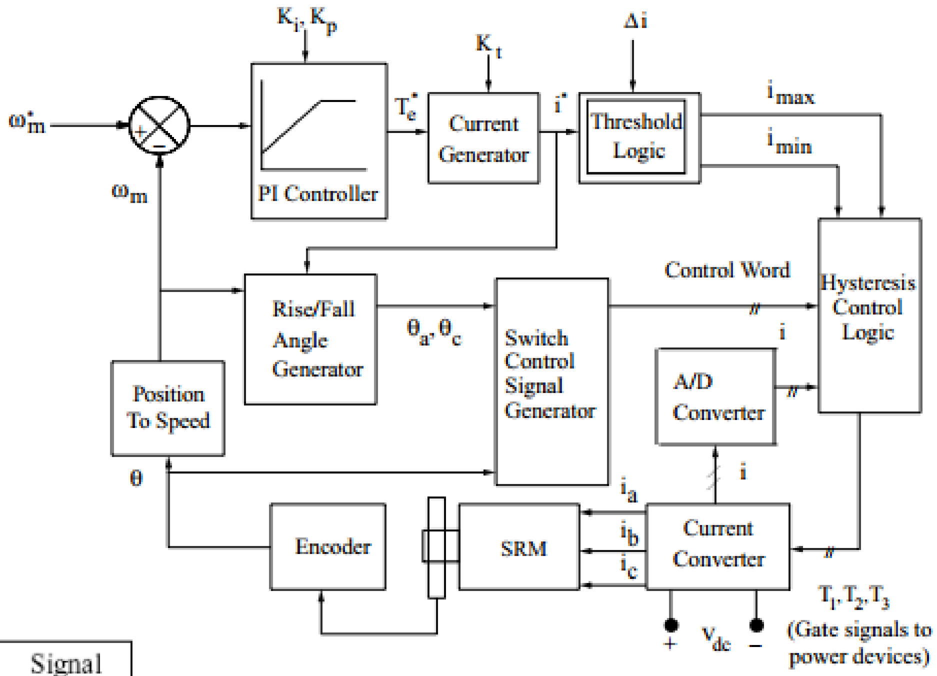


Fig. Sensorless Control of SRM with Artificial Neural Network

CLOSED-LOOP, SPEED-CONTROLLED SRM DRIVE

A closed-loop, speed-controlled SRM drive is shown in Figure 5.3. The speed error is processed through a proportional plus integral (PI) controller and a limiter to yield the torque command, T_e^*. From the torque command, the current command i^* is obtained using the torque constant, K_t. This torque constant is for the linearized inductance vs. rotor position characteristics for a particular value of current. The current command is added and subtracted from the hysteresis window, Δi, to obtain the i_{max} and i_{min} that determine the switching of the phase and main switches of any converter. The currents are injected into respective windings based on their position information obtained from an encoder or a resolver or position estimator. The rise and fall angles are calculated from the magnitude of the stator current, rotor speed, and minimum and maximum inductances. The rise and fall angles are incorporated with the rotor position information in the switch control signal generator block shown in the block diagram.



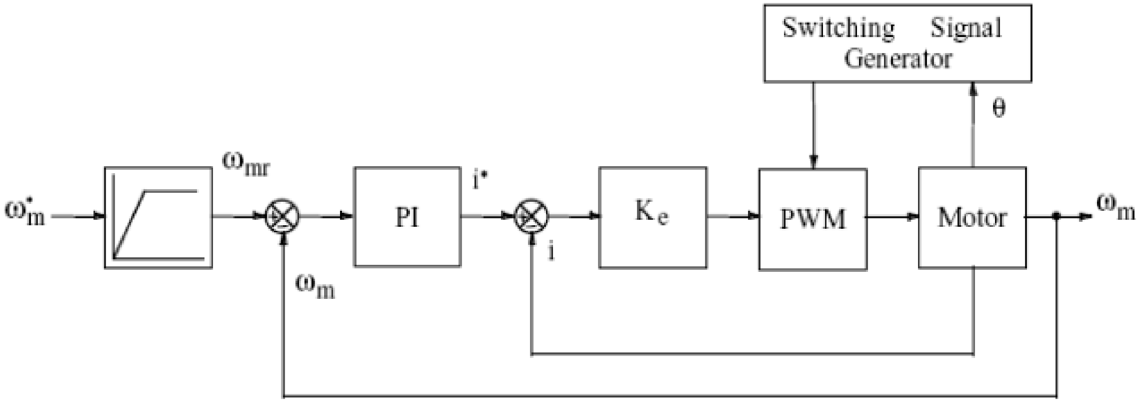
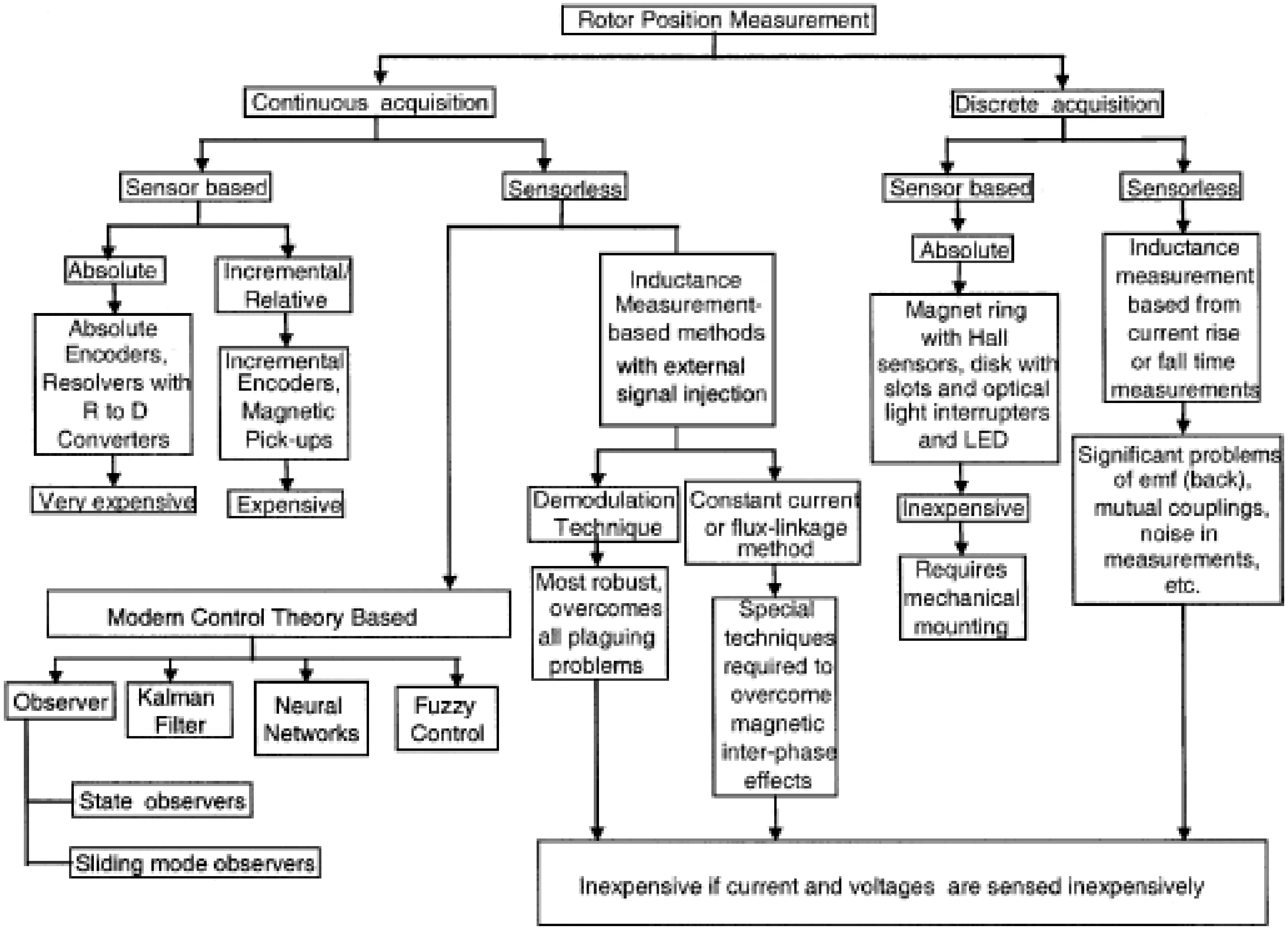


Fig. Simplified closed loop model of SRM

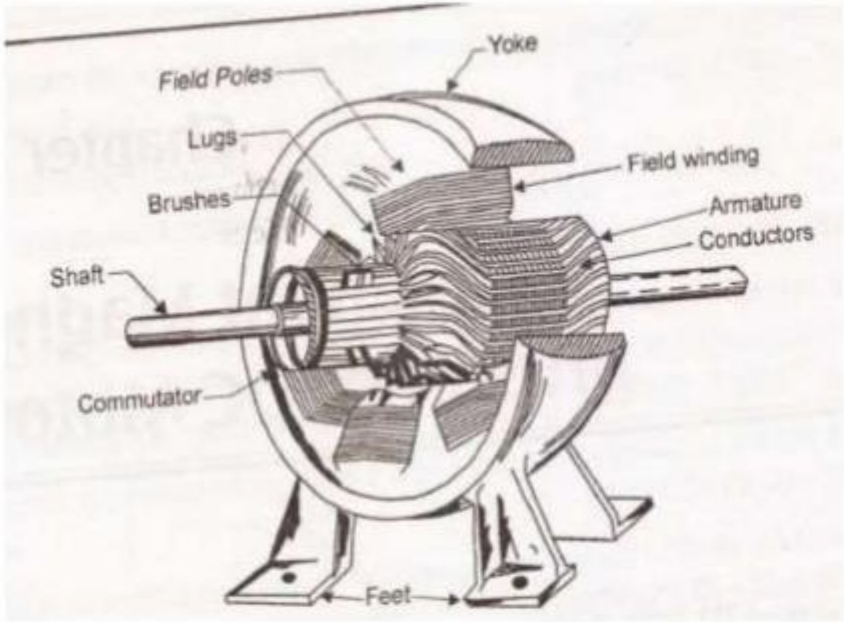
ROTOR POSITION MEASUREMENT AND ESTIMATION METHODS



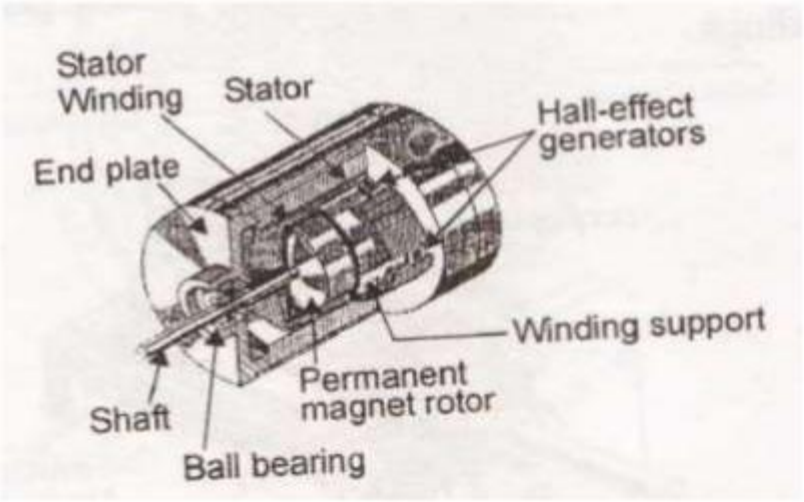
Unit 4

Brushless DC Motor

Conventional dc motor



PMBLDC MOTOR



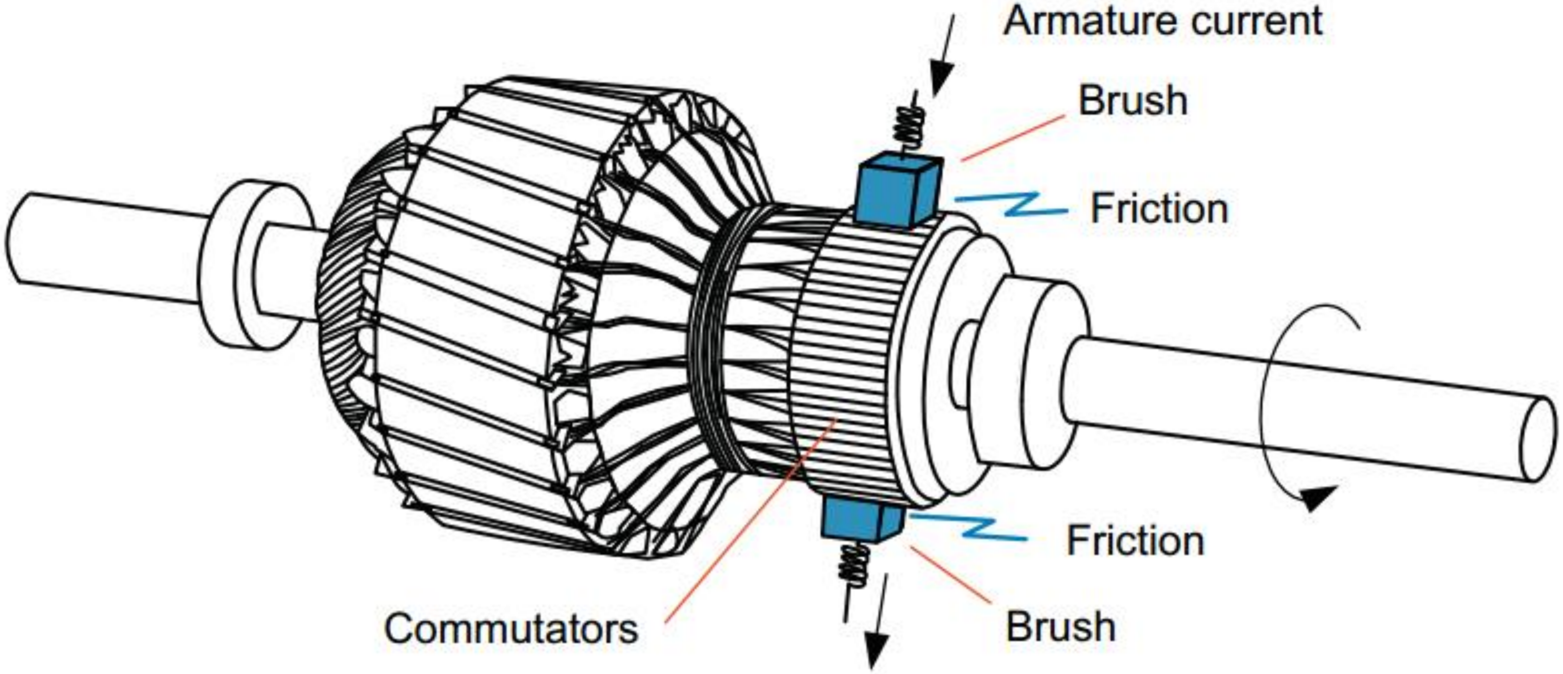


FIGURE ~~10.1~~

Mechanical commutation devices of a DC motor.

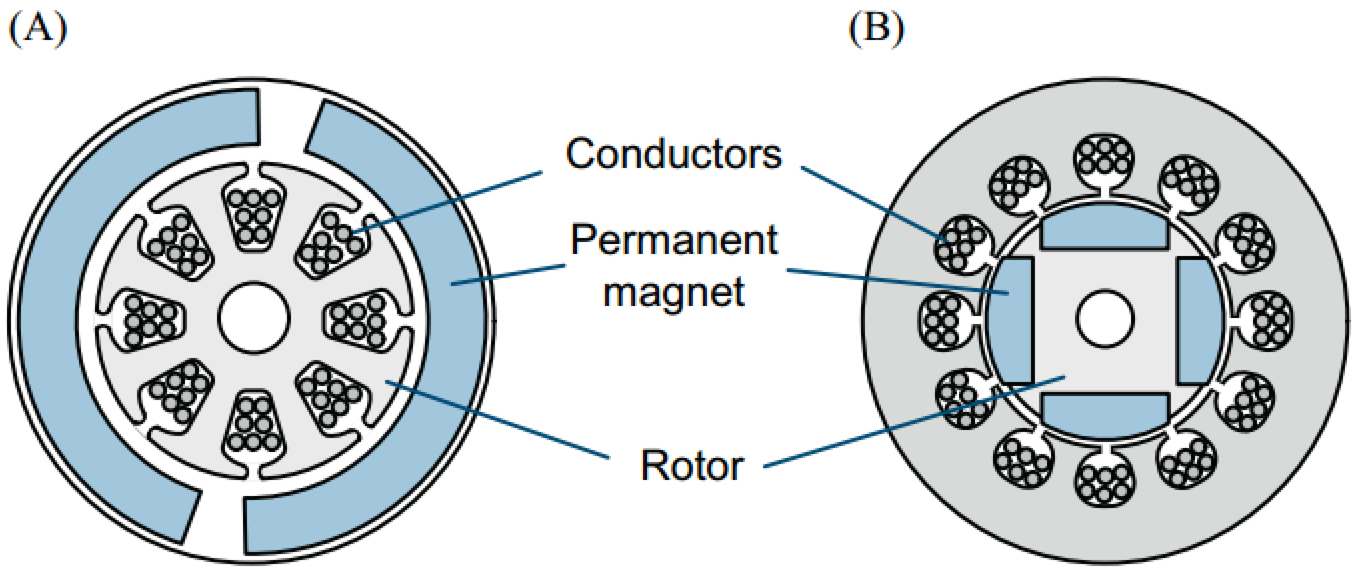


FIGURE ~~10~~.2

Configurations of (A) DC motor and (B) BLDC motor.

However, similar to the current flowing in armature windings of a DC motor, the current flowing in the windings of a BLDC motor is a quasi-square waveform.

Such configuration of BLDC motors has the following merits over DC motors. Compared to the heavy rotor of DC motors consisting of many conductors, BLDC motors have a low inertia rotor. Thus BLDC motors can provide a rapid speed response. Moreover, windings placed on the stator side can easily dissipate heat, allowing BLDC motors to have a better attainable peak torque capability compared to DC motors whose maximum current is limited to avoid the demagnetization of magnets. In addition, BLDC motors can operate at a higher speed because of nonmechanical commutation devices.

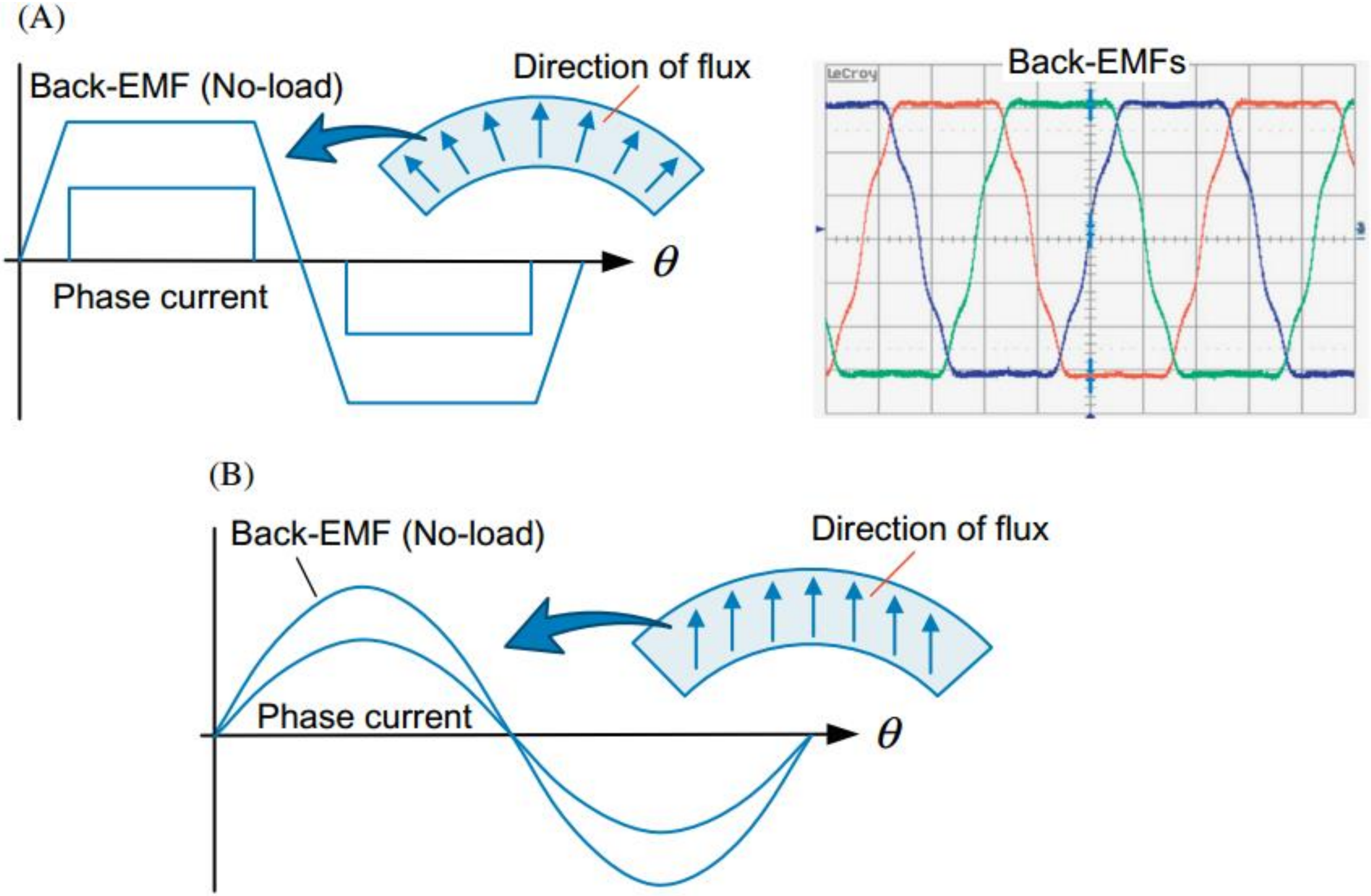


FIGURE [redacted]3

Comparison between (A) BLDC motors and (B) PMSMs.

Table 13.1 Comparison Between BLDC Motors and PMSMs

	BLDC Motor	PMSM
Back-EMF	Trapezoidal waveform	Sinusoidal waveform
Stator winding	Concentrated winding	Distributed winding
Stator current	Quasi-square waveform	Sinusoidal waveform
Driving circuit	Inverter (120° conduction)	Three-phase inverter (180° conduction)
Drive method	Simple, using low-cost Hall effect sensors	Complex (using high-resolution position sensor such as an encoder or a resolver)
Torque ripple	Significant torque ripple	Nearly constant torque
System cost	Low cost	High cost

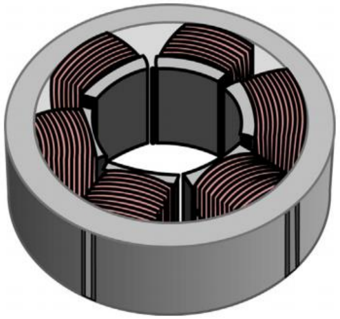


FIGURE 13.4

Concentrated stator windings of BLDC motors.

Construction of PMBLDC motor

- It consist of two parts mainly stator & rotor.
- Stator is made up of silicon steel stampings with slots.
- The slots are accomodated armature windings.
- This winding is wound with specified no.of poles.(even number).
- This winding connected a dc supply through a power electronic switching circuits.

- Rotor accomodates PM.
- The rotor shaft carries a rotor position sensor.
- Sensor provides information about the position of the shaft.
- This shaft position signal send to electronic commutator.

Advantages

- There is no mechanical commutator, so that size become very small.
- Speed can be easily controlled.
- Regenerative braking is possible.

Applications

- Automotive application.
- Textile and industries.
- Computer and robotics.
- Small appliances such as fans, mixers etc

Operating Principle

The schematic diagram of a brushless DC motor is shown in Fig. 12.15. It consists of a multi-phase winding wound on a non-salient stator and a permanent magnet (*PM*) rotor. The required voltage is applied to the individual phase winding through a sequential switching operation such that the necessary commutation is achieved to impart rotation to the motor. The necessary switching operation is achieved by using electronic circuit and devices such as transistors, thyristors, etc.

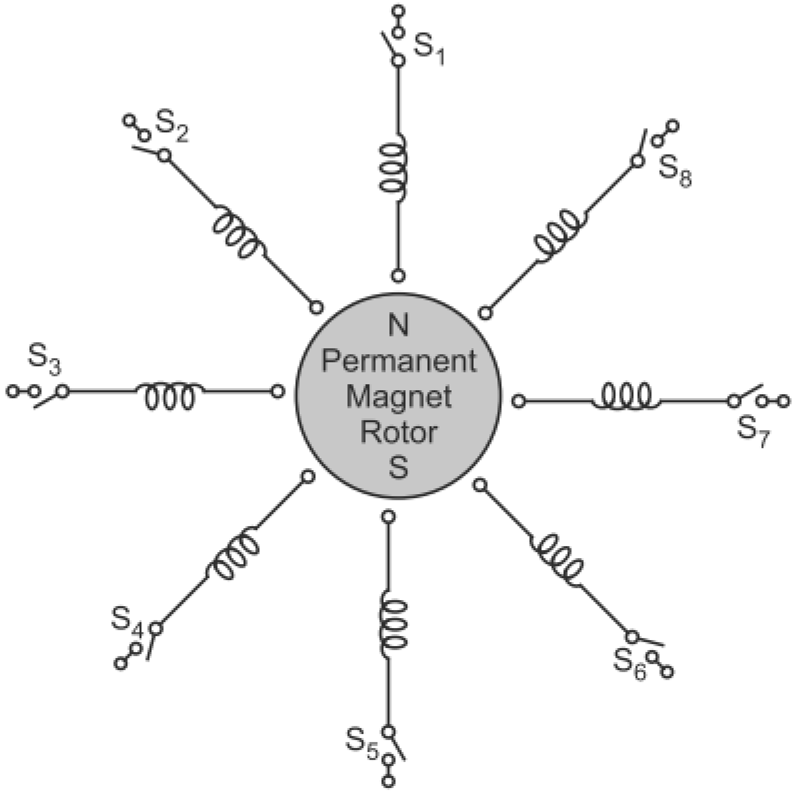


Fig. ~~12.15~~ Conventional diagram of a brush-less DC motor

Let us see how it operates

When switch S_1 is closed, winding-1 is energised, the *PM* rotor is aligned with its magnetic field. When switch S_1 is opened and S_2 is closed, winding-2 is energised, the *PM* rotor is aligned with its magnetic field and turn through a particular angle. When a number of such phase windings are energised sequentially, the rotor rotates.

In such motors, the windings can be designed for required voltage and very high speeds, as high as 40000 rpm.

Advantages

(*i*) Require little or no maintenance
(*ii*) Have longer operating life
(*iii*) Less losses, more operating efficiency
(*iv*) No sparking, hence can be used in the vicinity of combustible fluids and gases.
(*v*) These are very reliable and efficient (efficiency is more than 75%).
(*vi*) They are capable to run at very high speeds (more than 40000 rpm)

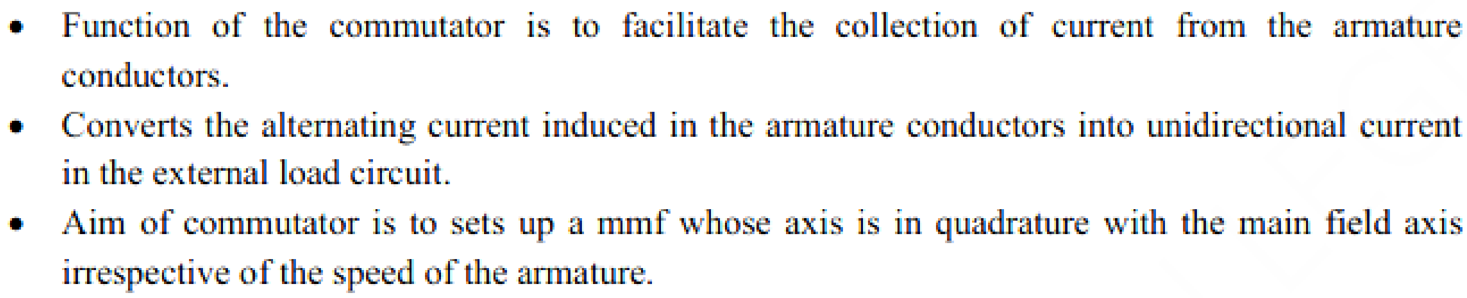
Disadvantages

(i) More expressive then conventional DC motors
(ii) Additional electronic circuit and devises are required that increases the overall size of the machine

Application

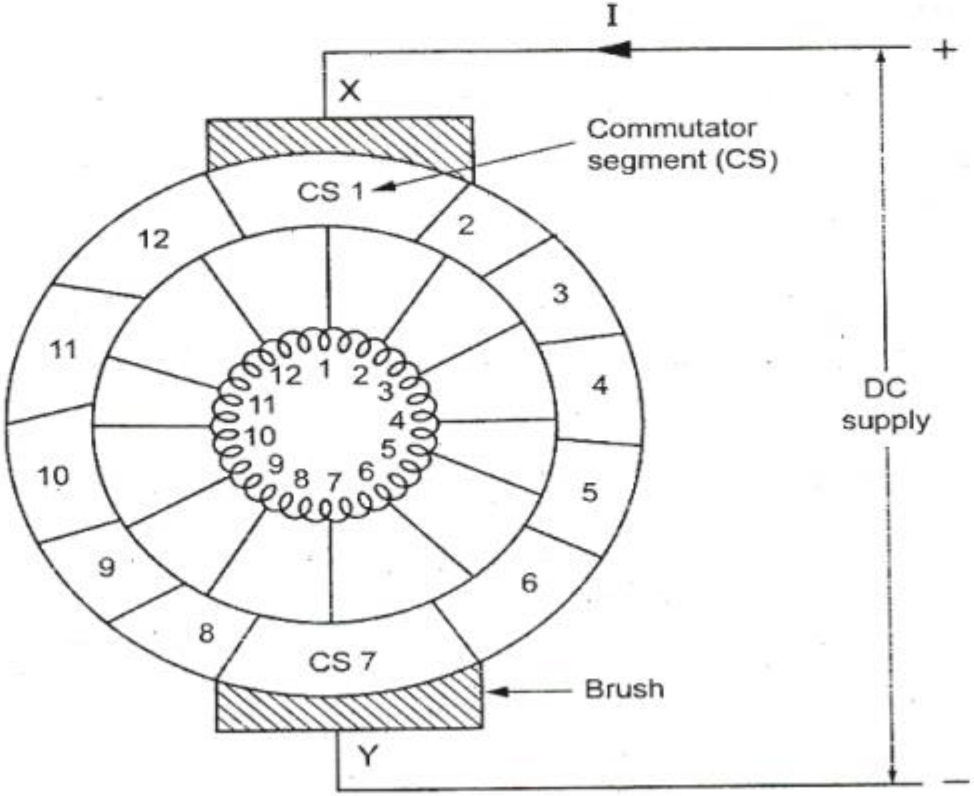
Due to high reliability and low maintenance these motors find their applications in aerospace industry. satellites, gyroscope and high efficiency robotic system. These motors are also suitable for artificial heart pumps, disc drives, video recorders and biomedical fields.

Commutator



- Function of the commutator is to facilitate the collection of current from the armature conductors.
- Converts the alternating current induced in the armature conductors into unidirectional current in the external load circuit.
- Aim of commutator is to sets up a mmf whose axis is in quadrature with the main field axis irrespective of the speed of the armature.

Mechanical commutator



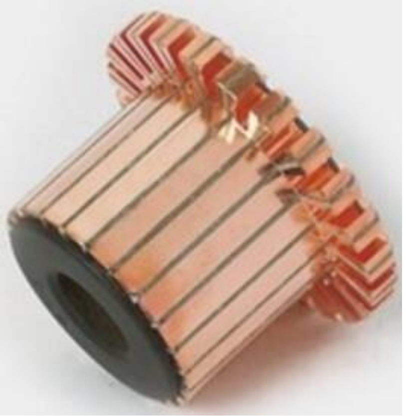


Fig. Mechanical commutator and brushes arrangement

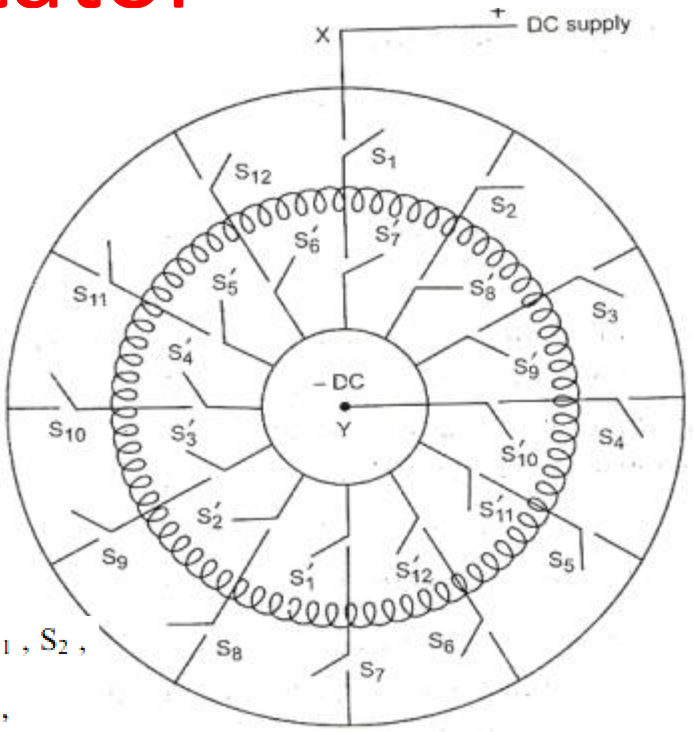
- Commutator segments are mechanically fixed to the shaft using V-shaped circular steel clamps, isolated electrically from the shaft.
- Schematic of a two pole machine with 12 commutator segments.
- Brushes of the two pole machine are X and Y.

Arrangement of mechanical Commutator:

- Brush X contacts with CS1 and Y contacts with CS7.
- The brushes X and Y are connected across the dc supply.
- A dc current is passing through the brush X, CS1, tapping 1, tapping 7, CS7 and brush Y.
- There are two armature parallel paths between the tappings1and 7.
    - ✓ 1-2-3-4-5-6-7
    - ✓ 1-12-11-10-9-8-7

- Current passing through the armature conductors sets up an mmf along the axis of the tappings 7 and 1 (ie), along the axis of Y and X.
- Commutator rotates along the anti-clockwise direction.
- Now the brush X contacts with CS2, & brush Y contacts with CS8.
- Hence the current passes through the tappings 2 and 8.
- There are two parallel paths,
    - ✓ 2-3-4-5-6-7-8
    - ✓ 2-1-12-11-10-9-8
- The mmf set up by the armature winding is from tapping 8 to 2 (ie), along the brush axis Y and X.
- Armature mmf direction is always along the brush axis Y and X.
- Function of the commutator and brushes arrangement in a conventional dc machine is to set up an armature mmf whose axis is always in quadrature with the main field irrespective of the speed of the rotation of the motor.

Electronics commutator



- Armature winding in the stator has 12 tappings.

- Each tapping is connected to the positive of the dc supply (ie) X through the switches S_1 , S_2 , S_3……..S_{12}
- Connected to the negative of the supply (ie) Y through the switches S_1', S_2' , S_3' …… S_{12}'.
- When the switches S1 and S_1' are closed, the dc supply is given to the tappings1 and 7.
- The current which is passing through the armature winding has two parallel paths.
  - ✓ 1-2-3-4-5-6-7
  - ✓ 1-12-11-10-9-8-7
- This current sets up an armature mmf, whose axis is along the axis of the tappings 1 &7.
- After a small interval of time, Switches S_1 and S_1' are kept open and S_2 and S_2' are closed.
- Current passes through the tappings 2, 8 and sets up an mmf along the axis of the tapping 2 &8.
- By operating the switches in sequential manner, it is possible to get revolving magnetic field in the airgap.

- These switches S_1 to S_{12} and S_1' and S_{12}' can be replaced by power electronic switching devices such as SCR, MOSFET, IGBT etc.

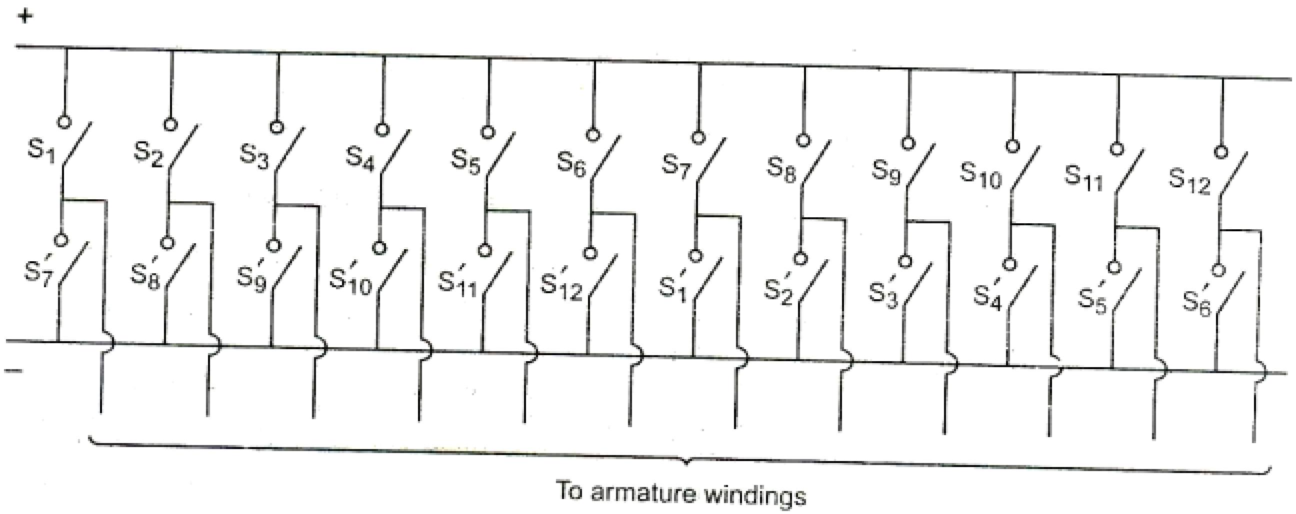
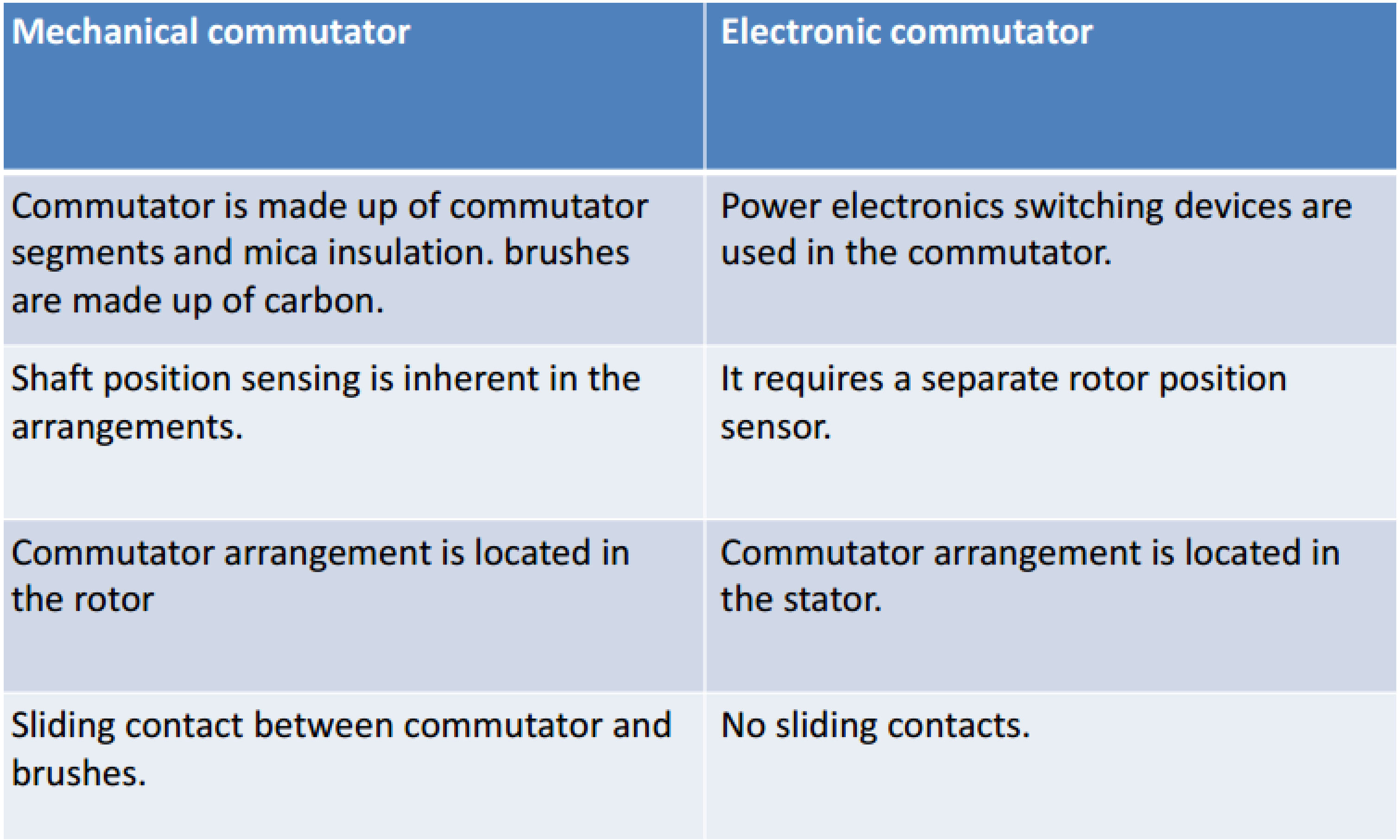


Fig. Switching circuit of electronic commutator

- More number of devices are employed, the circuit becomes complex.
- In normal electronic commutator, usually six switching devices are used. Hence, the armature should have three tappings, which can be connected either in star or delta.

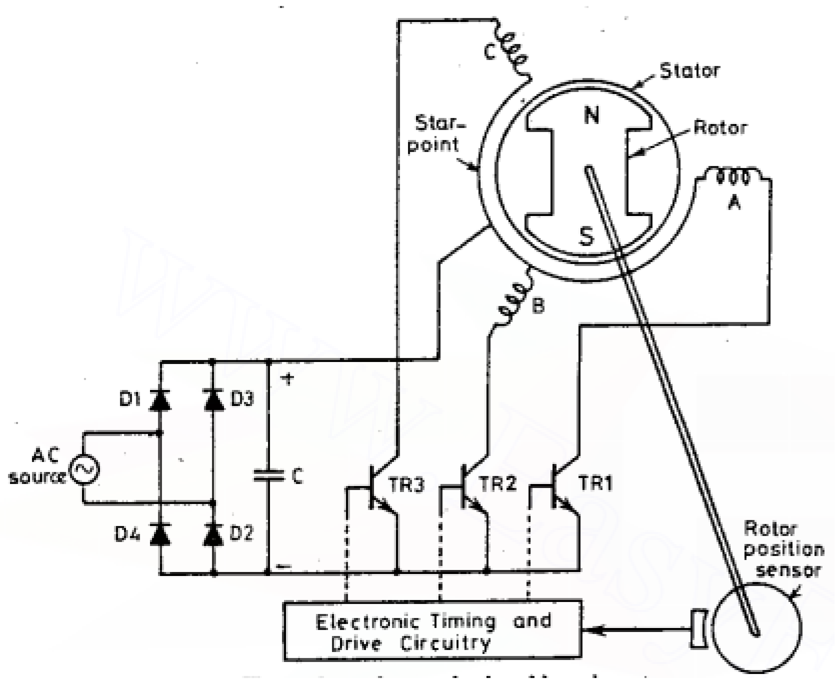
Comparison between mechanical and electronic commutator

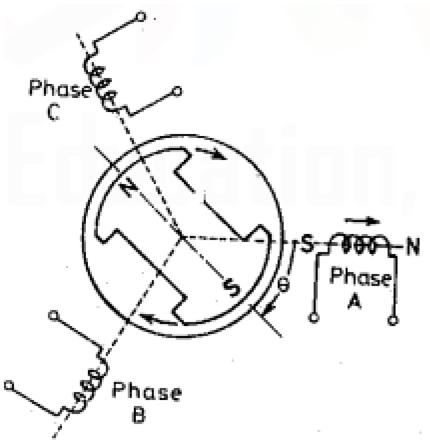


Mechanical commutator	Electronic commutator
Commutator is made up of commutator segments and mica insulation. brushes are made up of carbon.	Power electronics switching devices are used in the commutator.
Shaft position sensing is inherent in the arrangements.	It requires a separate rotor position sensor.
Commutator arrangement is located in the rotor	Commutator arrangement is located in the stator.
Sliding contact between commutator and brushes.	No sliding contacts.

Sparking takes place.	There is no sparking.
It requires a regular maintenance.	It requires less maintenance.
Number of commutator segments are very high.	Number of switching devices is limited to 6.
Difficult to control the voltage available across tapping	Voltage available across armature tappings can be controlled by PWM techniques.
Highly reliable	Reliability depends on the switching devices.

Operating principle of BLDC motor





$$T_e = K_1\, \phi_s\, \phi_r \sin\theta \qquad \ldots($$

where ϕ_s = stator field flux

ϕ_r = rotor field flux

θ = torque angle

and K_1 = torque constant

$$T_{ea} = K I_a \sin\theta$$

In case phase windings carry instantaneous currents i_a, i_b and i_c; the instantaneous torque, from Eq. (9.4) and Fig. 9.20, can be expressed as

$$T_{ea} = K\, i_a \sin\theta$$
$$T_{eb} = K\, i_b \sin(\theta - 120^\circ)$$
$$T_{ec} = K\, i_c \sin(\theta - 240^\circ)$$

If phase currents are assumed to vary sinusoidally with θ, then

$$i_a = I_m \sin\theta,\ i_b = I_m \sin(\theta - 120^\circ)$$

and
$$i_c = I_m \sin(\theta - 240^\circ)$$

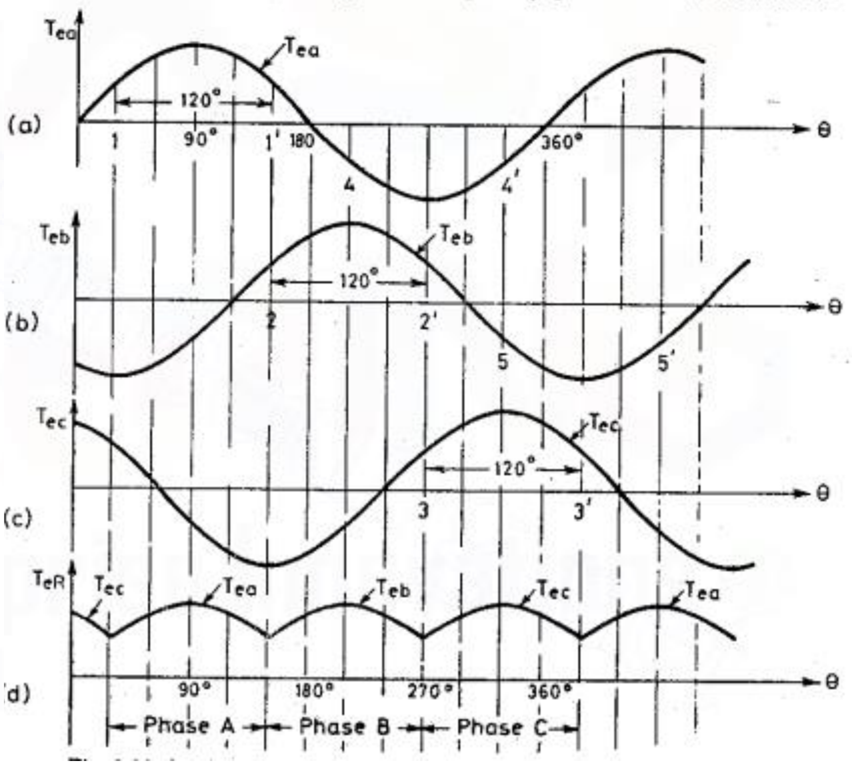
With these currents, the torque expressions for the three phases become,

$$T_{ea} = KI_m \sin^2\theta,\ T_{eb} = K I_m \sin^2(\theta - 120^\circ)$$

and
$$T_{ec} = K I_m \sin^2(\theta - 240^\circ)$$

Resultant torque, $T_{eR} = T_{ea} + T_{eb} + T_{ec}$

$$= KI_m [\sin^2\theta + \sin^2(\theta - 120^\circ) + \sin^2(\theta - 240^\circ)]$$
$$= KI_m \left[\frac{1-\cos 2\theta}{2} + \frac{1-\cos 2(\theta - 120^\circ)}{2} + \frac{1-\cos 2(\theta - 240^\circ)}{2}\right]$$
$$= \frac{3}{2} KI_m$$



Types of BLDC

Types of BLDC based on magnetic arc:

The BLPMDC motor can be classified on the basis of flux-density distribution in the air gap of the motor. They are,

- BLPM square wave DC motor.
- BLPM sine wave DC motor.

BLPM square wave dc motor:

There are two types of BLPM square wave DC motor

- 120° pole arc BLPM square wave DC motor
- 180° pole arc BLPM square wave DC motor

180° magnetic arc BLDC motor:

- 180° magnetic arc, 120° square wave phase current.
- Phase windings are star connected.

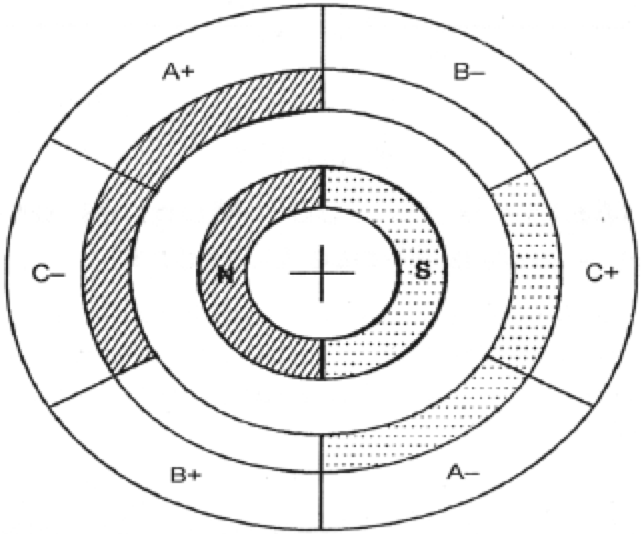


Fig. BLDC motor with 180° magnetic pole arc

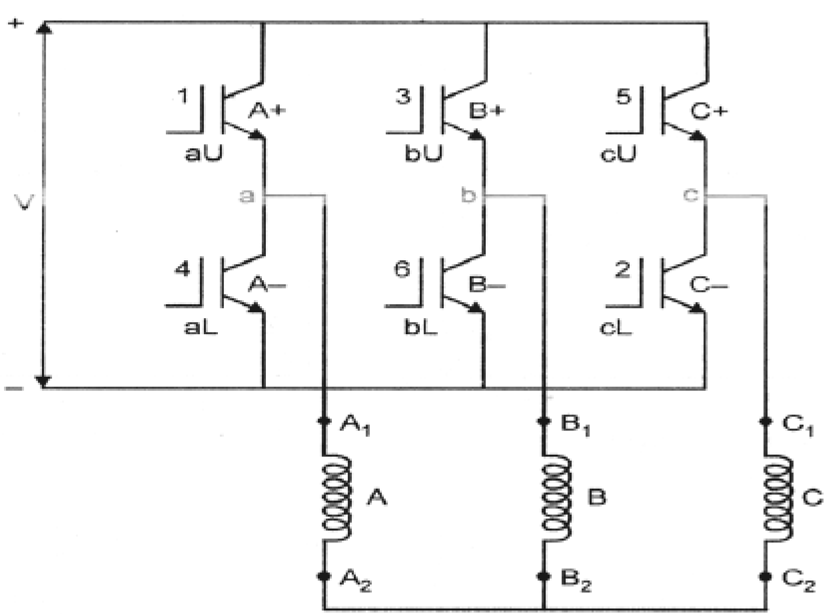
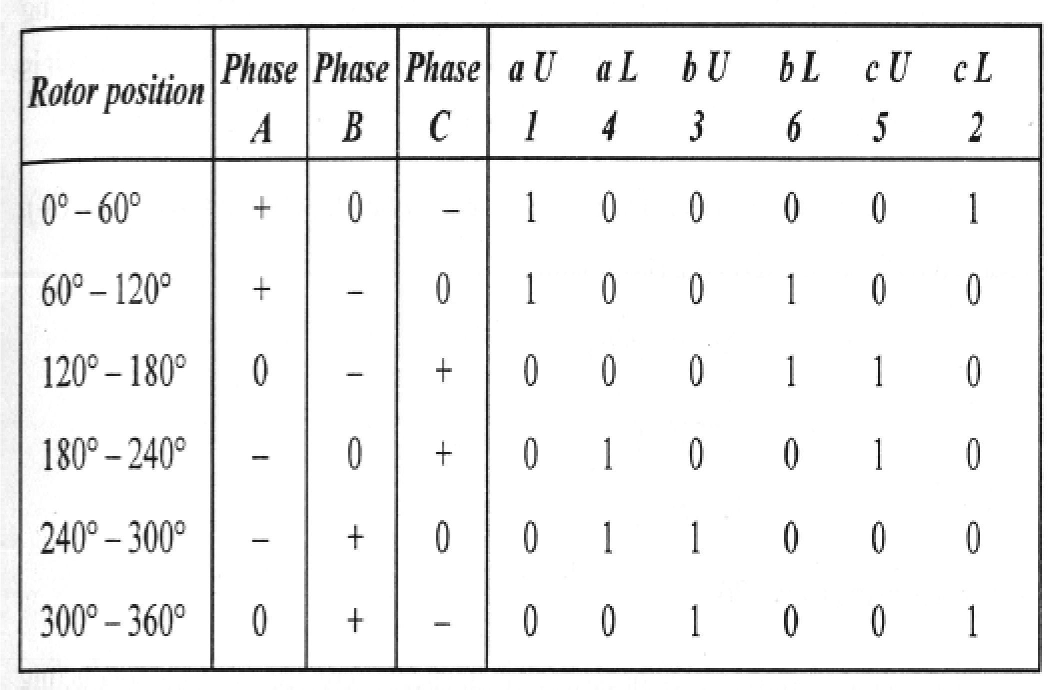


Fig. Converter with star connected phase winding

In the BLDC motor drive, only two of the three-phase windings are excited, while the other winding is left unexcited.

Table: Commutation logic table



Rotor position	*Phase A*	*Phase B*	*Phase C*	*a U 1*	*a L 4*	*b U 3*	*b L 6*	*c U 5*	*c L 2*
0° – 60°	+	0	–	1	0	0	0	0	1
60° – 120°	+	–	0	1	0	0	1	0	0
120° – 180°	0	–	+	0	0	0	1	1	0
180° – 240°	–	0	+	0	1	0	0	1	0
240° – 300°	–	+	0	0	1	1	0	0	0
300° – 360°	0	+	–	0	0	1	0	0	1

Rotor position 0° - 60° :

- A+, C- windings are energized.
- IGBT-1, IGBT-2 are conducting.

Rotor position 60° - 120° :

- A+, B- windings are energized.
- IGBT-1, IGBT-6 are conducting.

Rotor position 120° - 180° :

- B- , C+ windings are energized.
- IGBT-6, IGBT-5 are conducting.

Rotor position 180° - 240° :

- A-, C+ windings are energized.
- IGBT-4, IGBT-5 are conducting.

Rotor position 240° - 300° :

- A-, B+ windings are energized.
- IGBT-4, IGBT-3 are conducting.

Rotor position 300° - 360° :

- B+ , C- windings are energized.
- IGBT-3, IGBT-2 are conducting.

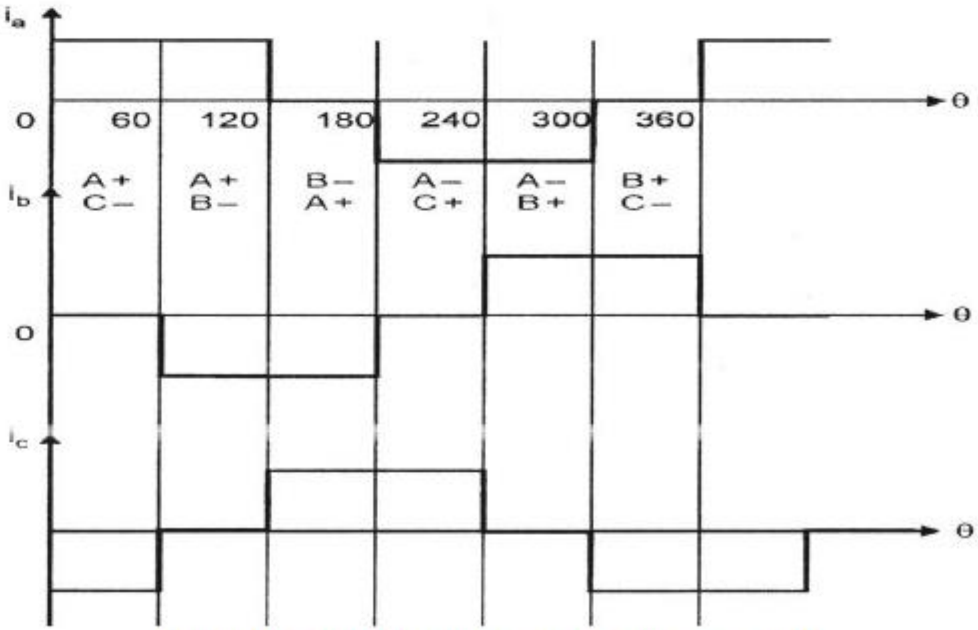


Fig. Phase current waveforms for 180° pole arc magnet

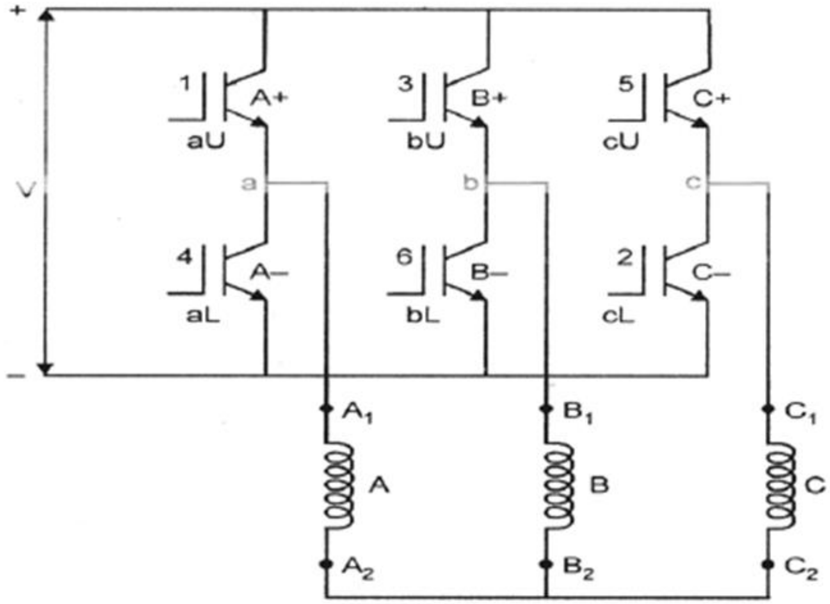


Fig. Converter with star connected phase winding

120° magnetic arc BLDC motor:

- 120˚ magnetic arc, 180˚ square wave phase current.
- Phase windings are delta connected.

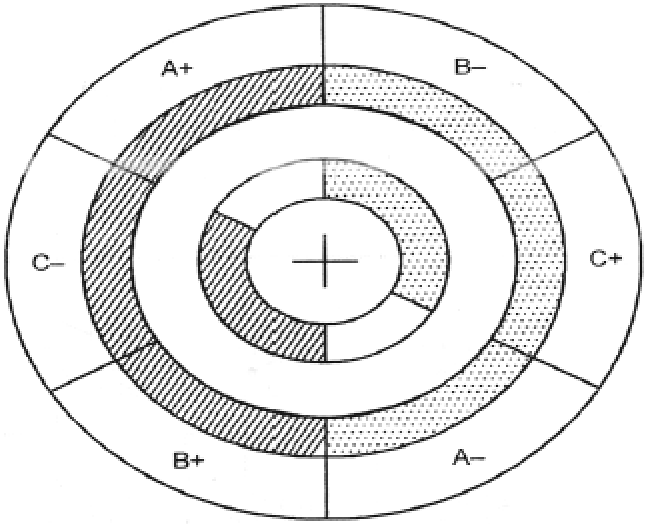


Fig. BLDC motor with 120˚ magnetic pole arc

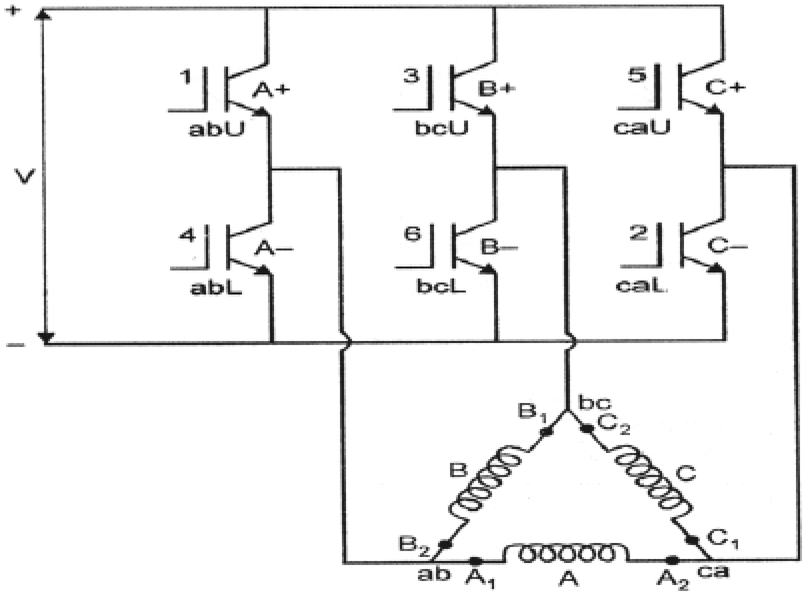


Fig. Converter with delta connected phase winding

- A+ - current flows from A_1 to A_2 A- - current flows from A_2 to A_1
- Same applicable for B and C windings.
- U – upper leg L – Lower leg

Table: Commutation logic table

Rotor position	*Phase A*	*Phase B*	*Phase C*	*ab U 1*	*ab L 4*	*bc U 3*	*bc L 6*	*ca U 5*	*ca L 2*
0° – 60°	+	+	–	0	0	1	0	0	1
60° – 120°	+	–	–	1	0	0	0	0	1
120° – 180°	+	–	+	1	0	0	1	0	0
180° – 240°	–	–	+	0	0	0	1	1	0
240° – 300°	–	+	+	0	1	0	0	1	0
300° – 360°	–	+	–	0	1	1	0	0	0

Rotor position 0° - 60° :

- A+, B+ , C- windings are energized.
- IGBT-3, IGBT-2 are conducting.

Rotor position 60° - 120° :

- A+, B- , C- windings are energized.
- IGBT-1, IGBT-2 are conducting.

Rotor position 120° - 180° :

- A+, B- , C+ windings are energized.
- IGBT-1, IGBT-6 are conducting.

Rotor position 180° - 240° :

- A-, B- , C+ windings are energized.
- IGBT-6, IGBT-5 are conducting.

Rotor position 240° - 300° :

- A-, B+ , C+ windings are energized.
- IGBT-4, IGBT-5 are conducting.

Rotor position 300° - 360° :

- A-, B+ , C- windings are energized.
- IGBT-4, IGBT-3 are conducting.

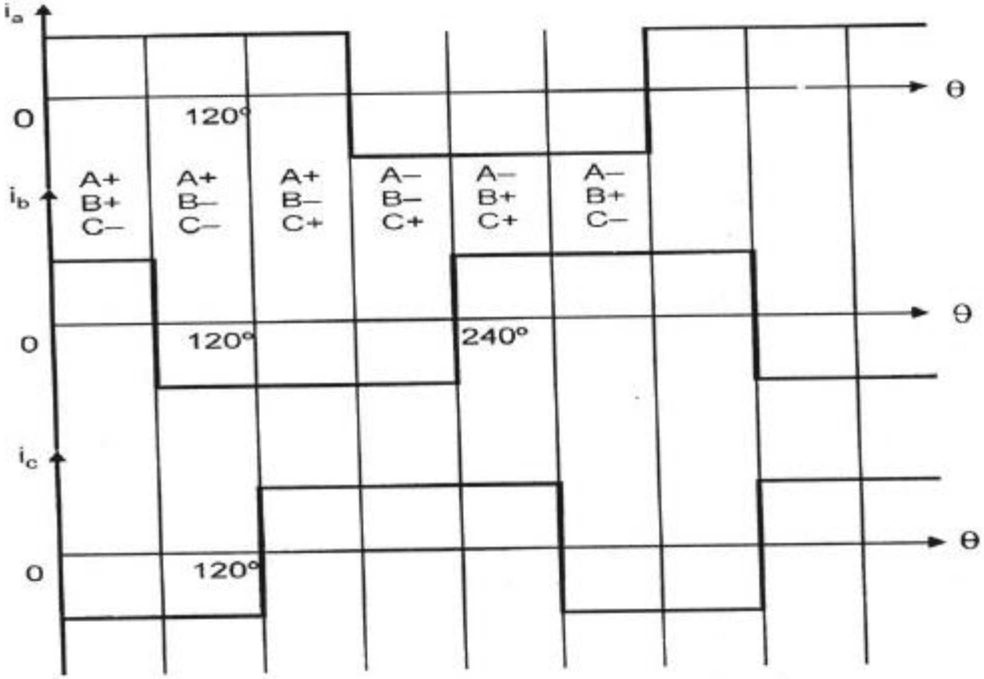


Fig. Phase current waveforms for 120° pole arc magnet

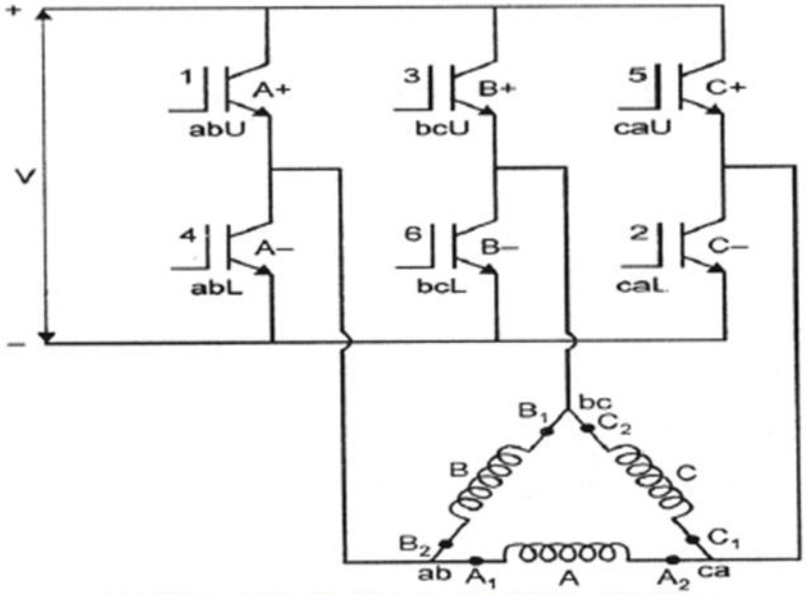
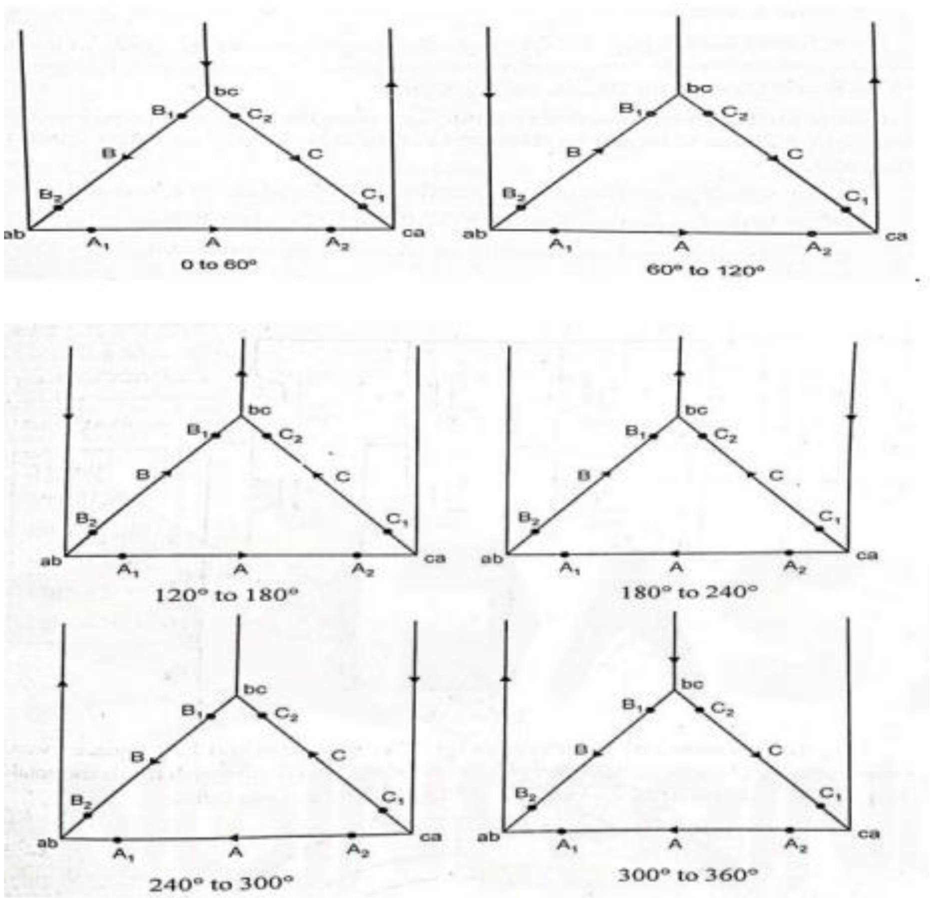


Fig. Converter with delta connected phase winding



0 to 60°

60° to 120°

120° to 180°

180° to 240°

240° to 300°

300° to 360°

Fig mmf vectors for BLDC motor with 120º magnet arcs.

power controller for BLPM SQW DC Motor

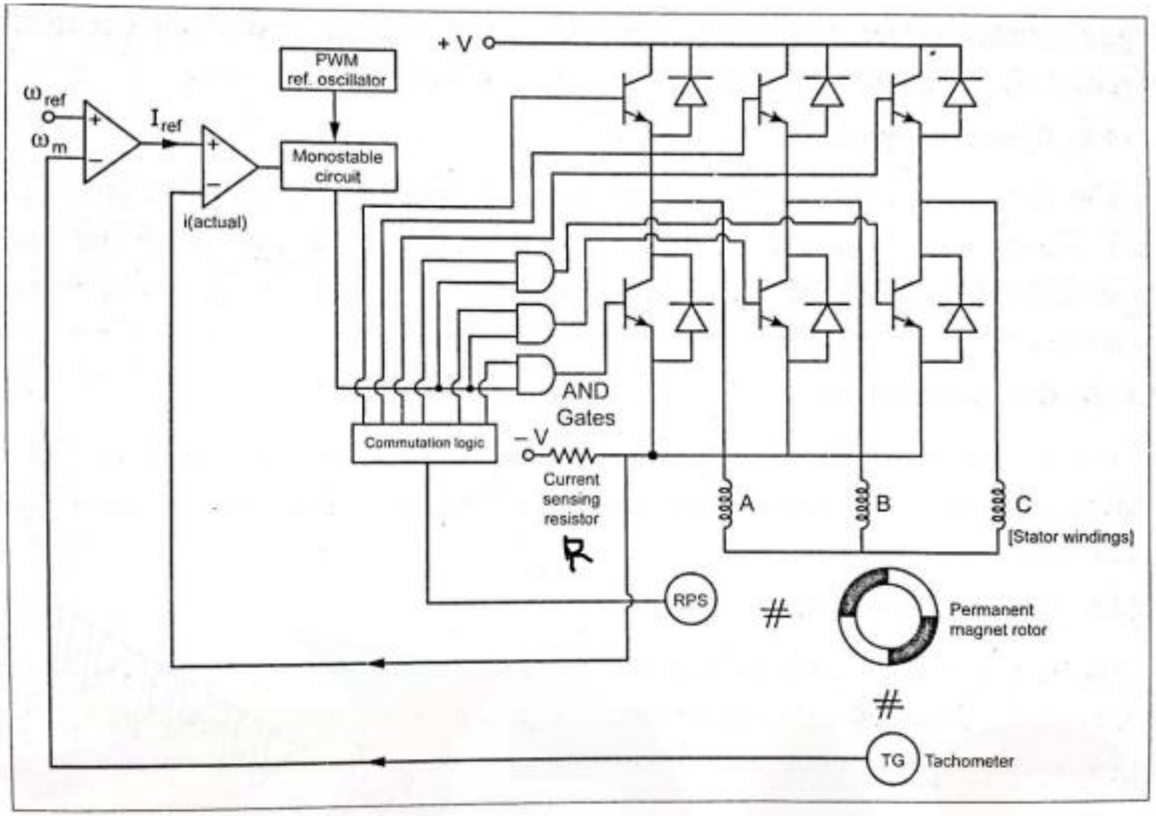


Fig structure of controller for brushless PM DC Motor

Power circuit:

- Consists of six power switching devices.
- Connected in bridge configuration across the DC supply.
- A shunt resistance ‘R’ is connected in series to get the current feedback signal.
- Feedback diodes are connected across the main devices.
- Armature winding is assumed to be star connected.
- Rotor carries rotor position sensor and shaft is coupled with tacho generator to get speed feedback signal.

Control circuit:

- Consists of commutation logic circuit which gets information about the rotor position
- Decides which devices are to be turned ON and OFF.

Commutation logic circuit:

- Provides six output signals out of which three signals are used as the base drive for the upper leg devices.
- Other three output signals are logically ANDed with high frequency pulses (PWM).
- The resultant signals are used to drive the lower leg devices.

Speed Comparator:

- Speed comparator compares the reference speed (W_{ref}) with the speed feed back signal (ω_m) obtained from the tachogenerator.
- Output of the speed comparator (ie) the speed error signal serves as the current reference for the current comparator.

Current Comparator:

- Compares the reference current (i_{ref}) with the actual current signal (i_{actual}) obtained from the current transducer.
- Resulting error signal is fed to the monostable circuit.

Monostable Circuit:

- The monostable circuit is excited by high frequency pulse signals.
- The duty cycle of the output of monostable multivibrator circuit is controlled by error signal.

Rotor Position Sensor:

- Rotor position sensor converts information of rotor shaft position into a suitable electrical signal.
- Signal from rotor position sensor is fed to the commutation logic circuit.
- It gives necessary output signals to switch ON and switch OFF the semiconductor devices of electronic switching and commutation circuit.
- Sensors : Optical position sensor & Hall effect position sensor.

Function of the controller:

- The rotor position is sensed by a hall effect sensor.
- These signals are decoded by commutation logic circuit to give the firing signals for 120° conduction.
- It has six outputs which control the upper and lower phase leg transistors.

- The PWM signal is applied only to the lower leg transistors.
- Reduces current ripple.
- The upper leg transistors need not be the same type as the lower leg transistor.
- The use of AND gate is a simple way of combining commutation signal and chopping signals.

- The monostable circuit is controlled by the error signal obtained from the comparator.
- The output of the monostable circuit and signal from the commutation logic circuit influences the conduction period and duty cycle of lower leg devices.
- Desired current for desired speed is obtained.

PERMANENT-MAGNET CHARACTERISTICS:

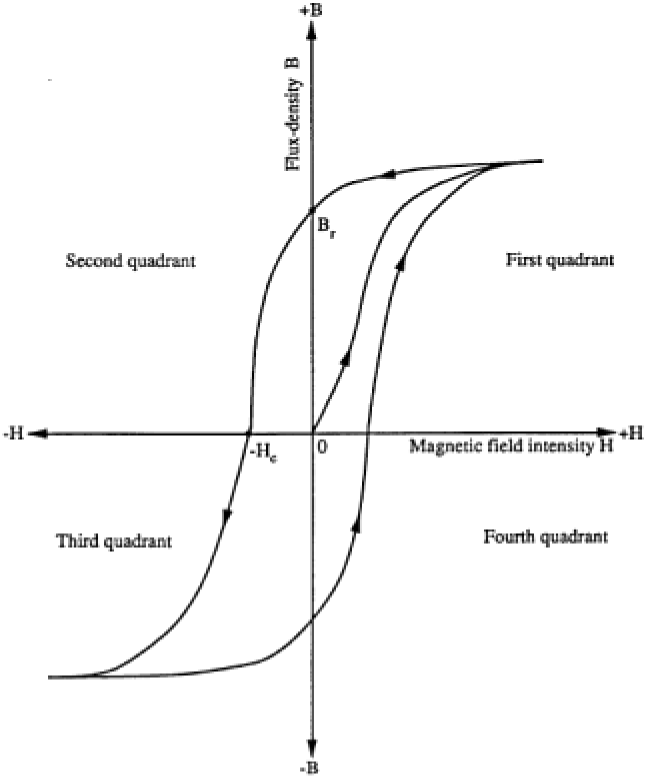


Fig. Hysteresis loop for a typical permanent-magnet material

The intrinsic loop represents that proportion of the flux-density which is intrinsic to the magnetic material itself for a particular magnetic field H, and it is of use in the design of a permanent-magnet machine.

The intrinsic flux-density B_i, the normal flux-density B and the magnetizing field intensity H are related by the equation

$$B=\mu_0 H+Bi$$

Several terms used in permanent-magnet studies are defined below.

(i) Magnetic Remanence
(ii) Normal Coercivity
(iii) Wrinsic Coercivity

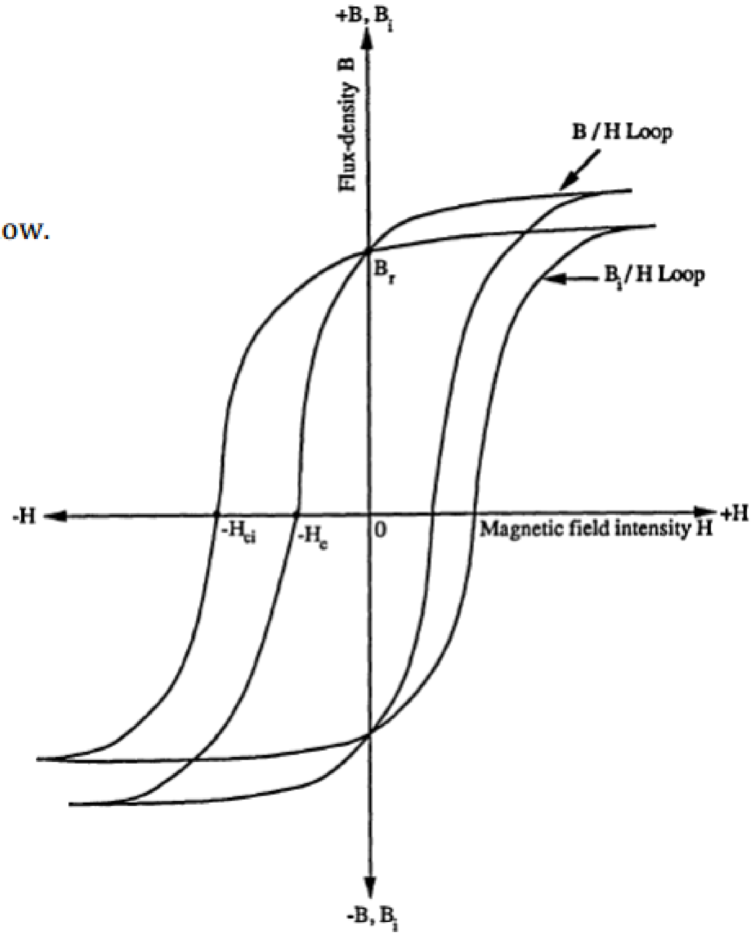


Fig. Normal and intrinsic hysteresis loops for a typical permanent-magnet material

MAGNETIC REMANENCE, B_r

If a magnetizing force is applied to an unmagnetized magnet and then removed, there remains a residual flux-density or remanence B_r due to the non-elastic displacement of the boundary walls between the magnetic domains.

The remanence is the same for both the B and B_i curves, since at this point H is zero.

NORMAL COERCIVITY, H_C

Increasing the negative magnetic field intensity H eventually reduces the normal flux-density B to zero. The value of the magnetic field intensity for this situation is termed the coercivity or coercive force H_c.

INTRINSIC COERCIVITY, H_{ci}

The intrinsic coercivity H_{ci} is the value of the magnetic field intensity at which the intrinsic flux-density B_i is zero.

It is a measure of the ability of the magnetic material to withstand demagnetizing forces without permanent changes in its magnetization. The magnitude of H_{ci} may be several times greater than that of H_c

PERMANENT-MAGNET TYPES

The main features of the three types of permanent-magnet materials used in small rotating machines are briefly summarized below.

(a) Ceramic-ferrite magnets

These have a relatively linear demagnetization characteristic, a low remanence of about 0.4T, a moderately high coercive force of up to 250kAm^{-1}, a maximum energy product of around 30kjm^{-3} and an extremely high electrical resistivity of about 10^{10} μΩm.

Ceramic magnets are relatively cheap and are used widely in small dc motors.

(b) Alnico magnet

This is a more expensive material than ceramic-ferrite. It has a non-linear demagnetization characteristic, a very high remanence of up to 1.2T, a low coercive force of below l20kAm^{-1} , a maximum energy product of around 60kjm^{-3}and a low electrical resistivity of about 0.5 μΩm.

(c) Samarium-cobalt rare-earth magnet

This material has an almost linear demagnetization characteristics, a high remanence (up to 0.9T), a very high coercive force (up to 750 kAm^{-1}) and a maximum energy product of round 400 kJm^{-3}.

It has a maximum energy product greater than that of other materials, which is a great advantage on a volumetric basis.

The disadvantage of the material is its high cost, and it was consequently first used only for aerospace and military equipment and for computer memory disks.

Samarium-cobalt rare-earth magnets are now becoming increasingly used, as the availability of the material becomes widespread and its cost falls.

Magnetic Circuit Analysis

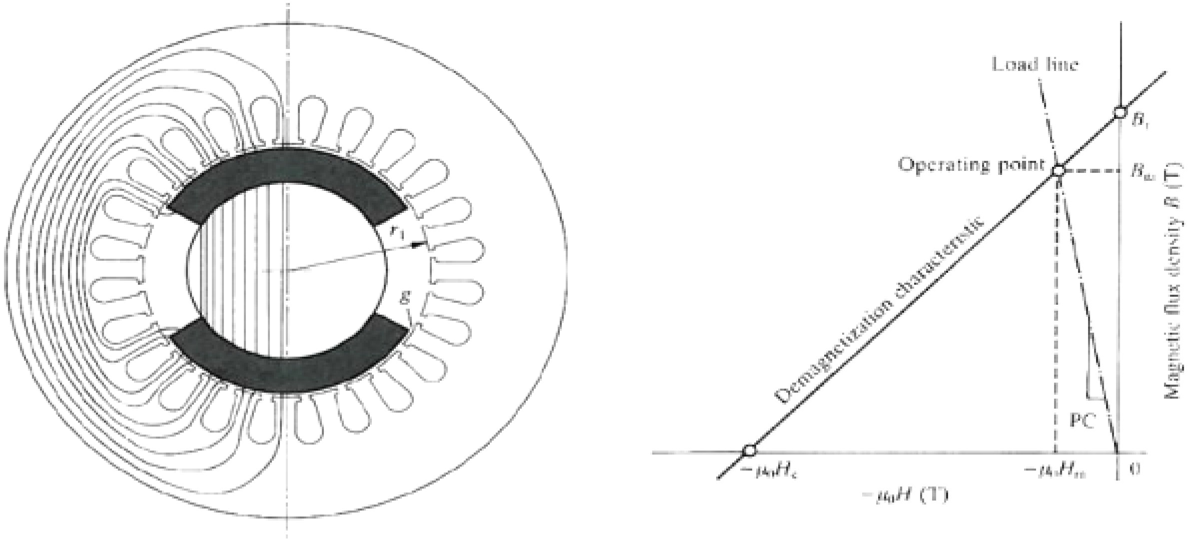


Fig. (a) Cross section and flux pattern of 2 pole BLDC motor
(b) Demagnetization curve

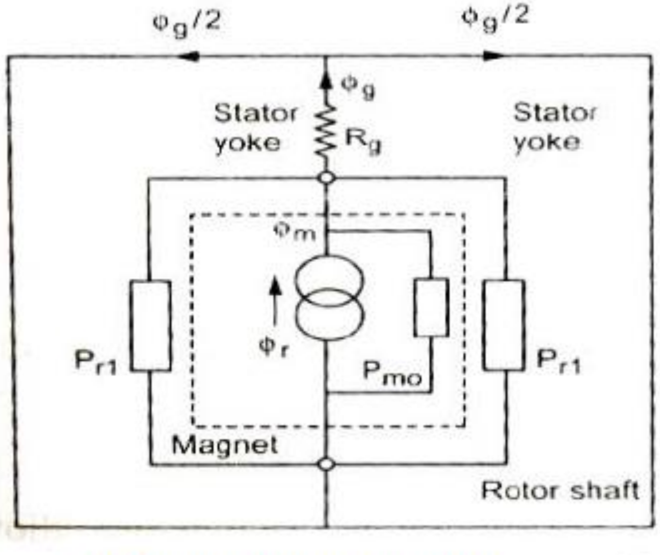


Fig. Magnetic equivalent circuit

$$\varphi_r = B_r \, A_m$$

$$P_{m0} = \frac{\mu_0 \, \mu_{rec} \, A_m}{l_m}$$

A_m – pole area of magnet

B_r – Remanent flux density

l_m – magnet length in the direction of magnetization

μ_{rec} – relative recoil permeability

120° magnet arc is considered.

$$A_m = \frac{2}{3} \, \pi \left[r_1 - g - \frac{l_m}{2} \right] l$$

$$R_g = \frac{g'}{\mu_0 \, A_g}$$

where, $g' = k_c \, g$ Kc: carter‘s coefficient,

By allowing fringing,

$$A_g = \left[\frac{2}{3} \, \pi \left(r_1 - \frac{g}{2} \right) + 2g \right] \, (l + 2\,g)$$

Ag- air gap area through which the flux passes as it crosses he gap

$$P_m = P_{m0} + P_{r1}$$

$$P_m = P_{m0} \, (1 + P_{r1})$$

By solving magnetic circuit,

$$F_m = \frac{\varphi_r - \varphi_g}{P_m} = \varphi_g \, R_g$$

$$\frac{\varphi_r - \varphi_g}{P_m} = \varphi_g \, R_g$$

$$\frac{\varphi_r}{P_m} - \frac{\varphi_g}{P_m} = \varphi_g \, R_g$$

$$\frac{\varphi_r}{P_m} = \varphi_g \, R_g + \frac{\varphi_g}{P_m} = \varphi_g \left(\frac{1}{P_m} + R_g \right) = \varphi_g \left(\frac{1 + R_g \, P_m}{P_m} \right)$$

$$\varphi_r = \varphi_g \left(1 + R_g P_m\right)$$

$$\varphi_g = \frac{\varphi_r}{1 + R_g P_m}$$

If we write the ratio of magnet pole area to air gap area as

$$\left.\begin{array}{c} Flux\ concenteration\ factor\ or \\ flux\ focusing\ factor \end{array}\right\} C_\varphi = \frac{magnet\ pole\ area}{airgap\ area} = \frac{A_m}{A_g}$$

$$airgap\ flux\ density, \qquad B_g = \frac{C_\varphi}{1 + R_g P_m} B_r$$

$$magnetic\ flux\ density, \qquad B_m = \frac{1 + P_{r1} R_g}{1 + P_m R_g} B_g$$

Magnetizing force H_m is solved using demagnetization characteristics.

$$-H_m = \frac{B_r - B_m}{\mu_0 \mu_{rec}} \quad A/m$$

- ve sign indicates demagnetizing force and magnet operates in second quadrant of B-H curve.

EMF equation of PMBLDC Motor.

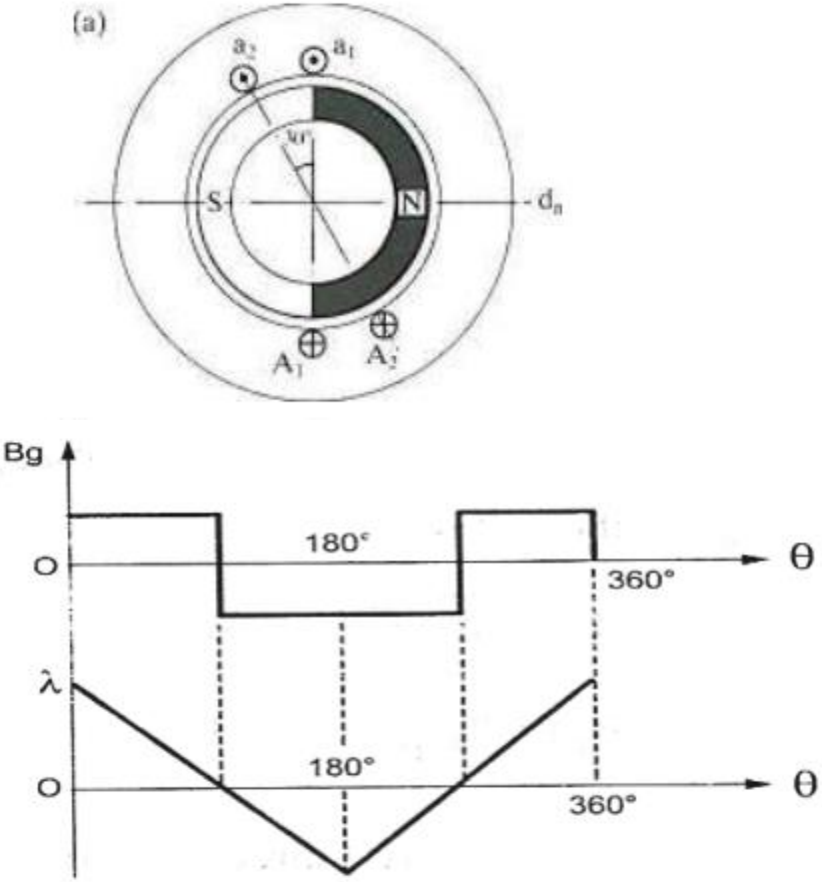


Fig. Magnetic flux density around the airgap

At θ = 0, center of N pole gets aligned with x-axis.

Flux enclosed with coil is,

$$\varphi_{max} = B_g \frac{2\,\pi\, r}{p}\, l$$

Flux linkage of coil is,

$$\lambda_{max} = \left(B_g \frac{2\,\pi\, r}{p}\, l\right) T_c \quad wb - T$$

λ varies with θ.

$$At\ \theta = 0^\circ\ or\ t = 0, \qquad \lambda = \frac{2\, B_g\, \pi\, r\, l\, T_c}{p}$$

$$At\ \theta = 90^\circ\ or\ t = \frac{\pi}{p\,\omega_m}, \qquad \lambda = 0$$

$$\therefore \frac{\Delta\lambda}{\Delta t} = \frac{final\ flux\ linkage - initial\ flux\ linkage}{final\ time - initial\ time}$$

$$= \frac{0 - \dfrac{2\, B_g\, \pi\, r\, l\, T_c}{p}}{\dfrac{\pi}{p\,\omega_m} - 0}$$

$$= \frac{-\dfrac{2\, B_g\, \pi\, r\, l\, T_c}{p}}{\dfrac{\pi}{p\,\omega_m}}$$

$$\frac{\Delta\lambda}{\Delta t} = -2\, B_g\, r\, l\, T_c\, \omega_m$$

$$e_c = -\frac{d\lambda}{dt} = -(-\,2\, B_g\, r\, l\, T_c\, \omega_m)$$

$$e_c = 2\, B_g\, r\, l\, T_c\, \omega_m \quad volts$$

Consider two coils, a_1A_1 & a_2A_2. Coils a_2A_2 is adjacent to a_1A_1 & connected in series, displaced by angle 30°.

Emf induced in coil a_1A_1 is,

$$e_{c1} = 2\, B_g\, r\, l\, T_c\, \omega_m \quad volts$$

Emf induced in coil a_2A_2 is,

$$e_{c2} = 2\, B_g\, r\, l\, T_c\, \omega_m \quad volts$$

Two coils are connected in series,

$$e_{c1} + e_{c2} = 4\, B_g\, r\, l\, T_c\, \omega_m \quad volts$$

If there are ‘ n_c ‘ no. of coils connected in series per phase,
Emf per phase,

$$e_{ph} = 2\, B_g\, r\, l\, T_c\, \omega_m\, n_c \qquad\qquad (n_c\, T_c = T_{ph})$$

$$\boxed{\boldsymbol{e_{ph} = 2\, B_g\, r\, l\, \omega_m\, T_{ph}\ volts}}$$

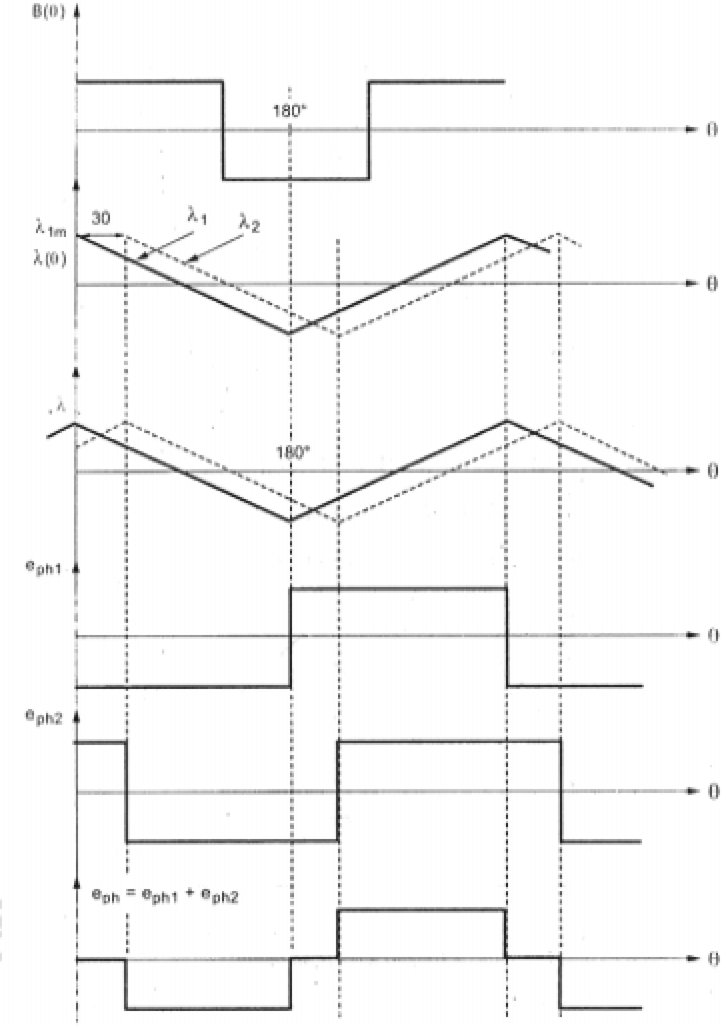
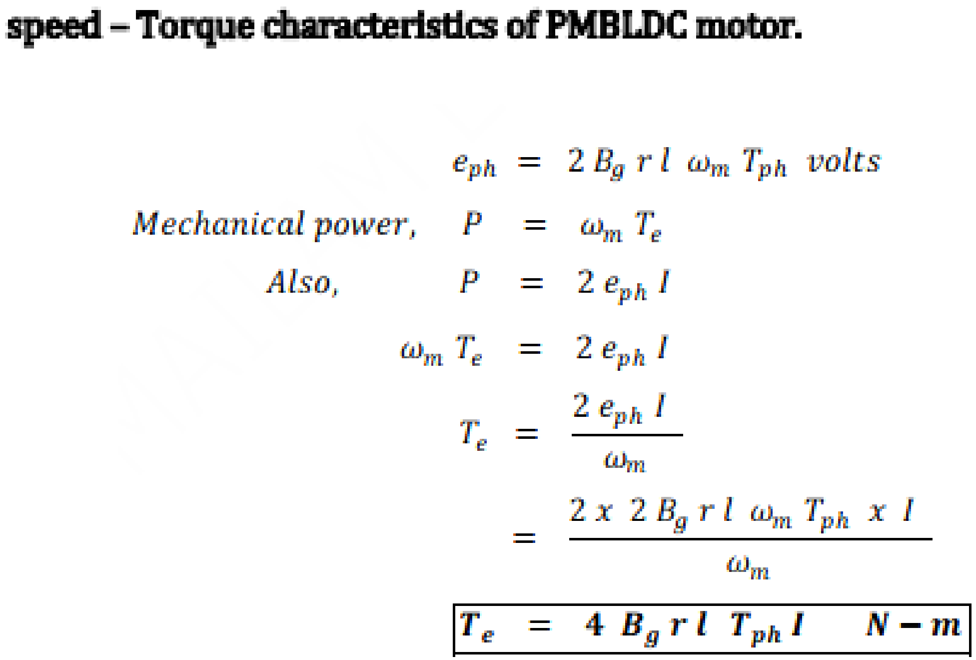


Fig. Waveform of flux density and flux linkage



speed – Torque characteristics of PMBLDC motor.

$$e_{ph} = 2\,B_g\,r\,l\;\omega_m\,T_{ph}\;\; volts$$

$$Mechanical\ power,\quad P = \omega_m\,T_e$$

$$Also,\quad P = 2\,e_{ph}\,I$$

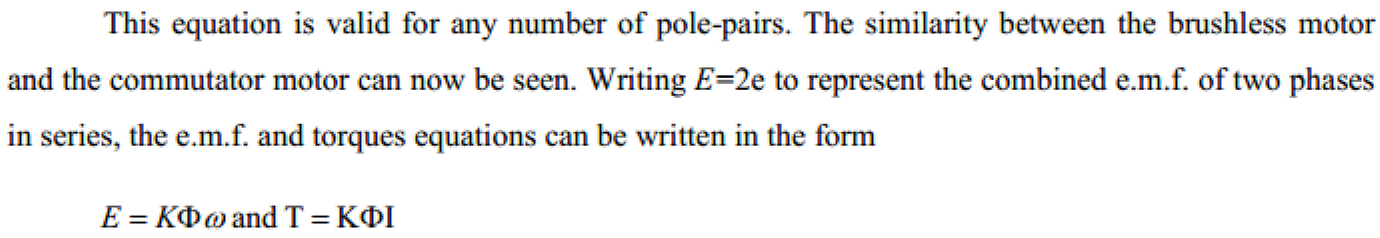
$$\omega_m\,T_e = 2\,e_{ph}\,I$$

$$T_e = \frac{2\,e_{ph}\,I}{\omega_m}$$

$$= \frac{2\ x\ \ 2\,B_g\,r\,l\;\omega_m\,T_{ph}\ \ x\ \ I}{\omega_m}$$

$$\boxed{\boldsymbol{T_e = 4\;B_g\,r\,l\;\;T_{ph}\,I \qquad N-m}}$$

If the phase windings are star-connected, then at any time there are just two phases and two transistors conducting.



This equation is valid for any number of pole-pairs. The similarity between the brushless motor and the commutator motor can now be seen. Writing E=2e to represent the combined e.m.f. of two phases in series, the e.m.f. and torques equations can be written in the form

$E = K\Phi\omega$ and $\mathrm{T} = \mathrm{K\Phi I}$

V = E + RI

Using this equation together with the e.m.f. and torque equations, the torque/speed characteristic can be derived as:

$$\omega = \omega_0 \left[1 - \frac{T}{T_0} \right].$$

$$\omega_0 = \frac{V}{k\Phi} rad / \sec$$

and the stall torque is given by

$$T_0 = k\Phi I_0$$

This is the torque with the motor stalled, i.e. at zero speed. The stall current is given by

$$I_0 = \frac{V}{R}$$

speed – Torque characteristics of PMBLDC motor.

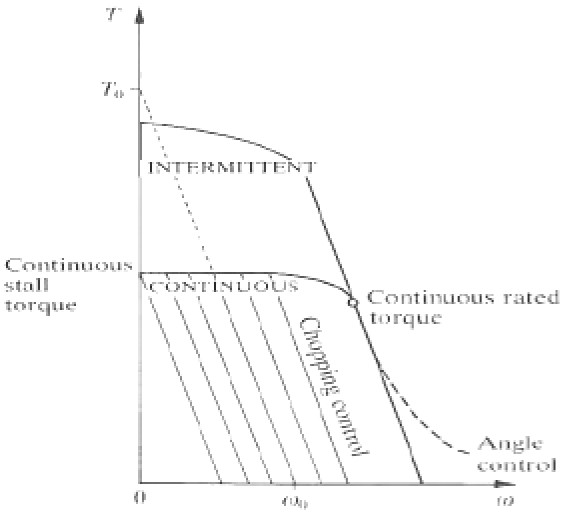


Fig. Speed-Torque characteristics of BLDC motors

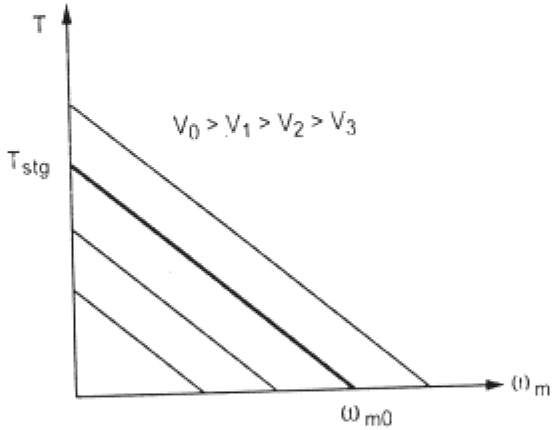
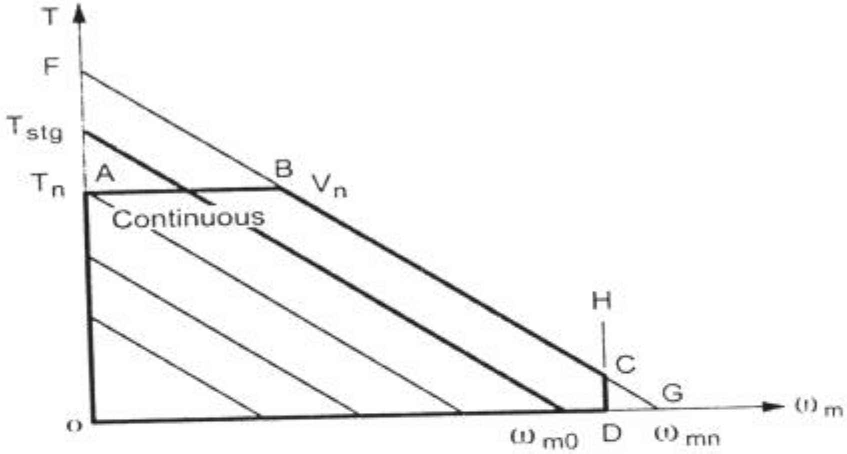


Fig. Family of T-N characteristics for various constant supply voltages.

Permissible region of operation:

- Current, torque, supply voltage, speed should be within limits.



Line AB:

- ✓ Parallel to x-axis
- ✓ Max. torque developed

Line FG:

- ✓ N-T characteristics for max. permissible voltage

Line DH:

- ✓ Perpendicular to x-axis
- ✓ Max. permissible speed

OABCDO – permissible region of operation

Power converter circuits and their controllers.

Classification of drive circuits:

- Classified based on the no. of phase windings and the no. of pulses given to the devices.
  - One phase winding and one pulse BLDC motor
  - One phase winding and two pulse BLDC motor
  - Two phase winding and two pulse BLDC motor
  - Three-phase winding and three pulse BLDC motor
  - Three-phase winding and six pulse BLDC motor

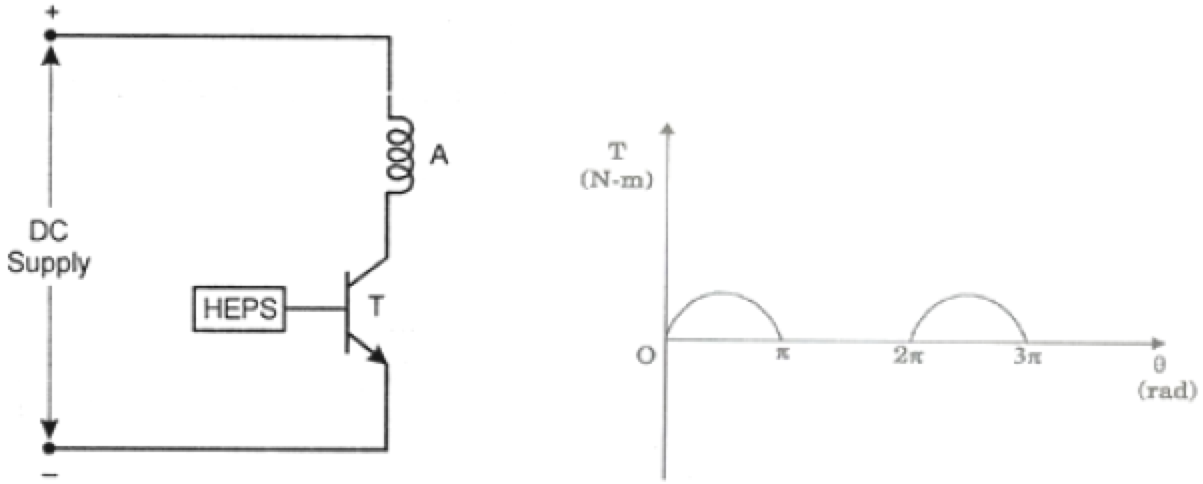


Fig. Stator with one phase winding

(a) One phase winding and one pulse BLDC motor

- Stator has one phase winding.
- Connected to the supply through a semi conductor switch.
- When rotor position sensor is influenced by north pole, it turns ON the switch.
- Current flows through the stator winding and develops torque.
- The current & torque are approximated as sinusoidally varying.

Advantages:

- Consists of the one transistor and one position sensor is sufficient.

De-merits:

- Inertia should be high, such that rotor rotates continuously.
- Utilization of Transistor & winding are less.

- The polarity of flux is altered depending upon the position of rotor.

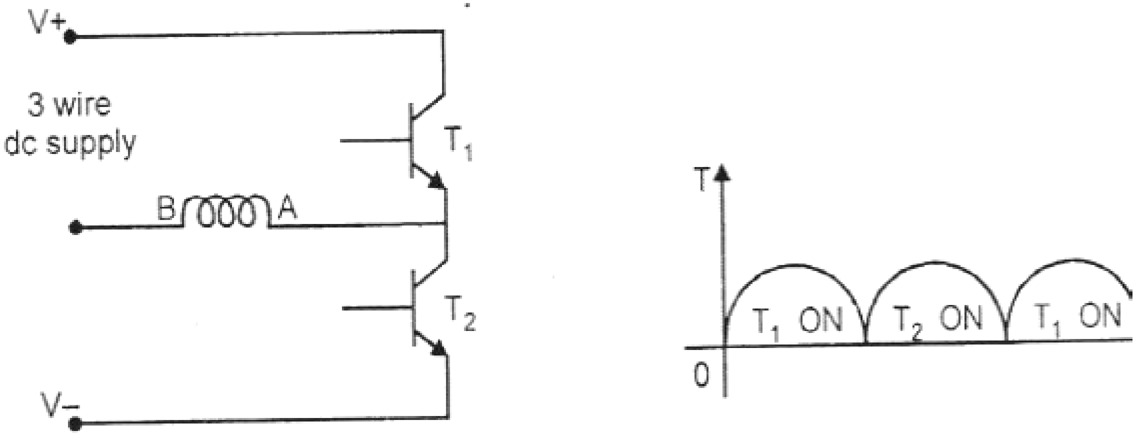


Fig. One phase winding and two pulse BLDC motor

(b) One phase winding and two pulse BLDC motor

- Stator has only one winding.
- Connected to the 3 wire dc supply through, two semi-conductor switches.
- Only one position sensor.

When the position sensor is under the influence of North pole:

T_1 is turned ON, phase winding carries current from A to B.

When the position sensor is under the influence of South pole:

T_2 is turned ON, phase winding carries a current from B to A.

- The polarity of flux is altered depending upon the position of rotor.

Features:

- Winding utilization is better, however transistor utilization is less.
- Torque developed is more uniform.

Disadvantages:

- Requires three wire DC supply.

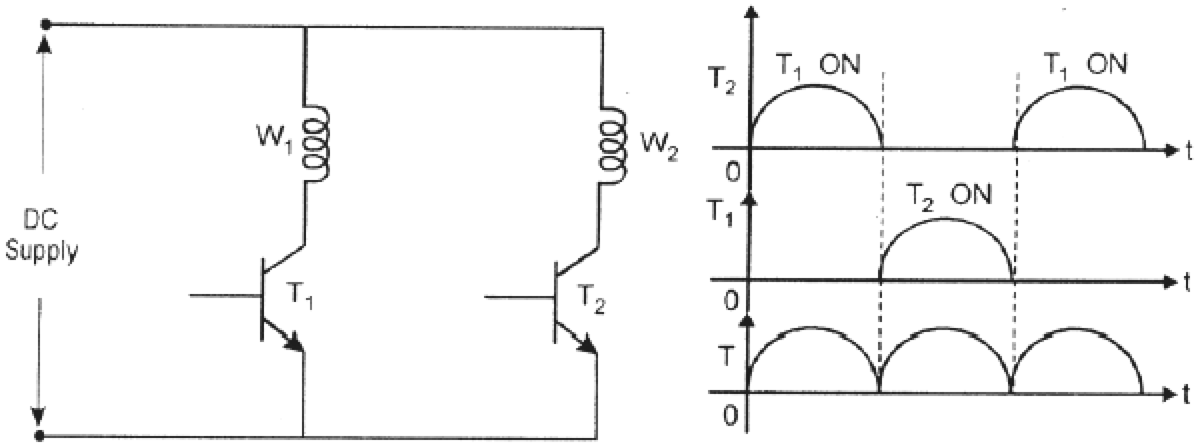


Fig. Two phase winding and two pulse BLDC motor

(c) Two phase winding and two pulse BLDC motor:

- Stator has two phase windings.
- Each phase winding is controlled by a switch depending on position of rotor.

Features:

- Windings utilization is only 50% which is less.
- It provides better torque waveforms.

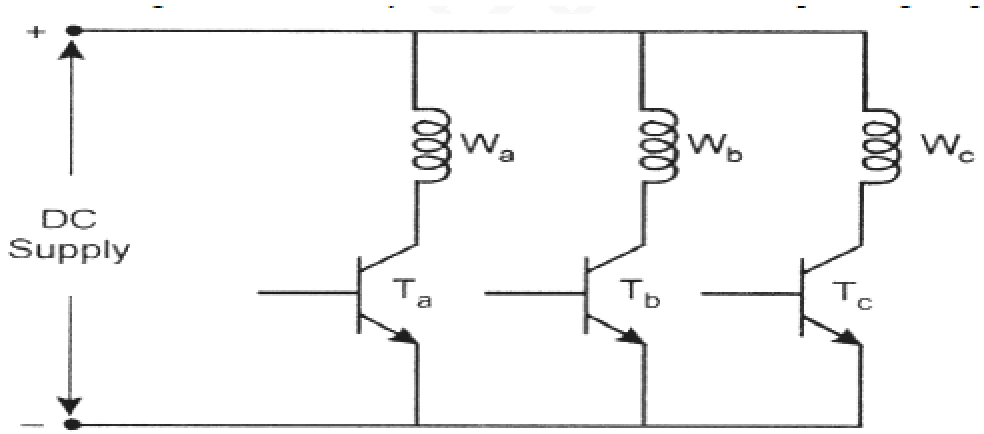


Fig. Three phase winding and three pulse BLDC motor

(d) Three phase winding and three pulse BLDC motor:

- The stator has three phase winding.
- Each phase winding is controlled by semi conductor switch, depending on position of rotor.

Disadvantage:

- 3 position sensors are required.

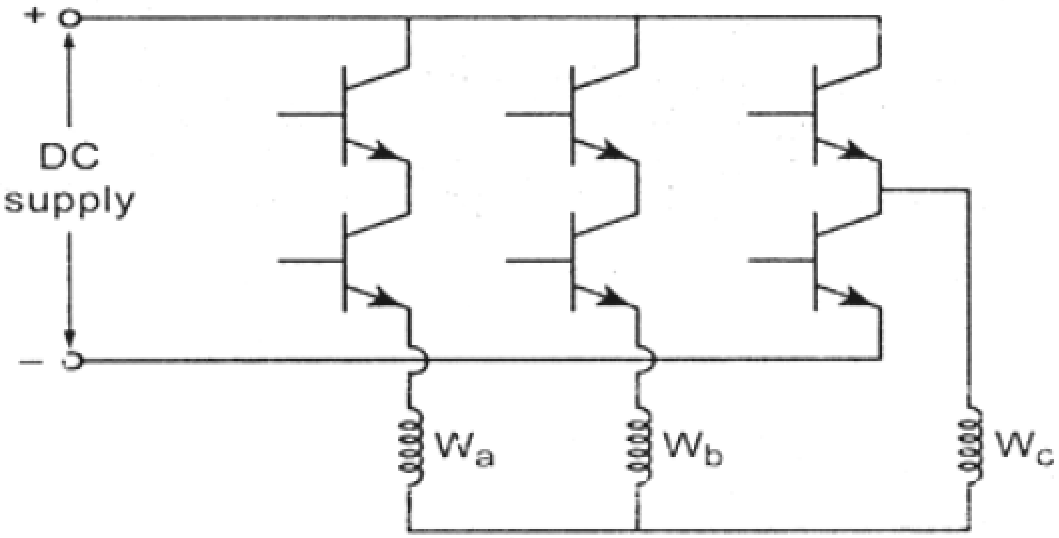


Fig. Three phase windings and six pulse BLDC motor

(e) Three phase windings and six pulse BLDC motor:

- Uses three phases and six switching devices.
- Usually 120 degree (or) 180 degree conduction is adopted.

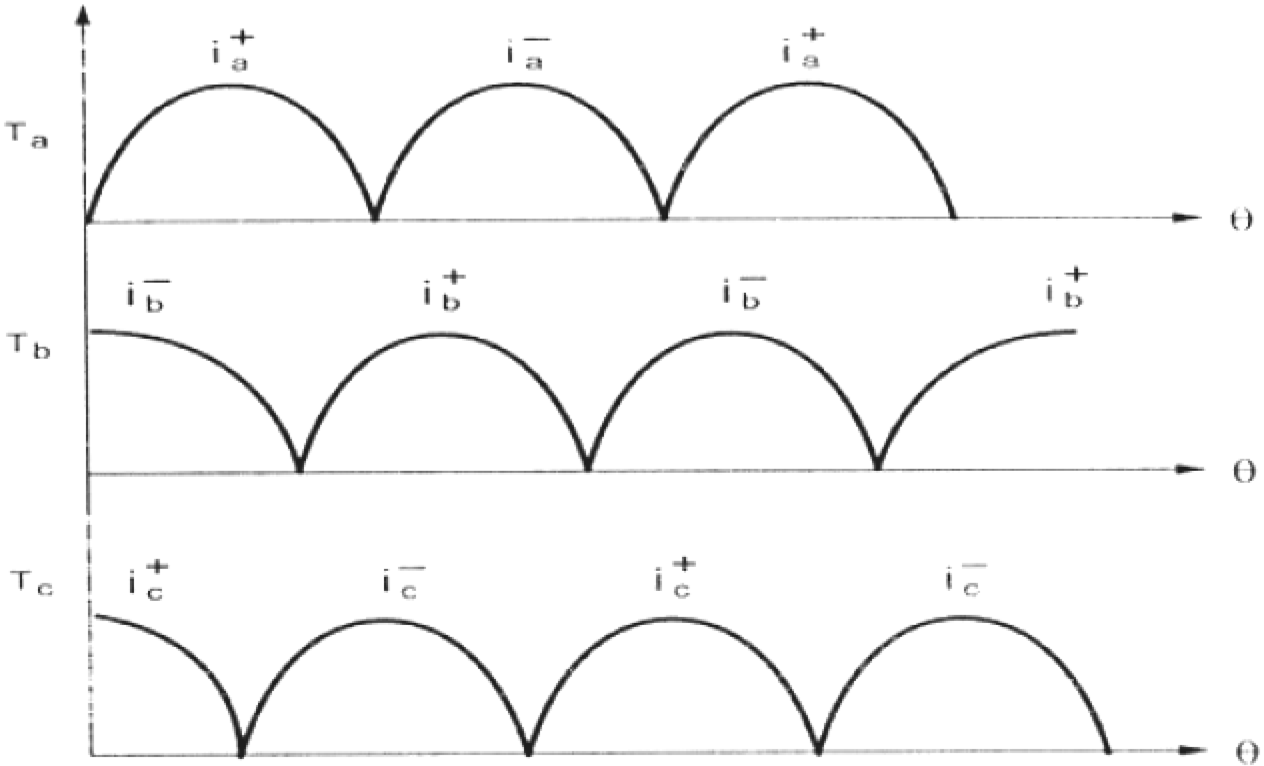


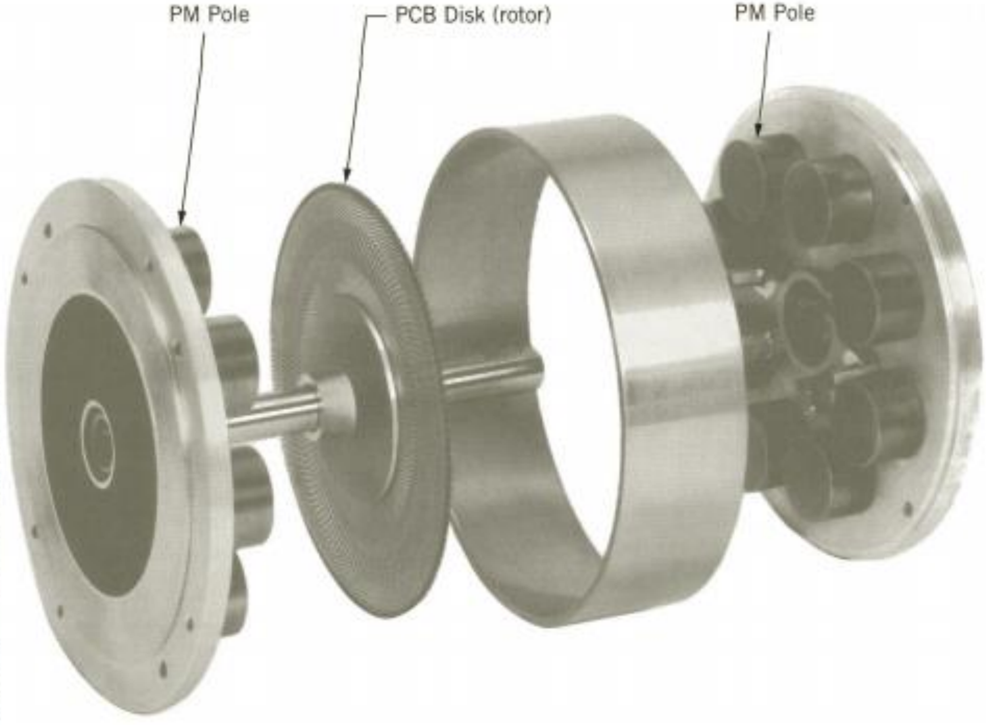
Fig. Torque waveforms

Features:

- Utilization factor of winding is better.
- Torque pulse and ripple frequency components are less.

Printed Circuit Board (PCB) Motor

The printed circuit board (or disk armature) motor, using permanent magnets, has a configuration radically different from that of the conventional dc motor. Figure 4.66 shows the construction of such a motor. The rotor has no iron and is formed of a disk of nonconducting, nonmagnetic material. The entire armature winding and the commutator are printed in copper on both sides of the disk. The brushes are placed around its inner periphery. The disk armature is placed between two sets of permanent magnets mounted on ferromagnetic end plates. This configuration



Courtesy of PMI Motion Technologies

provides axial flux through the armature. The radial current flowing through the disk armature interacts with the axial flux to produce torque that rotates the rotor, as in any dc motor.

This type of motor has several advantages:

Because of its low rotor inertia, it has a high torque/inertia ratio and thus can provide rapid acceleration and deceleration. The motor can accelerate from 0 to 4000 rpm in 10 milliseconds.

The armature inductance is low, because there is no iron in the rotor. Because of the low inductance, there is little arcing, which leads to longer brush life and high-speed capability. Low armature inductance makes the armature time constant low. Consequently, the armature current can build up very quickly (in less than 1 millisecond), which implies that full torque is available almost instantly, a key to quick motion response and accurate tracking.

The motor has no cogging torque, because the rotor is nonmagnetic.

These motors are particularly suitable for applications requiring high performance characteristics. Examples are high-speed tape readers, $X-Y$ recorders, point-to-point tool positioners, robots, and other servo drives. Typical sizes of these motors are in the fractional horsepower ranges. However, integral horsepower sizes are also available.

Advantages of Printed Circuit Board Motor

The various advantages of the PCB motor are as follows:-

- The motor provides quick acceleration and retardation. As the inertia of the motor is very low and therefore, the ratio of torque and inertia is very high.
- The rotor does not contain iron, thus the armature inductance is low.
- The lower inductance of the motor reduces sparking and as a result life of brushes is increased.
- Cogging torque is absent because of the non-magnetic rotor.
- A PCB motor has a high overload current capacity.
- There is a negligible armature reaction and flux distortion and hence the speed torque curve of the motor is linear.

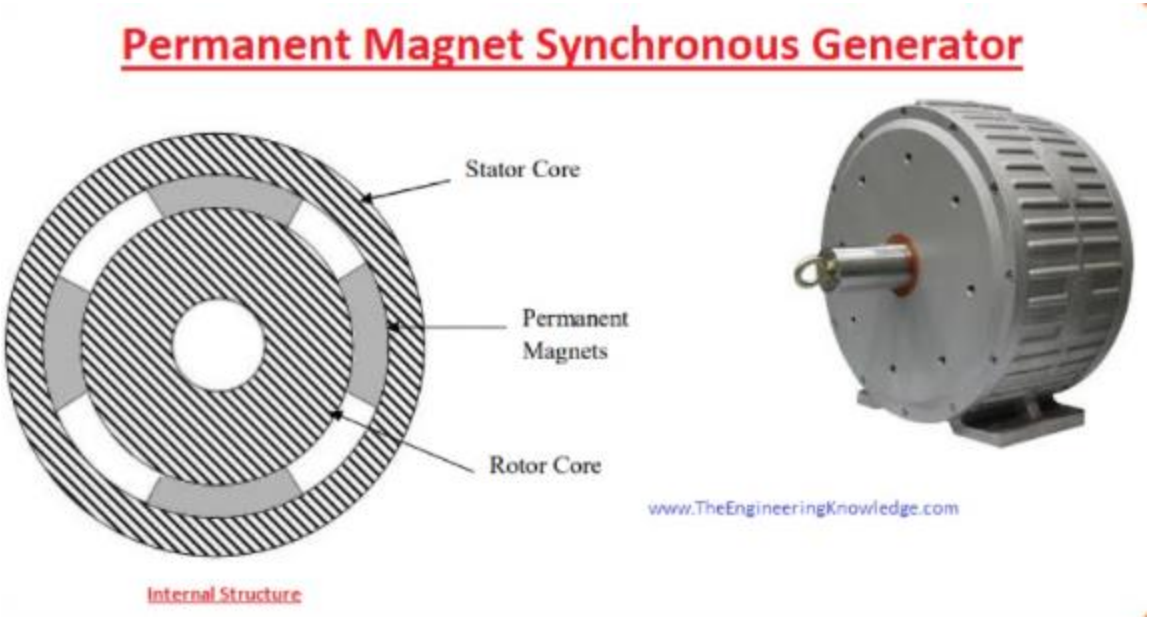
Applications of the Printed Circuit Board Motor

The characteristics of the PCB motor, i.e. the high torque and inertia ratio makes the motor suitable for controlling applications. The various usage of the motor is as follows:-

- Used in high-speed tape readers.
- PCB motor is used in X-Y recorders, point to point tool positioner.
- Used in robots and other servo drives.
- It is also suitable for heavy-duty drives such as lawn mowers.

As there is inbuilt optical position encoder thus it can be used in some place of the stepper motor.

Permanent Magnet Synchronous Generator



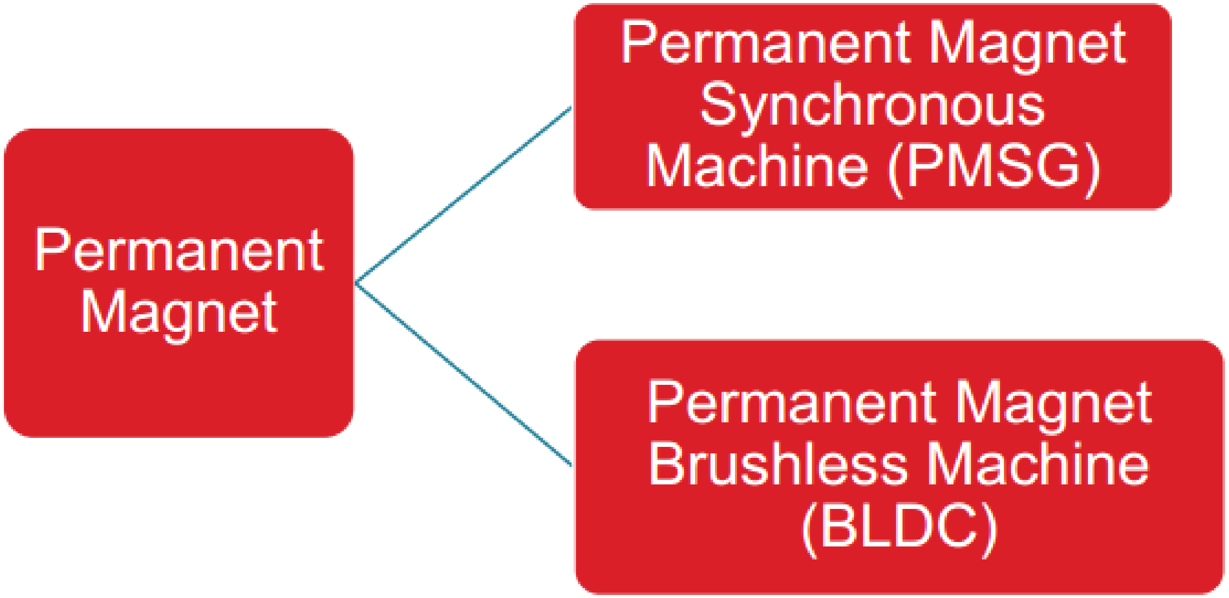
Permanent Magnet Technology

The use of permanent magnets (PMs) in construction of electrical machines

brings the following benefits:

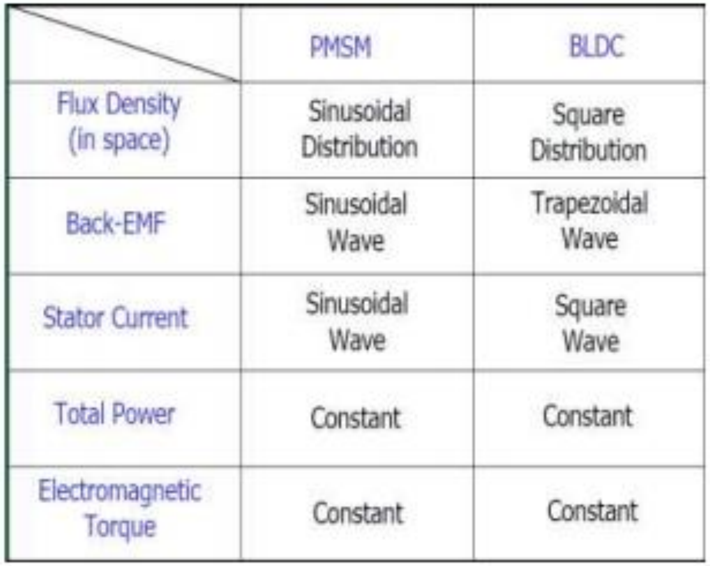
- *No electrical energy is absorbed by the field excitation system and thus there are no excitation losses which means substantial increase in the efficiency*,
- *Higher torque and/or output power per volume than when using electromagnetic excitation,*
- *Better dynamic performance than motors with electromagnetic excitation (higher magnetic flux density in the air gap),*
- *Simplification of construction and maintenance,*
- *Reduction of prices for some types of machines.*

Permanent Magnet Classification

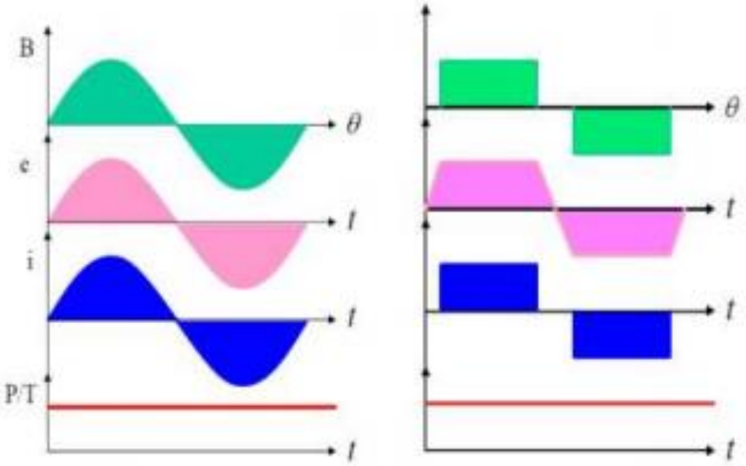


- Permanent Magnet
  - Permanent Magnet Synchronous Machine (PMSG)
  - Permanent Magnet Brushless Machine (BLDC)

Permanent Magnet Classification



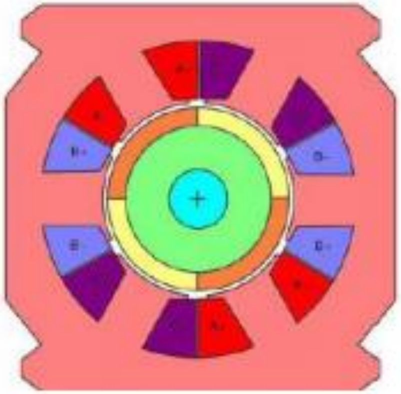
	PMSM	BLDC
Flux Density (in space)	Sinusoidal Distribution	Square Distribution
Back-EMF	Sinusoidal Wave	Trapezoidal Wave
Stator Current	Sinusoidal Wave	Square Wave
Total Power	Constant	Constant
Electromagnetic Torque	Constant	Constant



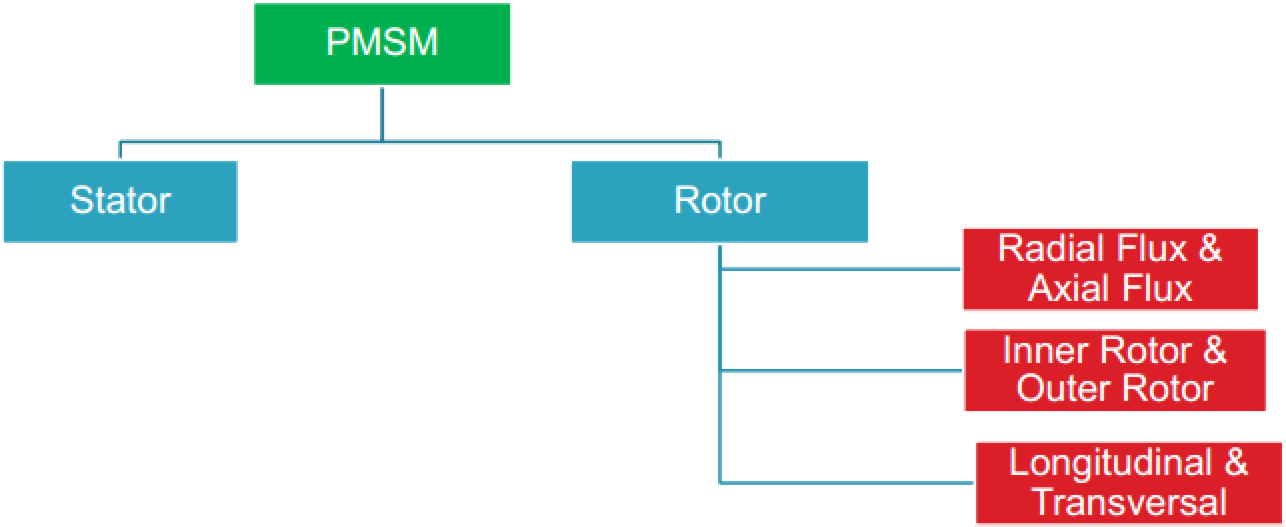
PMSM BLDC

Introduction

- PM Synchronous Machine are widely used in
  - Wind mile generation
  - Industrial servo-applications due to its high-performance characteristics.
- General characteristics
  - Compact
  - High efficiency (no excitation current)
  - Smooth torque
  - Low acoustic noise
  - Fast dynamic response (both torque and speed)
  - Expensive



Construction



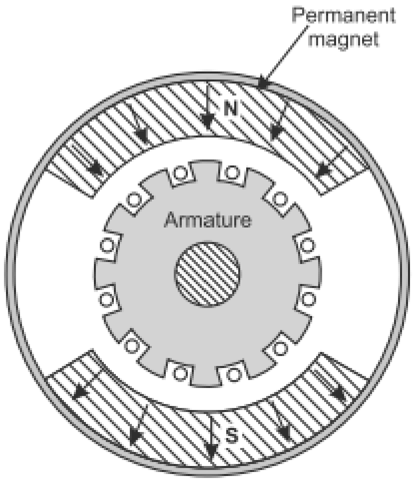
PMSM

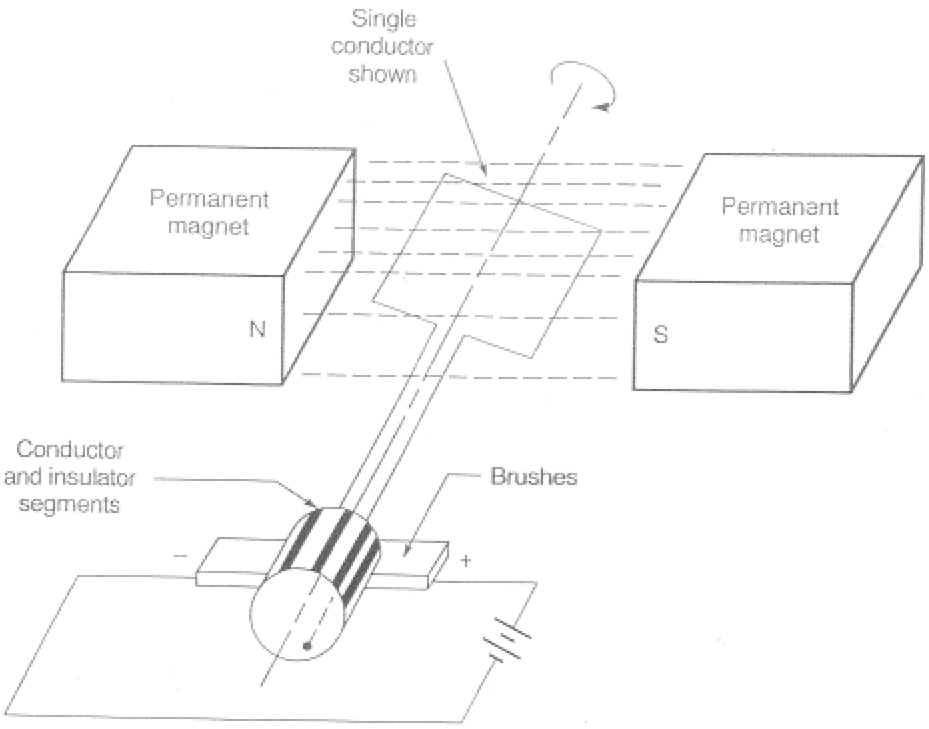
- Stator
- Rotor
  - Radial Flux & Axial Flux
  - Inner Rotor & Outer Rotor
  - Longitudinal & Transversal

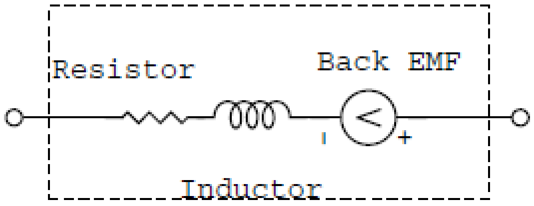
Permanent Magnet DC Motors

Construction

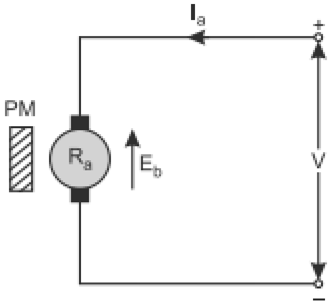
In these motors, the permanent magnets are supported by a cylindrical steel stator which provides an easy path for the magnetic flux. The cross-sectional view of a 2-pole permanent magnet DC motor is shown in Fig. [redacted] The permanent magnets occupy comparatively less space which reduces the overall size of the machine. The armature with conductors in its slots, commentator and brushes, etc, are the same as in a conventional DC machine.



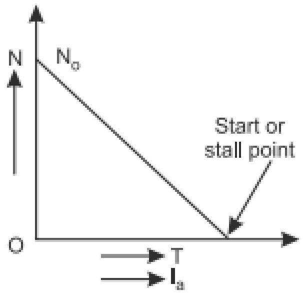




Electrical Model of a Permanent Magnet Brushed DC Motor



Schematic diagram



Speed-torque characteristics

$$E_b = V - I_a R_a$$

Motor speed, $N \alpha \dfrac{E_b}{\phi}$ Or $N \alpha \dfrac{V - I_a R_a}{\phi}$

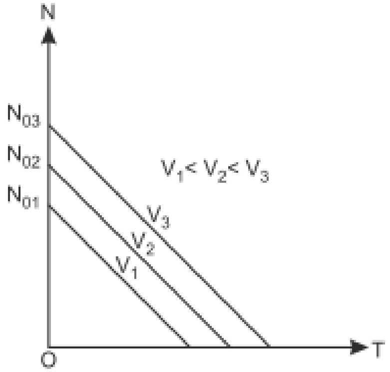
$N \alpha \; V - I_a R_a$ (since ϕ is constant)

$T \alpha I_a$ (as $T \alpha I_a \phi$ and ϕ is constant)

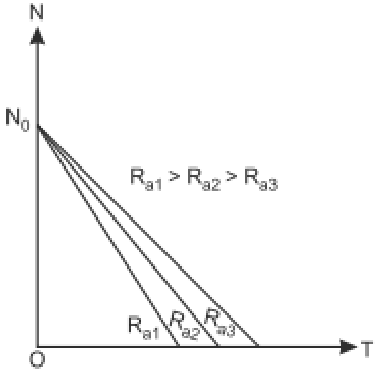
The characteristics of a permanent magnet (*PM*) DC motor can be changed by changing either

(*i*) the supply voltage, V or

(*ii*) resistance of the armature circuit



Effect of change in supply voltage



Effect of change in armature circuit resistance

By changing the supply voltage, the no-load speed N_o of the motor is changed without affecting the slope of the characteristic

By change the effective resistance of the armature circuit (inserting some resistance in series with the armature), the slope of the speed-torque characteristic changes without affecting the no-load speed N_o of the motor

Salient Features of PM DC Motor

(*i*) Generally, the size of this motor, for the same rating, is smaller than the excited field wound motors.

(*ii*) Since these motors do not have exciting field winding, losses are reduced and improved efficiency is obtained.

(*iii*) Usually, these motors are used for low voltage (less than 12V) and smaller power rating (0.1 to 5W).

(*iv*) Smaller life for small motors which is basically determined by the commutator and brushes used. Usually, it is limited to 1000 to 2000 hour.

While designing such motors, a compromise is made between cost, life span and efficiency. Since these motors are operated by battery, these are designed for an efficiency of 80%.

Applications

These motors are used for toys, windshield wipers, cordless tools, shavers, portable vacuum cleaners, automobile heaters, fans, wheel-chairs etc. For all these applications, millions of small *PM* DC motors with the ratings of 0.1 to 5W, 12 V or less than 12 V are produced every year.

A PMDC motor has armature resistance of 1 Ω. When fed from 48 V dc source, it runs at a speed of 2400 rpm while taking 0.8 A. Determine (a) the no-load rotational losses of the motor (b) the motor output when running at a speed of 1600 rpm and with source voltage of 40 V dc and (c) its stall torque when the source voltage is 20 V dc.

$$E_a = 48 - 0.8 \times 1 = 47.2 \text{ V}$$

At no load, all the electromagnetic power developed is used to supply the no-load rotational losses.

∴ No-load rotational losses

$$= E_a I_a = 47.8 \times 0.8 = 37.76 \text{ W}$$

(*b*) $$E_a = K_m \omega_m$$

$$\therefore \text{ Speed-voltage constant, } K_m = \frac{47.2 \times 60}{2\pi \times 2400} = 0.188 \text{ V-s/rad}$$

$$\text{For a speed of 1600 rpm,} \quad \omega_m = \frac{2\pi \times 1600}{60} \text{ rad/s}$$

$$\text{Generated emf,} \quad E_a = K_m \omega_m = 0.188 \times \frac{2\pi \times 80}{3} = 31.5 \text{ V}$$

$$\text{New armature current, } I_a = \frac{V_t - E_a}{r_a} = \frac{40 - 31.5}{1.0} = 8.5 \text{ A}$$

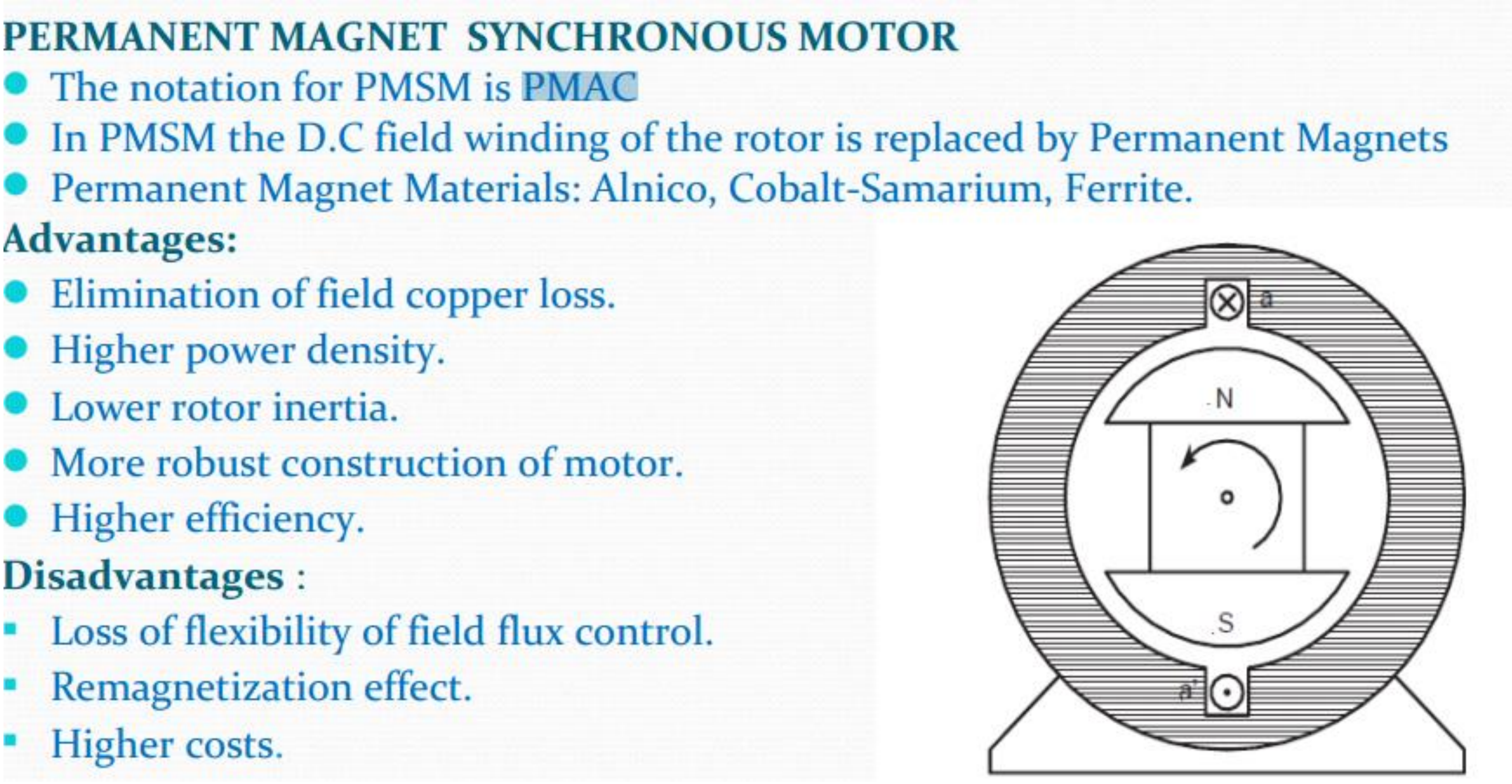
Electromagnetic power developed

$$= E_a I_a = 31.5 \times 8.5 = 267.75 \text{ W}$$

$$\therefore \text{ Output, or shaft power } = E_a I_a - \text{no-load rotational losses}$$

$$= 267.75 - 37.76 \cong 230.0 \text{ W}$$

Permanent Magnet AC Motor



PERMANENT MAGNET SYNCHRONOUS MOTOR

- The notation for PMSM is PMAC
- In PMSM the D.C field winding of the rotor is replaced by Permanent Magnets
- Permanent Magnet Materials: Alnico, Cobalt-Samarium, Ferrite.

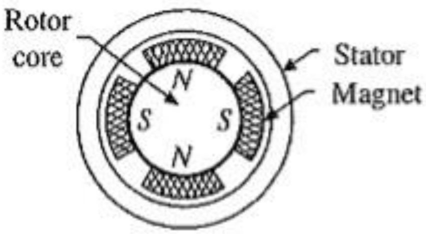
Advantages:

- Elimination of field copper loss.
- Higher power density.
- Lower rotor inertia.
- More robust construction of motor.
- Higher efficiency.

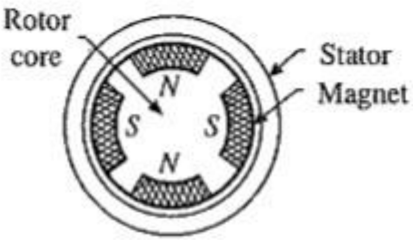
Disadvantages :

- Loss of flexibility of field flux control.
- Remagnetization effect.
- Higher costs.

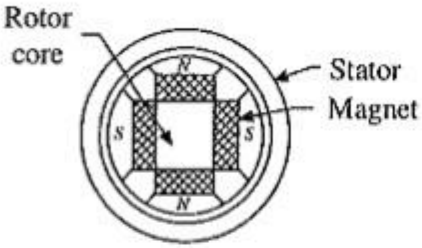
Permanent Magnet Synchronous Motors Types



(a) Projecting surface mounting



(b) Inset magnet



(a) Interior magnet

1.Surface Mounted-PMSM

- Projecting type
- Inset type

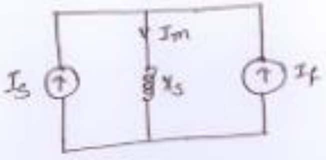
2.Interioror Buried-PMSM

SINUSOIDAL PMAC MOTOR DRIVES.

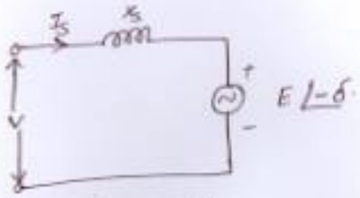
→ Let us examine the behavior of a sinusoidal PMAC motor with a Variable frequency current source.

→ since, it is a 3-Phase supply all the 3-Phase currents with a phase difference of 120° between them.

→ The Norton's and Thevinin's equivalent of the Synchronous motor are shown below.



Norton's ckt



Thevenin's ckt

→ for the variable frequency current source, Consider the Nortons equivalent circuit, where.

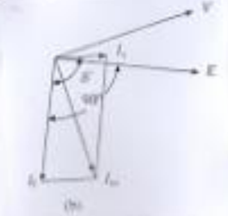
$$\bar{I}_f = \frac{\bar{E}}{jX_s} = \frac{E}{X_s}\angle -(\delta + \pi/2) \quad \text{①}$$

where X_s - synchronous reactance

E - excitation emf.

and $\bar{I}_m = \bar{I}_s + \bar{I}_f$ —②

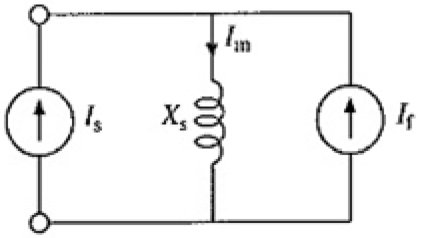
→ The Phasor diagram of motor, with "I_s" as a reference Phasor is shown in the figure



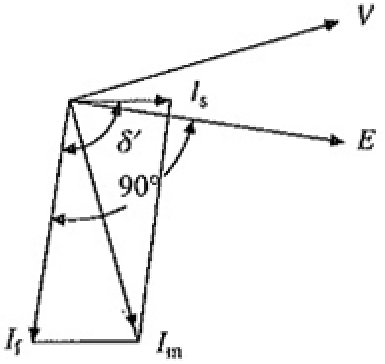
- Since the voltages induced in the stator phases of a sinusoidal PMAC motor are sinusoidal, ideally, the three stator phases must be supplied with variable frequency sinusoidal voltages or currents with a phase difference of 120° between them. Let us now examine its behavior from a variable frequency current source.
- Norton’s equivalent of the synchronous motor equivalent circuit

$$\bar{I}_f = \frac{\bar{E}}{jX_s} = \frac{E}{X_s} \angle -(\delta + \pi/2)$$

$$\bar{I}_m = \bar{I}_s + \bar{I}_f$$



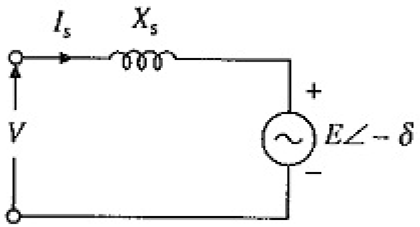
(a)



(b)

The mechanical power developed is

$$P_m = 3EI_s \cos(\delta' - \pi/2)$$



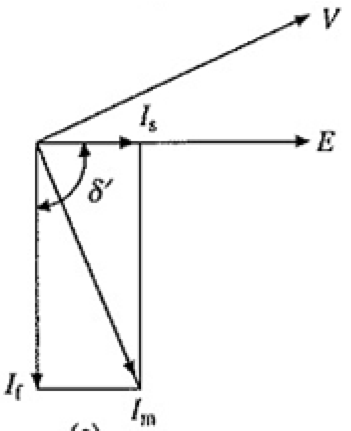
$$P_m = 3X_s\, I_s I_f \sin \delta'$$

$$T = \frac{P_m}{\omega_{ms}} = KI_s I_f \sin \delta'$$

where $K = 3X_s / \omega_{ms}$ = constant.

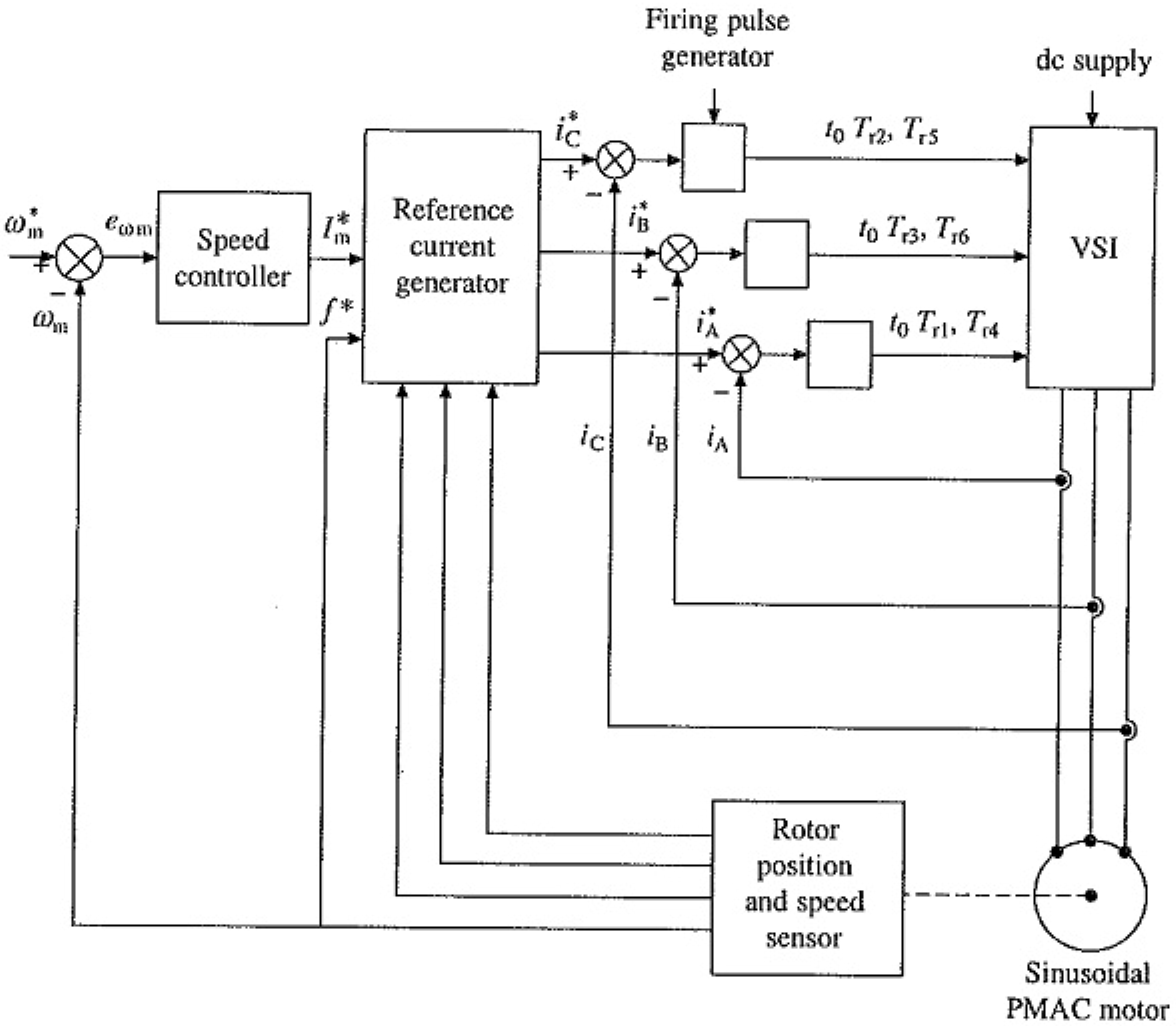
$$\delta' = \pm 90°$$

$$T = \pm KI_f I_s = \pm K_T I_s$$



(c)

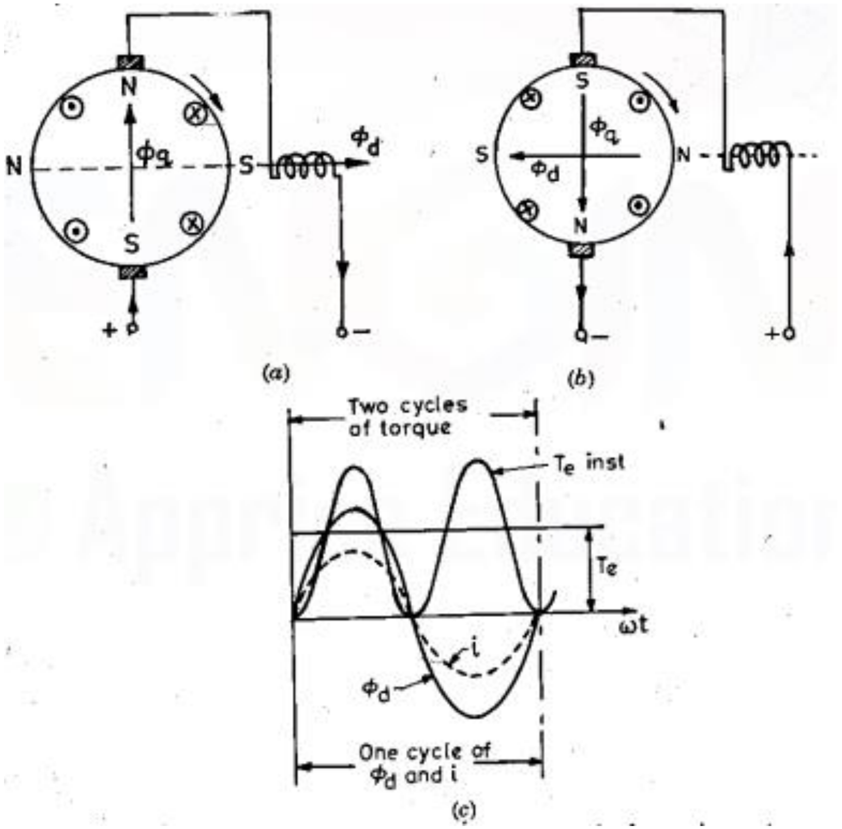
Hence torque is proportional to I_s.



Current regulated VSI fed sinusoidal PMAC motor drive for servo application

Unit 5

Single phase series motor (Universal Motor)



(a) (b)

(c)

Pertaining to the development of torque in d.c. series motor when fed from a.c. source.

1 $I_m \rightarrow$ No-load current

I_m → $I_m \cos\beta$ $\xrightarrow{\text{establishes}}$ ϕ_d

→ $I_m \sin\beta$ $\xrightarrow{\text{accounts for}}$ core losses + Friction & windage losses

2 Shorting End terminals

* Due to ϕ_d → Transformer emf E_{td} will be induced and lags by 90° (in armature winding)

* Due to ϕ_q → Rotational emf E_{rq} is induced in phase with ϕ_q

Note: ϕ_q is in time-phase with ϕ_d, but is in space displacement of 90°

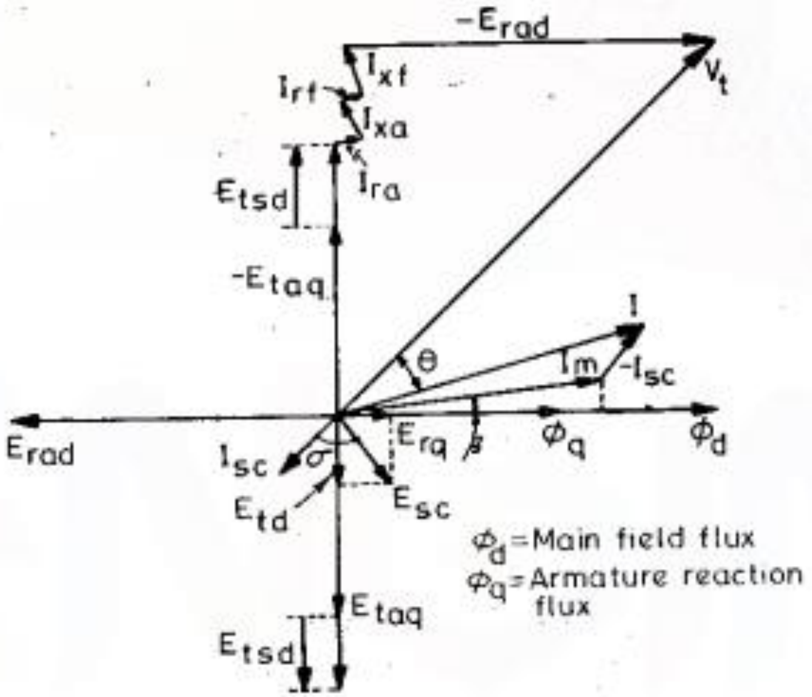
3: Hence, $E_{td} + E_{rq} = E_{sc}$ is resultant emf of commutating coils (neglecting reactance emf) due to which current I_{sc} (lagging by angle σ) will flow.

$$\therefore I_{sc} = \frac{E_{sc}}{\text{commutated coil-impedance}}$$

$$\sigma = \tan^{-1}\frac{\text{coil reactance}}{\text{coil resistance}}$$

4 ∴ current in coil

$$I = I_m + (-I_{sc}) \quad \text{(see phasor diagram)}$$
$$= I_m - I_{sc}$$



5 voltage across brushes.

- * Armature - reaction flux ϕ_q $\xrightarrow{\text{causes}}$ $\underline{E_{taq}}$.

  lagging by 90 w.r.t ϕ_q

- * Flux ϕ_d $\xrightarrow{\text{induces}}$ $\underline{E_{rad}}$ in time antiphase (ie 180°) w.r.t. ϕ_d (Rotational EMF)

  because it will oppose the current flow

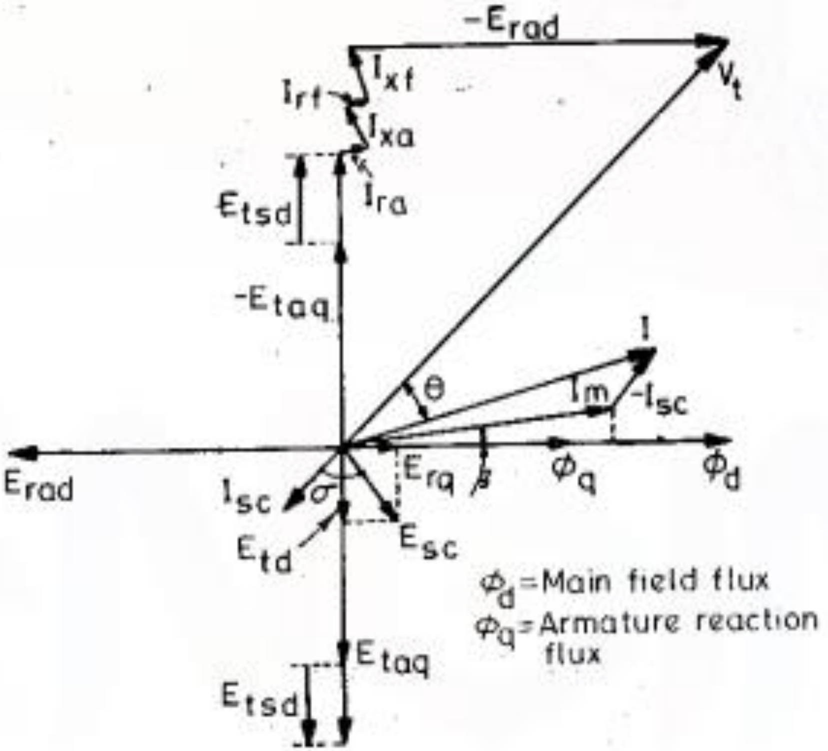
- * ϕ_d $\longrightarrow$ Transformer emf $\underline{E_{tsd}}$ lagging by 90° w.r.t ϕ_d (appears across FW)

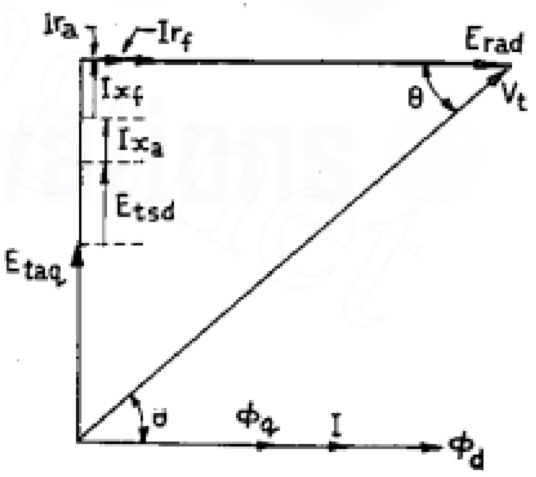
6 Armature current I $\xrightarrow{\text{produces drop}}$ $I(r_a + jx_a)$ $+$ $I(r_f + jx_f)$

$$\therefore V_t = I\left[(r_a + jx_a) + (r_f + jx_f)\right] + (-E_{taq}) + (-E_{tsd}) + (-E_{rad})$$

$$\therefore V_t = \left[\text{Field winding drop}\right] + \left[\text{Armature winding drop}\right]$$

$$\left[I(r_f + jx_f) + (-E_{tsd})\right] \qquad I(r_a + jx_a) + (-E_{taq}) + (-E_{rad})$$





Compensated A.C. Series Motors

- Conductively compensated motor
- Inductively compensated universal-motor

If compensated winding connected in series with armature and field winding in such a case motor is called a **conductively compensated motor.**

if compensating winding short-circuited on itself in such a case motor is called **inductively compensated** and received excitation voltage by transformer action since it is inductively coupled.

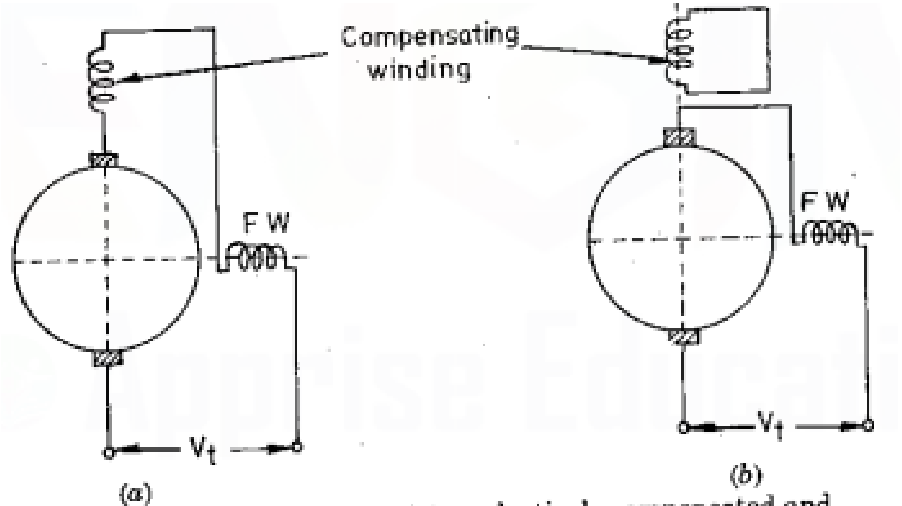
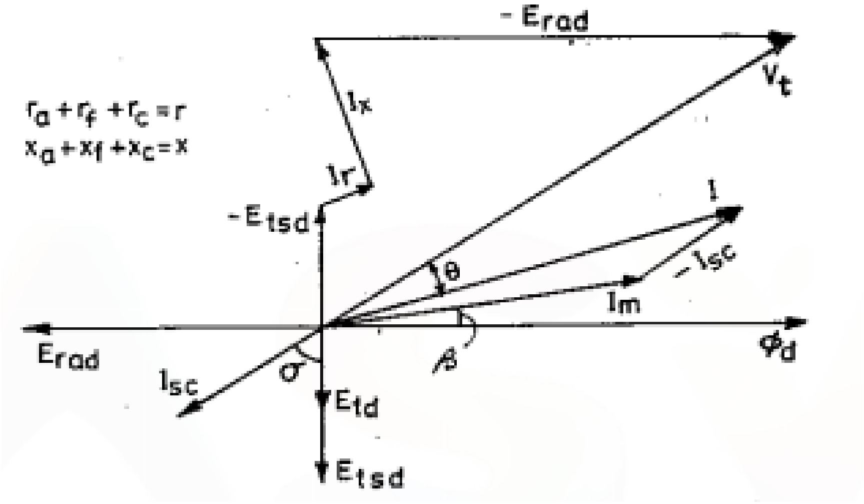
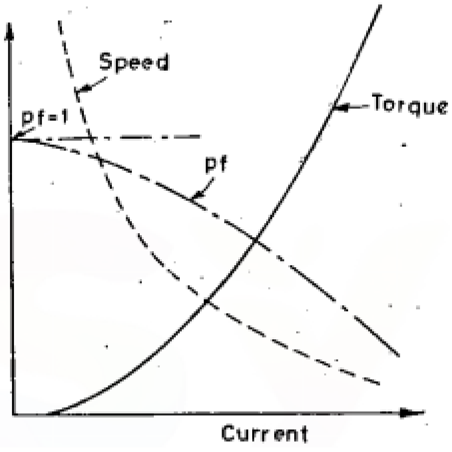


Fig. [redacted] A.C. series motor (*a*) conductively compensated and (*b*) inductively compensated. *FW* is field winding.

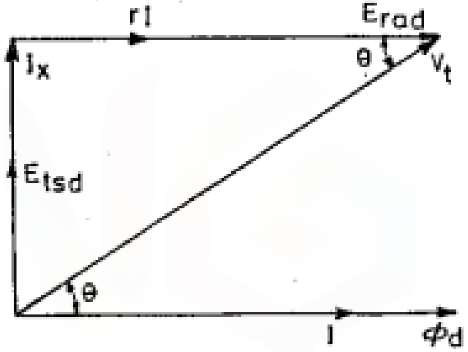


. Phasor diagram of a compensated A.C. series motor.



Typical characteristics of a 1-phase series motor.

It is seen from this figure that speed and power factor decrease whereas the electric torque increases as the motor is loaded.



Simplified phasor diagram

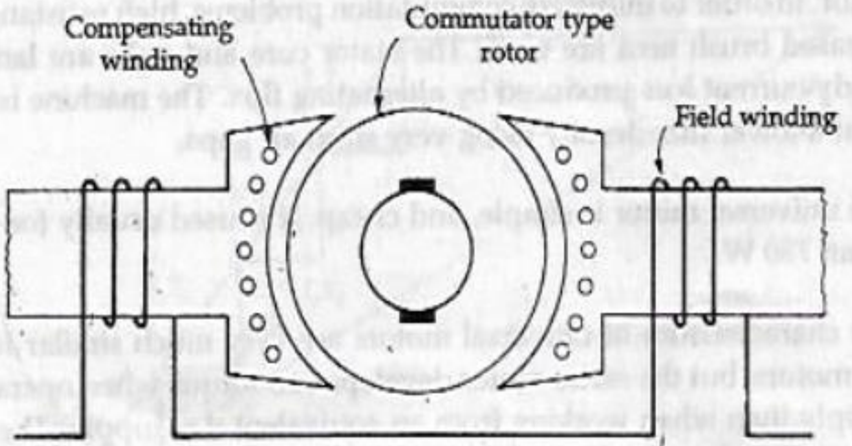
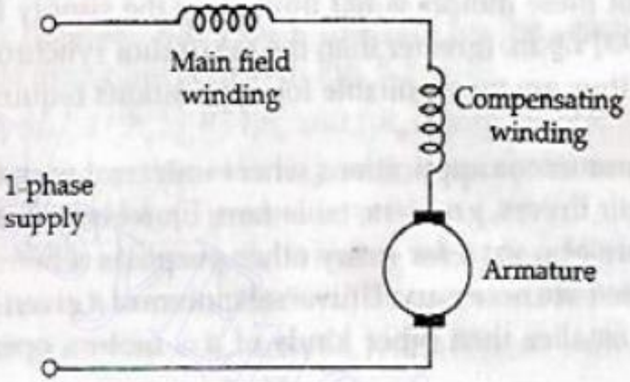


Fig. 8.17. *Series motor with conductively compensated winding.*



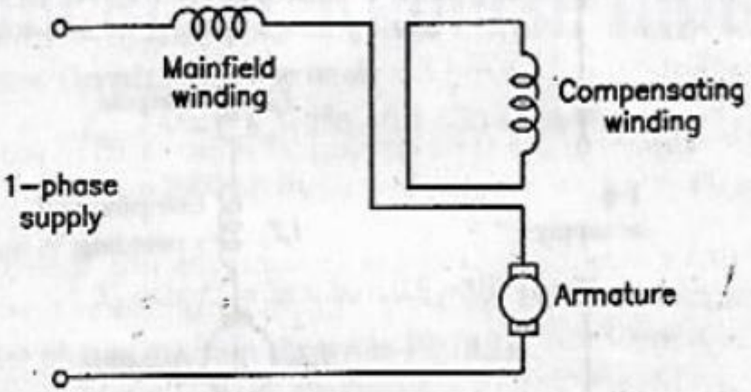
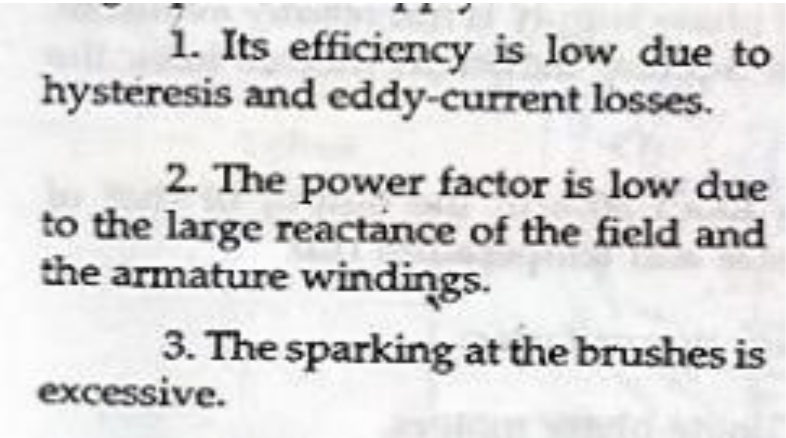


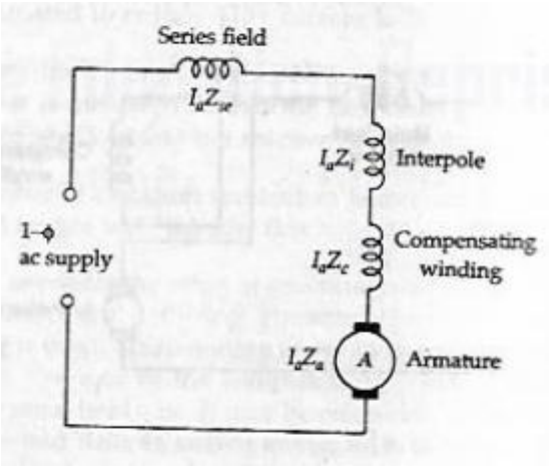
Fig. 8.19. *Series motor with inductively compensated winding.*

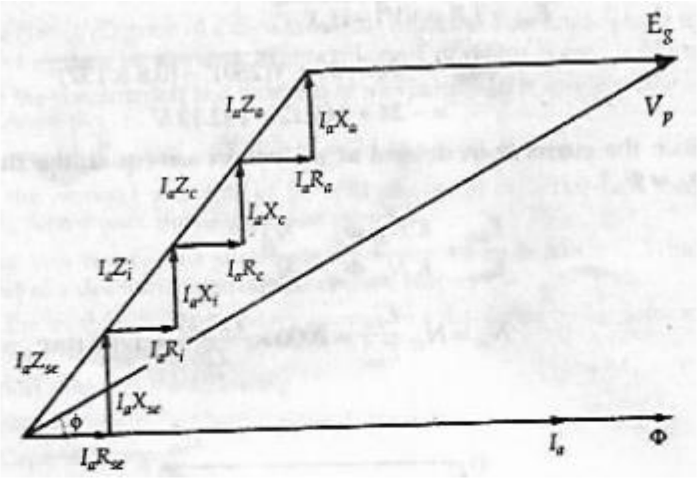


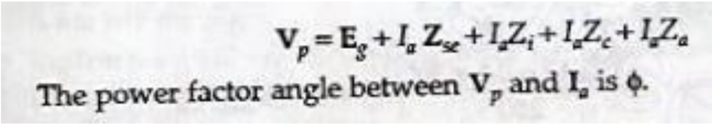
1. Its efficiency is low due to hysteresis and eddy-current losses.

2. The power factor is low due to the large reactance of the field and the armature windings.

3. The sparking at the brushes is excessive.

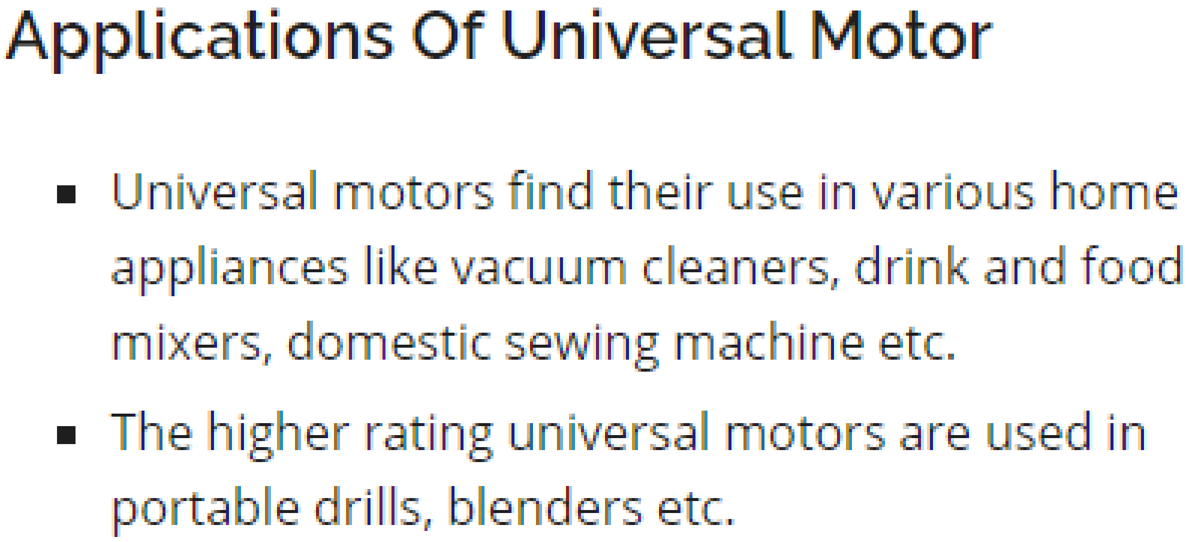






$$\mathbf{V}_p = \mathbf{E}_g + I_a \mathbf{Z}_{se} + I_a Z_i + I_a Z_c + I_a Z_a$$

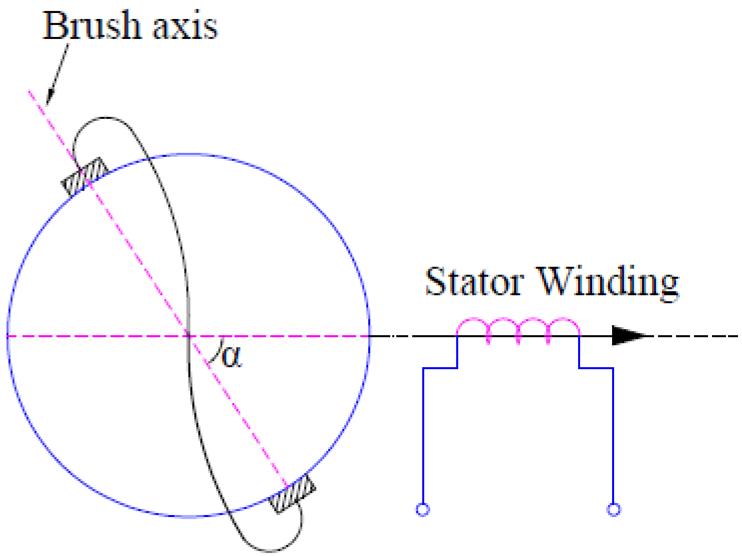
The power factor angle between $\mathbf{V}_p$ and $\mathbf{I}_a$ is ϕ.



Applications Of Universal Motor

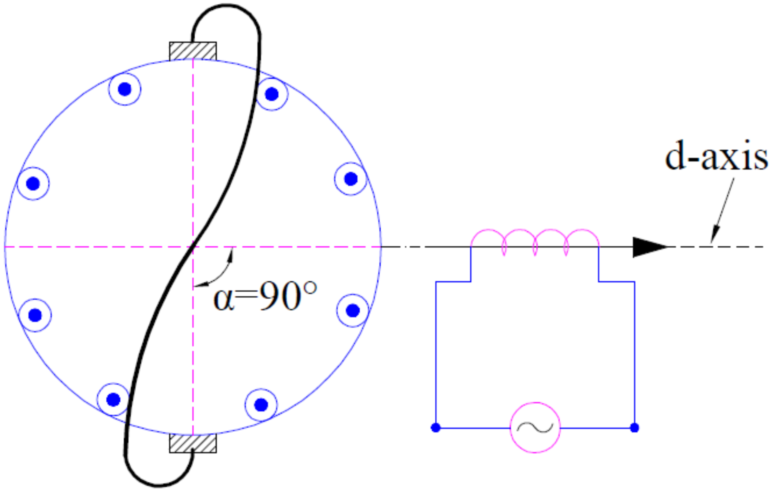
- Universal motors find their use in various home appliances like vacuum cleaners, drink and food mixers, domestic sewing machine etc.
- The higher rating universal motors are used in portable drills, blenders etc.

Repulsion Motor

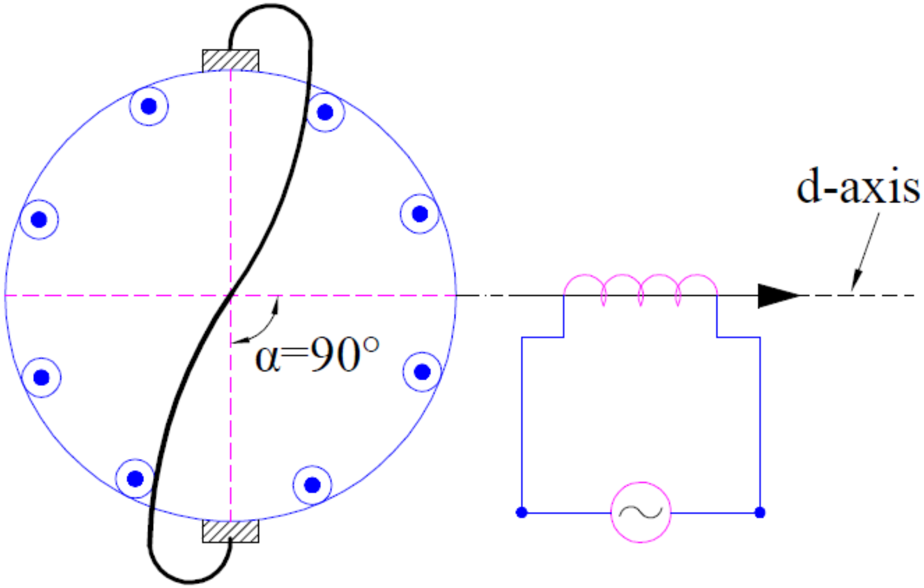


The main components of repulsion motor are stator, rotor and commutator brush assembly. The stator carries a single phase exciting winding similar to the main winding of single phase induction motor. The rotor has distributed DC winding connected to the commutator at one end just like in DC motor. The carbon brushes are short circuited on themselves.

- The basic principle behind the working of repulsion motor is that “similar poles repel each other.” This means two North poles will repel each other. Similarly, two South poles will repel each other.

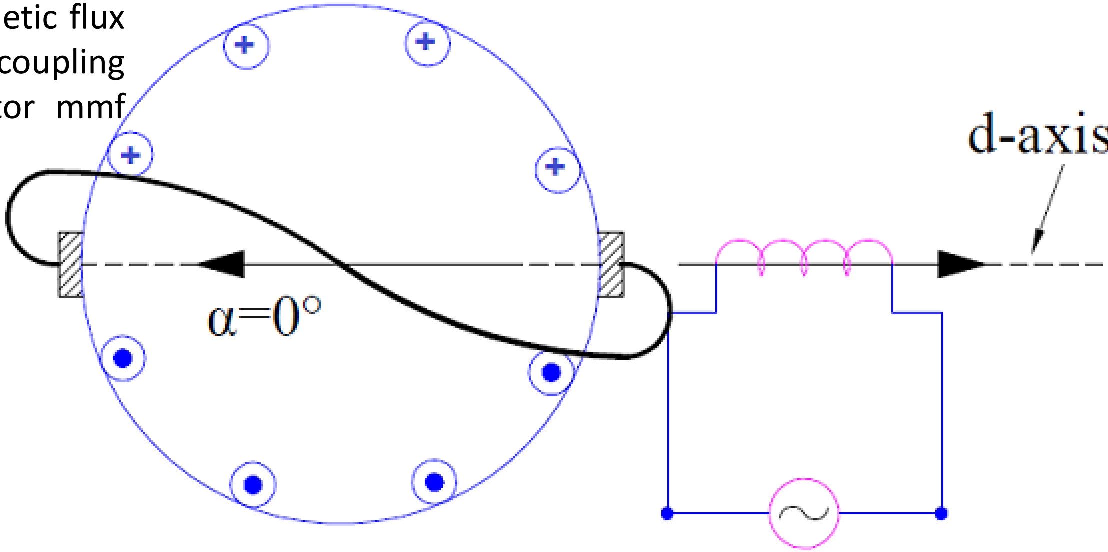


- When the stator winding of repulsion motor is supplied with single phase AC, it produces a magnetic flux along the direct axis as shown in figure above by arrow mark. This magnetic flux when link with the rotor winding, creates an emf. Due to this emf, a rotor current is produced. This rotor current in turn produces a magnetic flux which is directed along the brush axis due to commutator assembly. Due to the interaction of stator and rotor produced fluxes, an electromagnetic torque is produced. Let us discuss this aspect in detail.



Under this condition, there will not be any mutual induction between the stator and rotor windings. Therefore, no emf and hence no rotor current is produced. Thus no electromagnetic torque is developed.

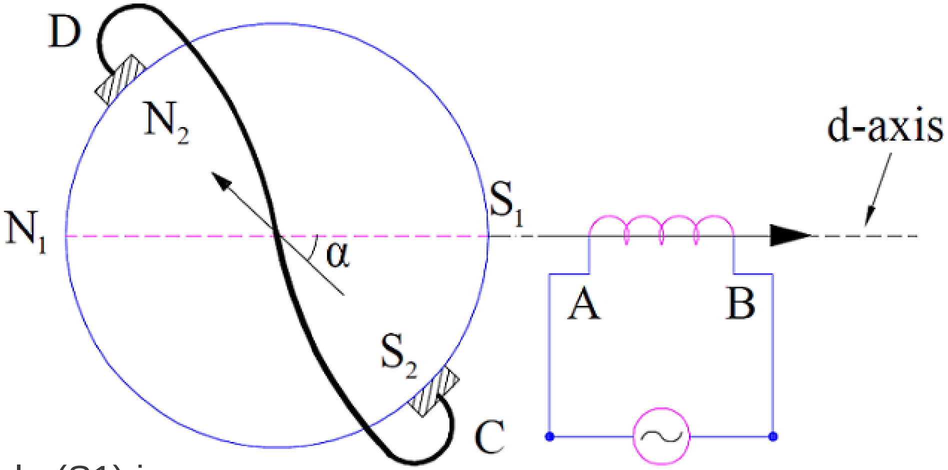
In this condition, a maximum emf is induced across the brushes. This is because, the rotor and stator magnetic flux coincides and hence there is a perfect mutual coupling between them. Large rotor currents produce rotor mmf opposite to stator mmf.



- No electromagnetic torque is developed as α = 0°. Thus in repulsion motor, no electromagnetic torque is developed when the angle between the stator and rotor magnetic flux axis is either 0 or 90°.

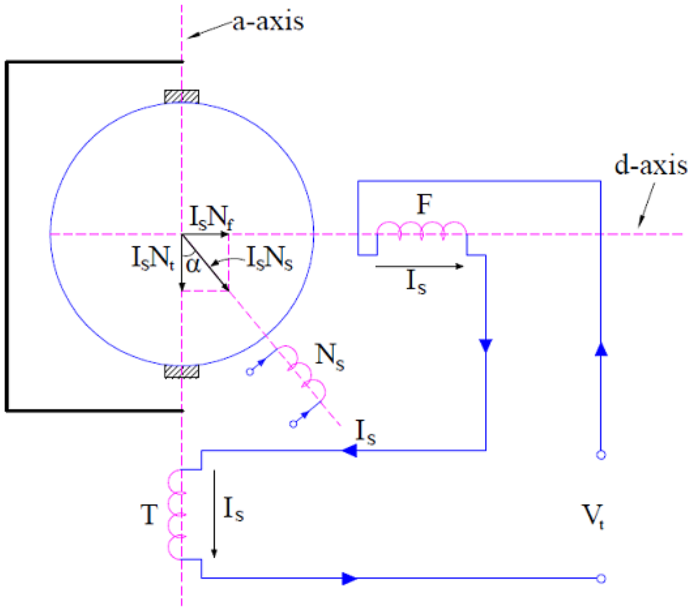
- Electromagnetic torque T is given as

T_e = k (Stator Field Strength) (Rotor Field Strength) Sinα
where is k is a constant.



- Since stator flux is toward A to B, South Pole (S1) is generated at A. Similarly South Pole (S2) is generated on rotor at C. Since similar poles repel each other, S1 will repel S2. Due to this repulsion between the like poles, motor will rotate in clockwise direction. ***This is the reason; this motor is called Repulsion Motor***.

Torque Equation of Repulsion Motor



The number of turns N_t of coil T can be found as below.
Mmf of coil T = I_sN_t
Component of stator mmf along the brush axis = $I_sN_s\text{Cos}\alpha$
$I_sN_t = I_sN_s\text{Cos}\alpha$
$N_t = N_s\text{Cos}\alpha$
Similarly, the number of turns of coil F is given as
$N_f = N_s\text{Sin}\alpha$

Now, the electromagnetic torque

T_e = k (Stator Field Strength) (Rotor Field Strength) Sinα

where is k is a constant.

$= k\,(I_sN_s)(I_sN_s\text{Cos}\alpha)\text{Sin}\alpha$

$= (k/2)(I_sN_s)^2(2\text{Cos}\alpha\text{Sin}\alpha)$

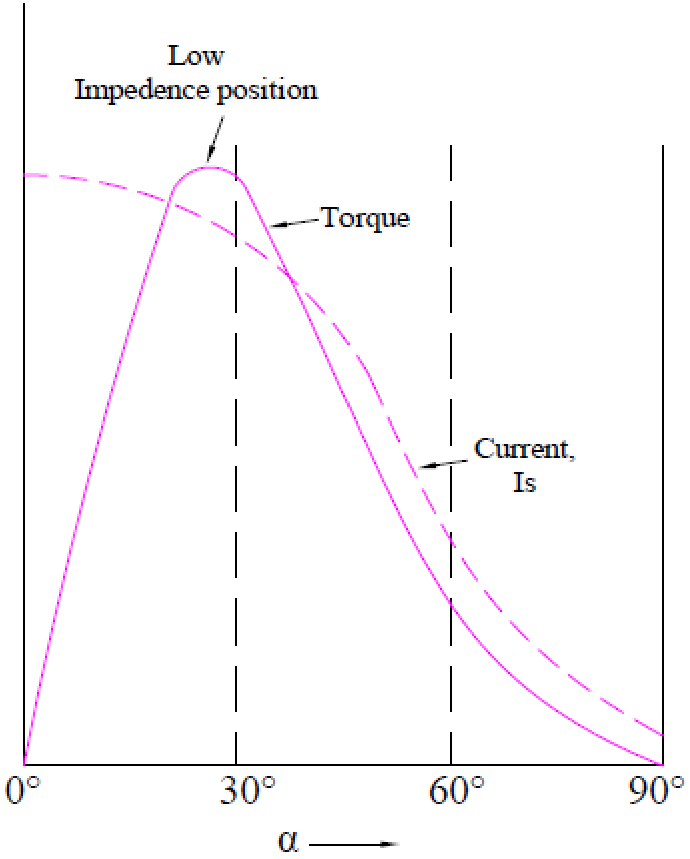
$= (k/2)(I_sN_s)^2\text{Sin}2\alpha$ …. [$\text{Sin}2\alpha = 2\text{Cos}\alpha\text{Sin}\alpha$]

Therefore, the torque in repulsion motor is given as

$T_e = (k/2)(I_sN_s)^2\text{Sin}2\alpha$

Maximum torque is achieved when stator and rotor magnetic axis are displaced from each other by 45°

The variation of current and torque with respect to different positions of brush is shown



Following points regarding must be noted from the above curve:

- Rotor current is maximum when the brush axis and direct axis coincides.
- Rotor current is zero when the brush occupies a position in quadrature with the direct axis.
- Maximum torque in repulsion motor is achieved when stator and rotor field axis are 45° apart.

Explanation of phasor diagram of Repulsion Motor

1: Flux ϕ_t is taken along ref-axis.
It is produced by winding T.

2: ϕ_t results induced emf E_{tr} in rotor winding (armature), lagging behind by 90°. (due to transformer action)

3: ϕ_t also induces emf E_{ts} in stator winding (field winding) lagging behind by 90°.

4: current I_s will lag behind by phasor $(-E_{ts})$ and produces flux ϕ_f in winding F, aligning with I_s (ideal case)

5: Flux ϕ_f causes a secondary induced emf E_{far} across brushes of rotor, due to rotation of rotor (rotational emf, E_{far}), in the direction of ϕ_f and I_s

6: Emf on armature (rotor winding) $\nearrow E_{far}$
$\searrow E_{tr}$

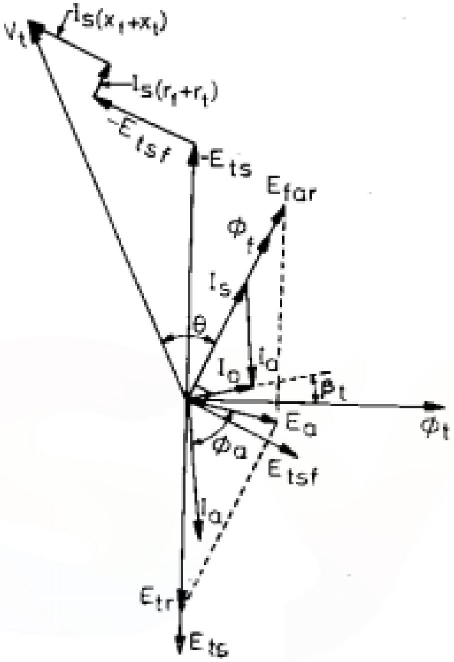
$\therefore$ Resultant emf $\bar{E}_a (= \bar{E}_{far} + \bar{E}_{tr})$ results current $\bar{I}_a$ in armature, lagging by angle ϕ_a

$$\phi_a = \tan^{-1}(x_a/r_a)$$

7: No-load current or Exciting current
$\bar{I}_o = \bar{I}_a + I_s$ → similar to transformer/induction motor
I_o leads ϕ_t by β_t (hysteretic angle of advance)

8: Similar to transformer. ϕ_f induces emf E_{tsf} (i.e. E_1) in rotor (i.e. primary winding). Lagging by 90°

$$\therefore V_t = I_s[(r_f+r_t)+j(x_f+x_t)] + (-E_{ts}) + (-E_{tsf})$$



1 Armature impedance $(r_a + jx_a)$ is neglected then,

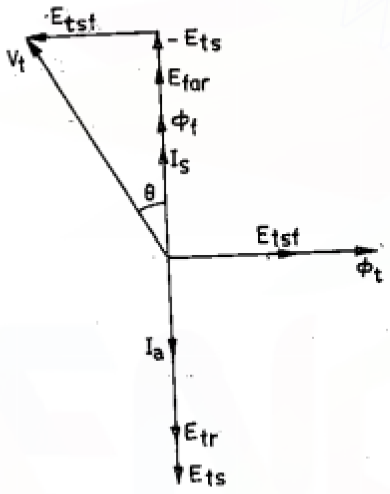
E_r and $E_{far} = E_{tr}$ (in phase opposition)

2 No-load current I_0 is ignored then,

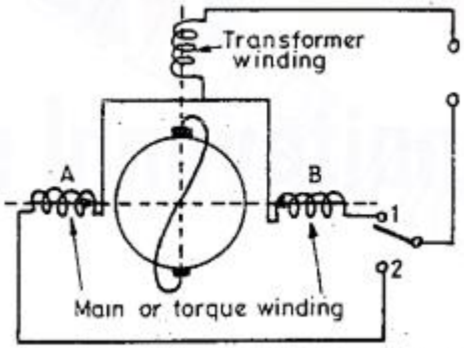
I_s and I_a will be in phase opposition then,

ϕ_t and ϕ_f are in time-phase displaced by 90°

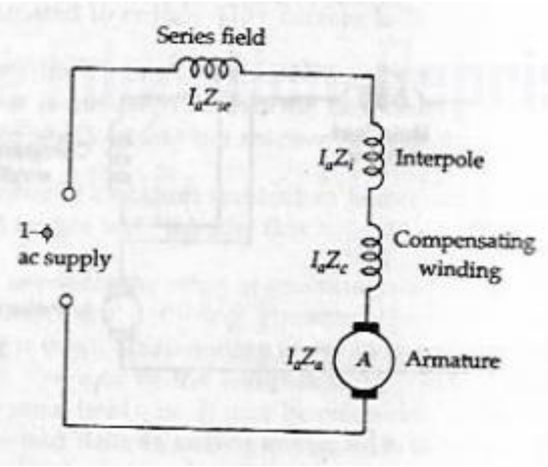
∴ and are elliptical field

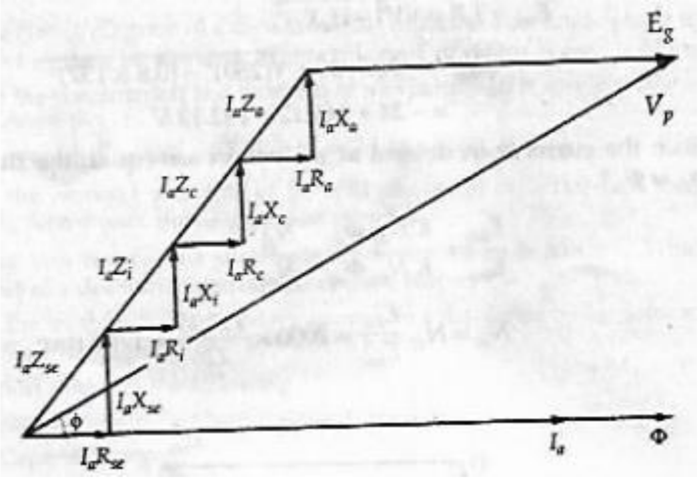


Simplified phasor diagram of a repulsion motor.

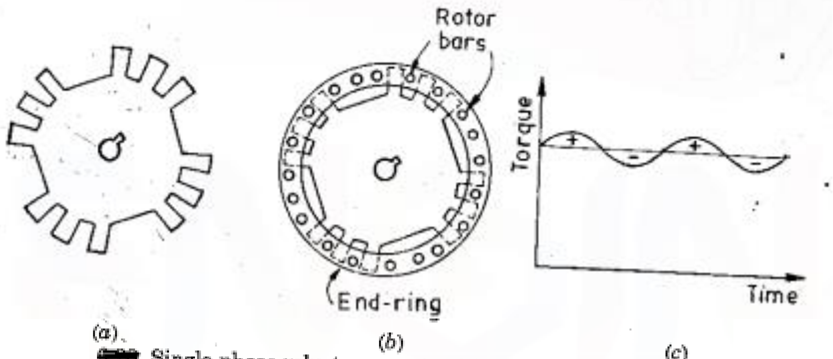


. Scheme for reversing the direction of rotation of a repulsion motor.

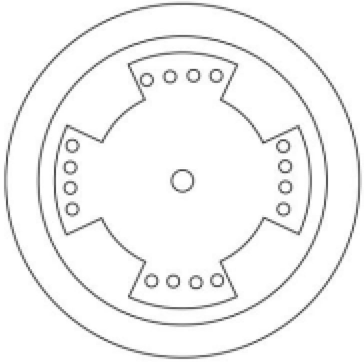

Series field
I_aZ_{se}
I_aZ_i
Interpole
1–ϕ
ac supply
I_aZ_c
Compensating winding
I_aZ_a
A
Armature


E_g
V_p
I_aZ_a
I_aX_a
I_aR_a
I_aZ_c
I_aX_c
I_aR_c
I_aZ_i
I_aX_i
I_aR_i
I_aZ_{se}
I_aX_{se}
I_aR_{se}
ϕ
I_a
Φ

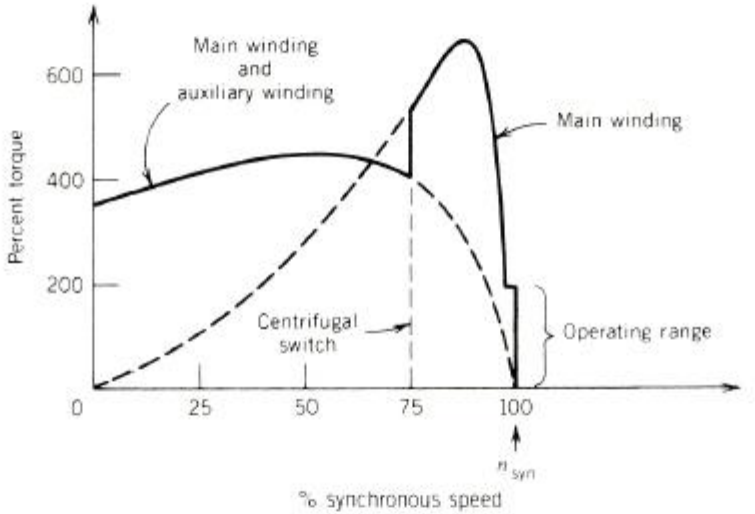
Reluctance Motor



[redacted]. Single-phase reluctance motor (*a*) 4-pole rotor laminations (*b*) 4-pole rotor laminations with bars and end rings (*c*) reluctance torque pulsations at small slip.



(a) Constructional features of 4-pole reluctance motor

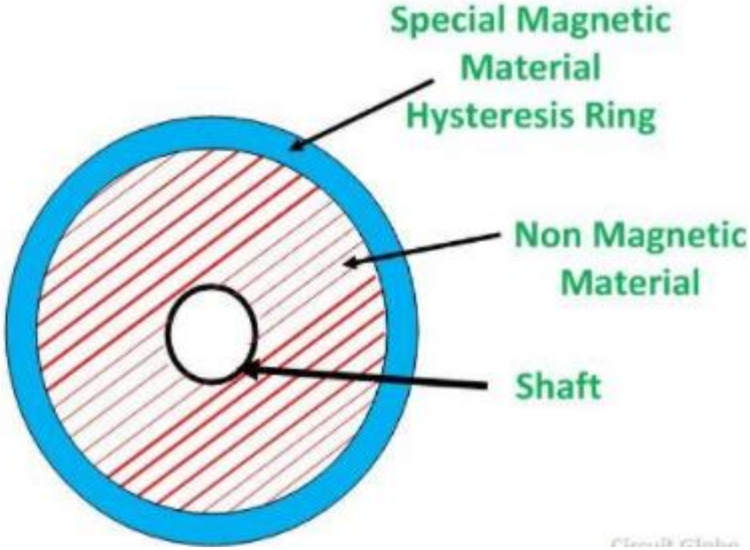


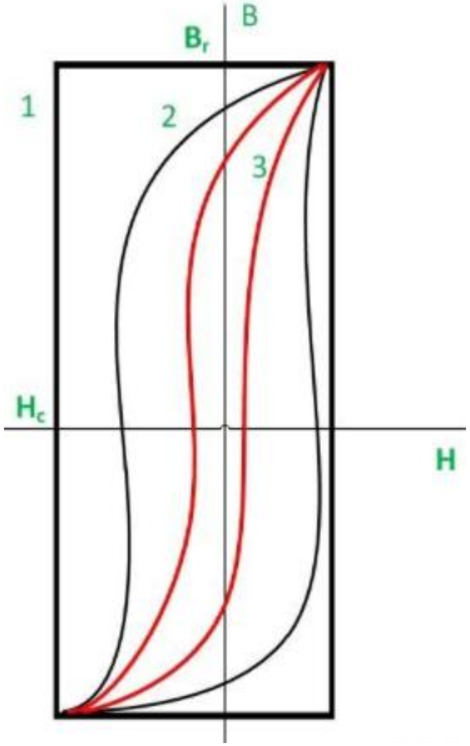
(b) Reluctance motor torque-speed characteristics

CONSTRUCTION

- The structure of reluctance motor is same as that of salient pole synchronous machine
- The stator has three phase symmetrical winding, which creates sinusoidal rotating magnetic field in the air gap, and the reluctance torque is developed because the induced magnetic field in the rotor has a tendency to cause the rotor to align with the stator field at a minimum reluctance position
- The rotor does not have any field winding
- The rotor of the modern reluctance machine is designed with iron laminations in the axial direction separated by non-magnetic material.

HYSTERESIS MOTORS





Supply is given applied to the stator

↓

A rotating magnetic field is produced

↓

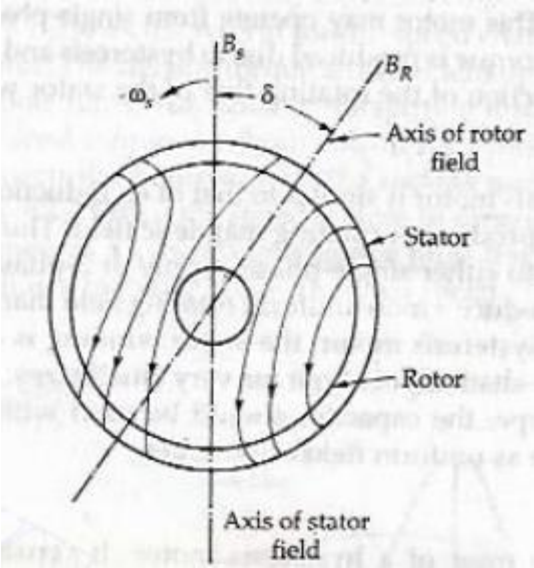
Magnetises the rotor ring and induces pole

↓

Induced rotor flux lags behind the rotating stator flux

↓

Production of the torque



The eddy current loss is given by the equation shown below.

$$p_e = k_e f_2^2 B^2$$

Where,

- k_e is a constant
- f_2 is the eddy current frequency
- B is the flux density

As we know,

$$f_2 = sf_1$$

Where s is the slip and f1 are the frequency of the stator.

Therefore,

$$p_e = k_e s^2 f_1^2 B^2$$

The torque is given by the equation shown below.

$$T_e = \frac{p_e}{s\,\omega_s} \quad \text{or}$$

$$T_e = k's \quad \dots\dots\dots(1)$$

Where,

$$k' = \frac{k_e f_1^2 B^2}{\omega_s} = \text{constant}$$

Now, the torque due to hysteresis loss is given by the equation shown below.

$$p_h = k_h f_2 B^{1.6} \quad \text{or}$$

$$p_h = k_h s f_1 B^{1.6} \quad \dots\dots\dots (2)$$

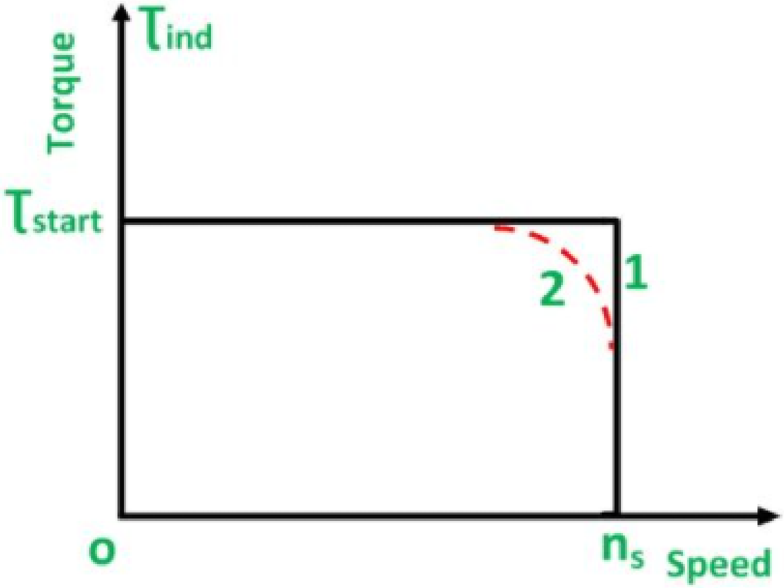
The Torque due to hysteresis is given as

$$T_h = \frac{p_h}{s\,\omega_s} \quad \text{or}$$

$$T_h = \frac{k_h f_1 B^{1.6}}{\omega_s} = k'' = \text{constant} \quad \dots\dots\dots.(3)$$

Torque Speed characteristic of Hysteresis Motor

The speed torque curve of the motor is shown below.



Energy Stored in a magnetic field

Applied input voltage

$$v = iR + e$$

Inst. power input

$$p = vi = i^2R + ei$$

Energy input to the system from $t=0$ to $t=T$

$$w_i = \int_0^T p\,dt = \int_0^T i^2R\,dt + \int_0^T ei\,dt$$

$\downarrow$ dissipated in R.

Energy stored in magnetic field

$$w_f = \int_0^T ei\,dt$$

Also, by Faraday's law

$$e = \frac{d\psi}{dt} = N\frac{d\phi}{dt}$$

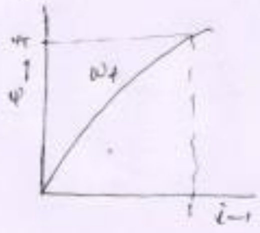
$$\therefore \quad w_f = \int_0^T \frac{d\psi}{dt} i\,dt = \int_0^T i\,d\psi \quad — \quad Ⓐ$$

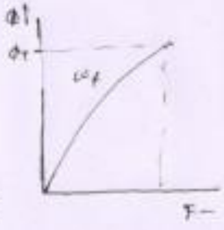
$$w_f = \int_0^T N\frac{d\phi}{dt} i\,dt = \int_0^T Ni\,d\phi = \int_0^{\phi_T} F\,d\phi \quad — Ⓑ$$

Where. $F = Ni \rightarrow$ mmf

Ⓐ → Area under $\psi - i$ curve

Ⓑ → Area under $\phi - F$ curve





Linear System

$$w_f = \int_0^{\psi} i\,d\psi = \int_0^{\psi} \frac{\psi}{L}\,d\psi = \frac{\psi^2}{2L} \quad (\because \psi = Li)$$

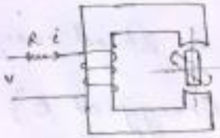
$$\therefore w_f = \frac{(Li)^2}{2L} = \frac{1}{2}Li^2$$

co-energy

$$w_f' = \int_0^{i} \psi\,di = \int_0^{i} Li\,di = \frac{1}{2}Li^2$$

$\therefore \quad w_f = w_f'$ ie Field energy = co-energy.

Single Excited System



$$v = iR + \frac{d\psi}{dt}$$

$$vi = i^2R + i\frac{d\psi}{dt}$$

On integrating from t = 0 to t = t

$$\int_0^t vi\,dt = \int_0^t Ri^2 dt + \int_0^\psi i\,d\psi$$

$$\therefore \quad W_e = W_{le} + \left[W_{fe} + W_{em}\right]$$

$$W_{fe} + W_{em} = \int_0^\psi i\,d\psi$$

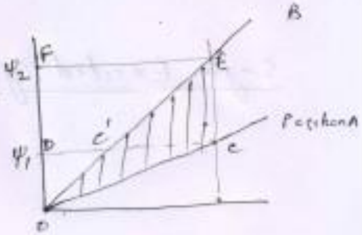
Static Energization → Mech. output is zero

$$\therefore W_m = 0$$

$$\therefore \quad W_f = \int_0^\psi i\,d\psi = \int_0^\psi \frac{\psi}{L}\,d\psi$$

$$= \frac{1}{2}Li^2 \quad (\because \psi = Li)$$

Slow Movement



change in stored energy

$$W_f = (\text{Energy at } B) - (\text{Energy at } A)$$

Electrical energy input

$$W_e = \int_{\psi_1}^{\psi_2} i_1\,d\psi = i(\psi_2 - \psi_1)$$

$$= \text{area } (cc'DFEC)$$

But $W_e = W_f + W_m$

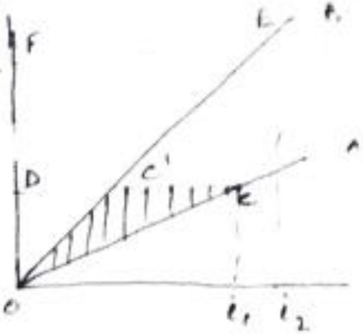
$$\text{Hence, area } (cc'DFEC) = \overbrace{\text{area}(Oc'EFDO) - \text{area}(Occ'DO)}^{W_f} + W_m$$

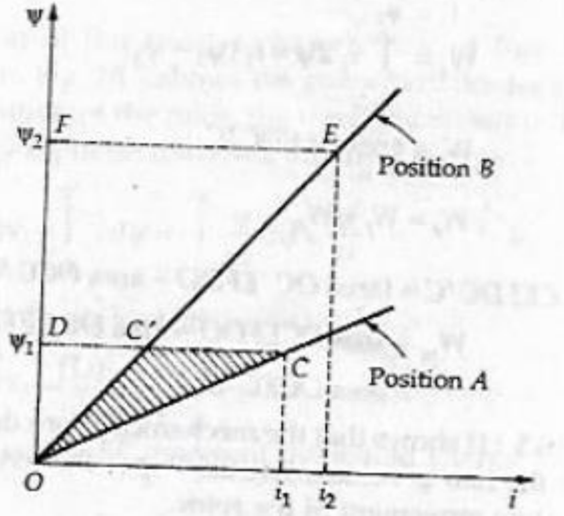
$$\therefore \quad W_m = \text{area}(OcEFDO) - \text{area}(Oc'EFDO)$$

$$= \text{area}(OcEc'O)$$

∴ Mech. work done = Area enclosed between two $(\psi - i)$ curve and vertical $(\psi\text{-}i)$ locus during slow movement.

(B) Instantaneous Movement





- Due to constant-flux linkage theorem, flux linkage will not change suddenly

Input electrical energy

$$w_e = \int_{\psi_1}^{\psi_2} i\,d\psi$$

$$\because \psi_2 = \psi_1$$

$$\therefore w_e = \int_{\psi_1}^{\psi_1} i\,d\psi = 0$$

Hence, no energy is taken from supply and mechanical output is obtained from stored magnetic field.

- Electromagnetic torque

$$T_e = \lim_{\Delta\theta \to 0} \left[-\frac{\Delta W_{fe}}{\Delta \theta} \right]_{\text{constant } \psi}$$

$$= -\frac{\partial W_{fe}}{\partial \theta} \bigg|_{\text{constant } \psi}$$

$$= -\frac{\partial}{\partial \theta} \frac{\psi^2}{2L} \bigg|_{\text{constant } \psi}$$

$$= \frac{-\psi^2}{2L^2} \frac{\partial L}{\partial \theta}$$

$$T_e = \frac{i^2}{2} \frac{dL}{d\theta} \longrightarrow \text{for linear system}$$

Transient Movement

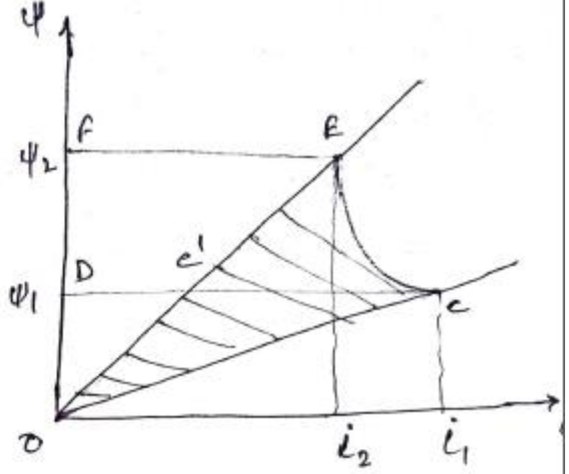
W_e = area (CEFDC)

W_f = area (OEFO) − area (OCDO)

$W_m = W_e - W_f$

= area (CEFDC) − area (OEFO) + area (OCDO)

= area (OCEO)



Doubly - Excited system

$$\begin{cases}\psi_1 = L_1 i_1 + M i_2 \\ \psi_2 = L_2 i_2 + M i_1\end{cases}$$

$$\begin{cases}v_1 = R_1 i_1 + \dfrac{d\psi_1}{dt} \\ v_2 = R_2 i_2 + \dfrac{d\psi_2}{dt}\end{cases}$$

$$\therefore \begin{cases}v_1 = R_1 i_1 + \dfrac{d}{dt}(L_1 i_1) + \dfrac{d}{dt}(M i_2) \\ v_2 = R_2 i_2 + \dfrac{d}{dt}(L_2 i_2) + \dfrac{d}{dt}(M i_1)\end{cases}$$

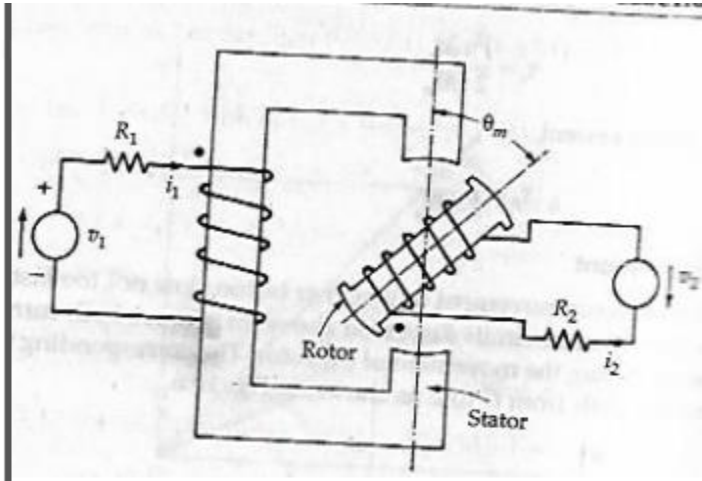
$L = f(\theta)$

$i = f(t)$

$$\begin{cases}v_1 = R_1 i_1 + L_1 \dfrac{di_1}{dt} + i_1 \dfrac{dL_1}{dt} + M \dfrac{di_2}{dt} + i_2 \dfrac{dM}{dt} \\ v_2 = R_2 i_2 + L_2 \dfrac{di_2}{dt} + i_2 \dfrac{dL_2}{dt} + M \dfrac{di_1}{dt} + i_1 \dfrac{dM}{dt}\end{cases}$$

$$\begin{cases}v_1 i_1 = R_1 i_1^2 + L_1 i_1 \dfrac{di_1}{dt} + i_1^2 \dfrac{dL}{dt} + i_1 M \dfrac{di_2}{dt} + i_1 i_2 \dfrac{dM}{dt} \\ v_2 i_2 = R_2 i_2^2 + L_2 i_2 \dfrac{di_2}{dt} + i_2^2 \dfrac{dL_2}{dt} + i_2 M \dfrac{di_1}{dt} + i_1 i_2 \dfrac{dM}{dt}\end{cases}$$

$$\int (v_1 i_1 + v_2 i_2)\,dt = \int (R_1 i_1^2 + R_2 i_2^2)\,dt + \int \Big(L_1 i_1\,di_1 + L_2 i_2\,di_2 + i_1^2\,dL + i_2^2\,dL_2 + i_1 M\,di_2 + i_2 M\,di_1 + 2 i_1 i_2\,dM\Big)$$



$$v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + i_1 \frac{dL_1}{dt} + M \frac{di_2}{dt} + i_2 \frac{dM}{dt}$$

$$v_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + i_2 \frac{dL_2}{dt} + M \frac{di_1}{dt} + i_1 \frac{dM}{dt}$$

$$v_1 i_1 = R_1 i_1^2 + L_1 i_1 \frac{di_1}{dt} + i_1^2 \frac{dL}{dt} + i_1 M \frac{di_2}{dt} + i_1 i_2 \frac{dM}{dt}$$

$$v_2 i_2 = R_2 i_2^2 + L_2 i_2 \frac{di_2}{dt} + i_2^2 \frac{dL_2}{dt} + i_2 M \frac{di_1}{dt} + i_1 i_2 \frac{dM}{dt}$$

Useful input energy = Stored energy + Electrical to Mech. energy

$$\therefore \int (v_1 i_1 + v_2 i_2)\,dt - \int (R_1 i_1^2 + R_2 i_2^2)\,dt =$$
$$\int \left(L_1 i_1 di_1 + L_2 i_2 di_2 + i_1 M di_2 + i_2 M di_1 + i_1^2 dL_1 + i_2^2 dL_2 + 2 i_1 i_2 dM \right) dt$$

Stored Energy in Magnetic field

Assumptions: 1 No mechanical output

2 Inductance is constant

$$\therefore \int dW_{fe} = \int_0^{i_1} L_1 i_1 di_1 + \int_0^{i_2} L_2 i_2 di_2 + \int_0^{i_1, i_2} (i_2 M di_1 + i_1 M di_2]$$

$$W_{fe} = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$$

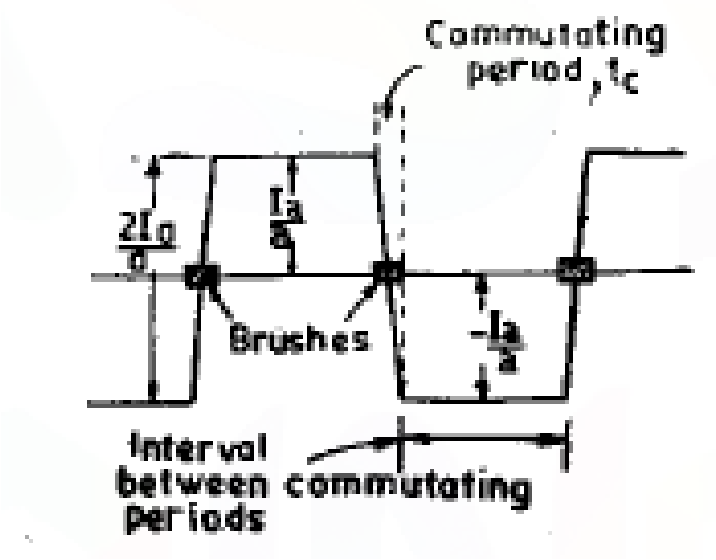
Electromagnetic Torque

$$T = \frac{\partial W_f'}{\partial \theta} \quad (\because W_f' = W_f \text{ for linear system})$$

$$= \frac{1}{2} i_1^2 \frac{dL_1}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_2}{d\theta} + i_1 i_2 \frac{dM}{d\theta}$$

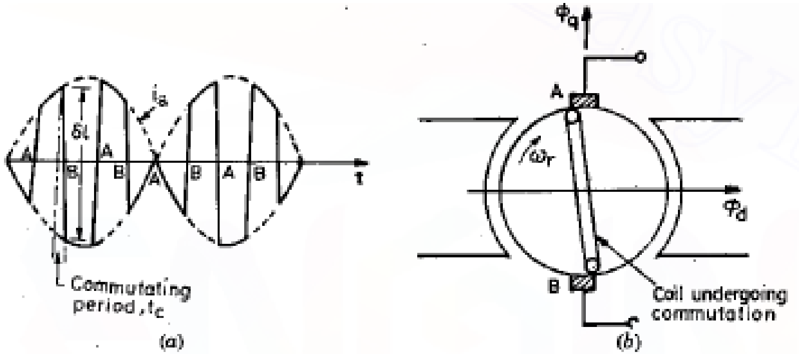
Commutation in AC motor

$$e_c = L_c \cdot \frac{1}{t_c}\left(\frac{2I_a}{a}\right)$$



If proper steps are not taken, this reactance e.m.f. delays the reversal of current in the commutated coil. As a result, the current reversal is not completed by the time the commutated-coil leaves the brush and sparking occurs giving rise to poor commutation.

Reactance emf E_1.



reactance emf e_1 is in phase with the armature current. The reactance voltage e_1 in a.c. commutator machines is given by

$$e_1 = L_c \frac{\delta i}{t_c}$$

where L_c = self-inductance of commutated coil

δi = change of commutated (not of alternating) current as shown in Fig. 7.9 (*a*).

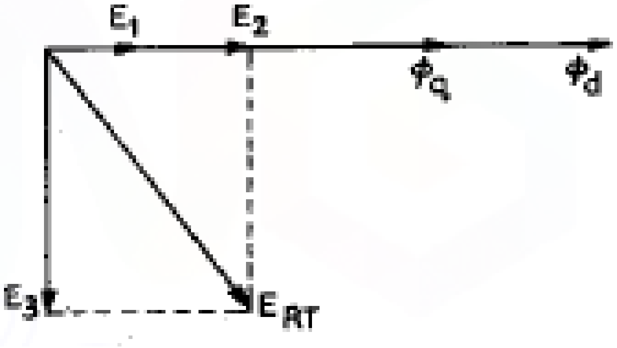
t_c = commutating period, inversely proportional to the armature speed and directly proportional to brush width.

Rotational emf E_2.

$$E_2 = \sqrt{2}\,\pi f_r N_c \phi_q$$

Transformer emf E_3.

$$E_3 = \sqrt{2}\,\pi f N_c \phi_{dm}$$



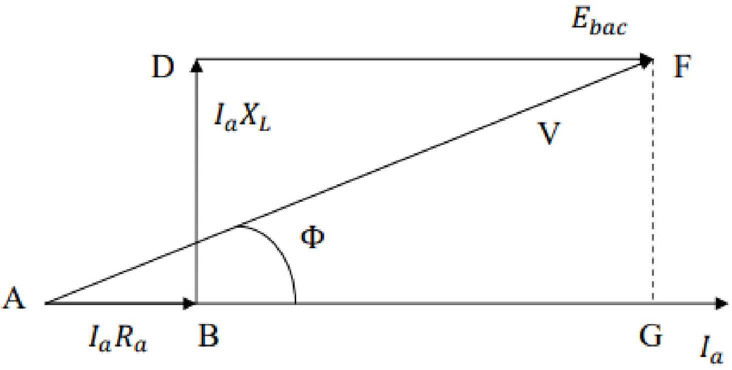
The total resultant voltage in the commutated coil, in a series motor, is given by

$$E_{RT} = \sqrt{(E_1 + E_2)^2 + (E_3)^2}$$

and it lags ϕ_d by and angle

$$\theta = \tan^{-1} \frac{E_3}{E_1 + E_2}$$

A universal series motor has a resistance of 30Ω and an inductance of 0.5 H. when connected to a 250 V dc supply and loaded to take 0.8 A it runs at 2000 rpm. Determine the speed, torque and power factor, when connected to a 250 V, 50 Hz ac supply and loaded to take the same current.



$$E = \frac{P \phi Z N}{60 A} = K \phi \boxed{N} \rightarrow dc \rightarrow ac \; ?$$

Also $\phi_{dc} = \phi_{ac}$

$$\frac{E_{bdc}}{E_{bac}} = \frac{K \phi_{dc} N_{dc}}{K \phi_{ac} N_{ac}} = \frac{N_{dc}}{N_{ac}}$$

N_{dc} = 2000 rpm (given)

N_{ac} : ??

Step I : calculate E_{bdc}

Step 2 : Draw phasor diagram and calculate E_{bac} from V-I equation.

Step 3 : calculate N_{ac} = ?

Step 4 : calculate p.f., cos φ from phasor diagram

Step 5 : calculate power developed

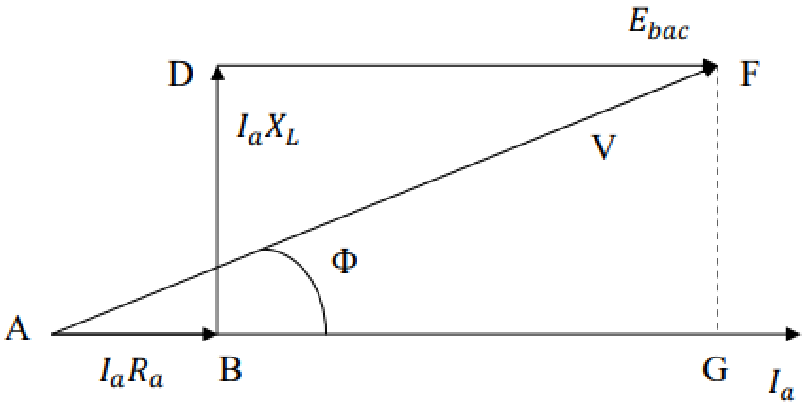
Step 6 : calculate torque developed

Solution Operation of motor on dc

$$E_{bdc} = V - I_a R_a = 250 - 0.8 \times 30 = 226\ V$$

$$N_{dc} = 2000\ r.p.m$$

Operation motor on ac



$$X_L = 2\pi f L = 2\pi \times 50 \times 0.5 = 157\ \Omega$$

From the phasor diagram shown in fig. 4,

$$AF^2 = AG^2 + GF^2$$

$$V^2 = (AB + BG)^2 + GF^2 = (AB + DF)^2 + GF^2$$

$$= (I_a R_a + E_{bac})^2 + (I_a X_L)^2$$

$$E_{bac} + I_a R = \sqrt{V^2 - (I_a X_L)^2}$$

$$E_{bac} = -0.8 \times 30 + \sqrt{(250)^2 - (0.8 \times 157)^2}$$

$$= -24 + 216.12 = 192.12\ V$$

Since the currents in dc and ac operation are equal, the flux will also be equal ($\Phi_{ac} = \Phi_{dc}$)

$$\frac{E_{bdc}}{E_{bac}} = \frac{KN_{dc}\Phi_{dc}}{KN_{ac}\Phi_{ac}} = \frac{N_{dc}}{N_{ac}}$$

$$\mathrm{N_{ac}} = \mathrm{N_{dc}}\frac{\mathrm{E_{bac}}}{\mathrm{E_{bdc}}} = 2000 \times \frac{192.12}{226} = 1700 \text{ rpm}$$

Power factor, $\cos\phi = \frac{AG}{AF} = \frac{E_{bac+}I_aR_a}{V}$

$$= \frac{192.12 + 0.8 \times 30}{250} = 0.8645\ (lagging)$$

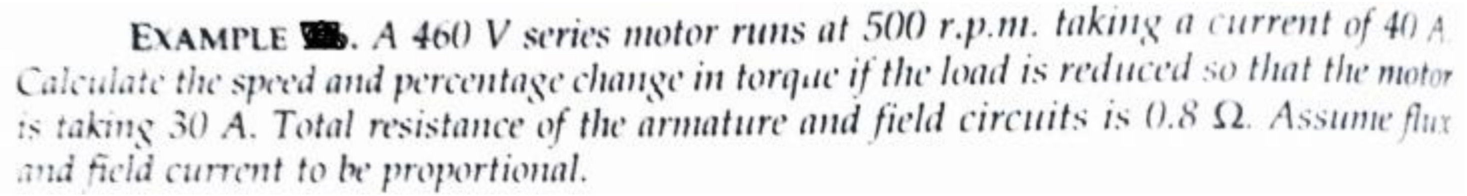
Mechanical power developed

$$\mathrm{P_{mech}} = \mathrm{E_{bac}I_a} = 192.12 \times 0.8 = 153.7\ \mathrm{W}$$

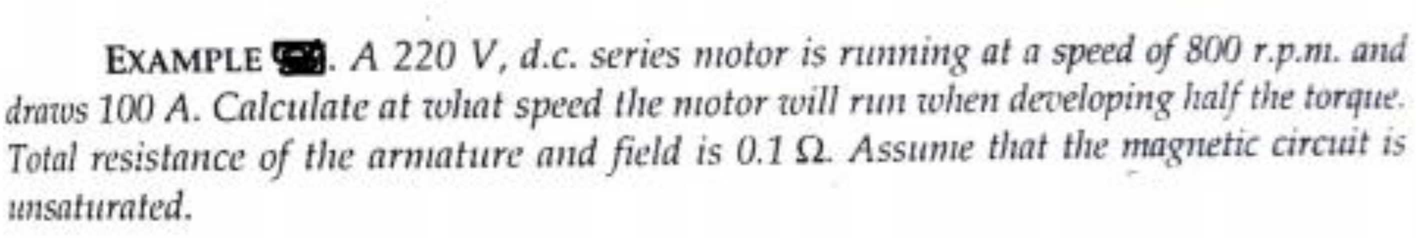
Torque developed

$$\tau = \frac{P_{mech}}{\omega_m} = \frac{P_{mech}}{2\pi n_{ac}}$$

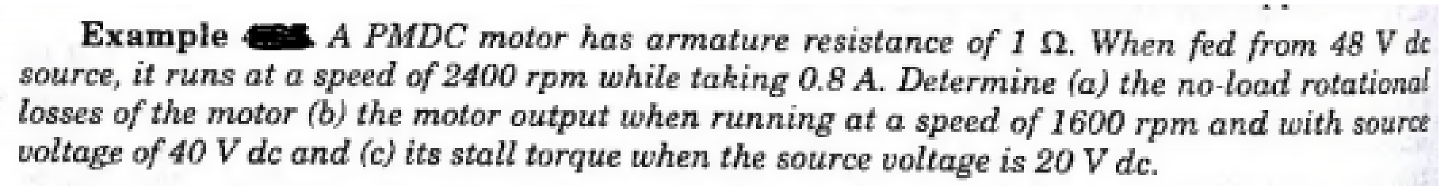
$$= \frac{153.7}{2\pi \times (1700/60)} = 0.8633\ Nm$$



EXAMPLE [redacted]. *A 460 V series motor runs at 500 r.p.m. taking a current of 40 A. Calculate the speed and percentage change in torque if the load is reduced so that the motor is taking 30 A. Total resistance of the armature and field circuits is 0.8 Ω. Assume flux and field current to be proportional.*



EXAMPLE [redacted]. *A 220 V, d.c. series motor is running at a speed of 800 r.p.m. and draws 100 A. Calculate at what speed the motor will run when developing half the torque. Total resistance of the armature and field is 0.1 Ω. Assume that the magnetic circuit is unsaturated.*



Example [redacted] *A PMDC motor has armature resistance of 1 Ω. When fed from 48 V dc source, it runs at a speed of 2400 rpm while taking 0.8 A. Determine (a) the no-load rotational losses of the motor (b) the motor output when running at a speed of 1600 rpm and with source voltage of 40 V dc and (c) its stall torque when the source voltage is 20 V dc.*

SOLUTION. $E_1 = V - I_{a_1} R_a = 460 - 40 \times 0.8 = 428 \text{ A}$

$$E_2 = V - I_{a_2} R_a = 460 - 30 \times 0.8 = 436 \text{ V}$$

Since $\Phi \propto I_a$, $\quad \dfrac{\Phi_1}{\Phi_2} = \dfrac{I_{a_1}}{I_{a_2}}$

$$N_2 = \frac{E_2}{E_1} \times \frac{\Phi_1}{\Phi_2} N_1$$

$$= \frac{E_2}{E_1} \times \frac{I_{a_1}}{I_{a_2}} N_1 = \frac{436}{428} \times \frac{40}{30} \times 500 = \textbf{679 r.p.m.}$$

$$\tau \propto \Phi I_a$$

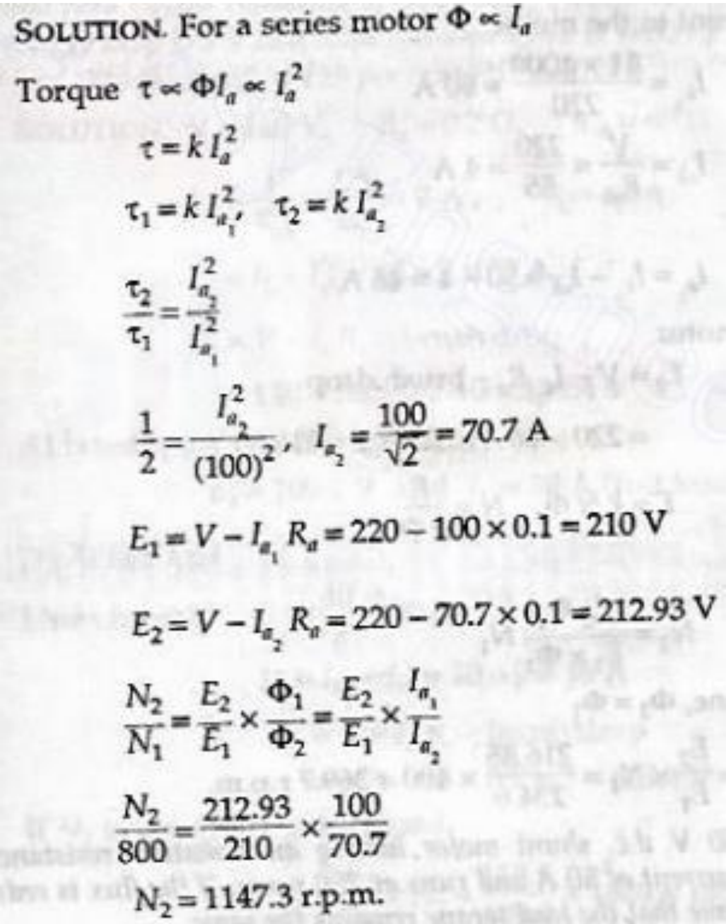
$$\tau \propto I_a^2, \quad \tau_1 = k\, I_{a_1}^2, \;\; \tau_2 = k\, I_{a_2}^2$$

$$\frac{\tau_2}{\tau_1} = \frac{k\, I_{a_2}^2}{k\, I_{a_1}^2} = \frac{I_{a_2}^2}{I_{a_1}^2} = \frac{(30)^2}{(40)^2} = \frac{9}{16}$$

Percentage change in torque

$$= \frac{\tau_1 - \tau_2}{\tau_1} \times 100$$

$$= \frac{\tau_1 - \frac{9}{16}\tau_1}{\tau_1} \times 100 = \frac{7}{16} \times 100 = 43.75\%$$



SOLUTION. For a series motor $\Phi \propto I_a$

Torque $\tau \propto \Phi I_a \propto I_a^2$

$$\tau = k\, I_a^2$$

$$\tau_1 = k\, I_{a_1}^2, \quad \tau_2 = k\, I_{a_2}^2$$

$$\frac{\tau_2}{\tau_1} = \frac{I_{a_2}^2}{I_{a_1}^2}$$

$$\frac{1}{2} = \frac{I_{a_2}^2}{(100)^2}, \; I_{a_2} = \frac{100}{\sqrt{2}} = 70.7 \text{ A}$$

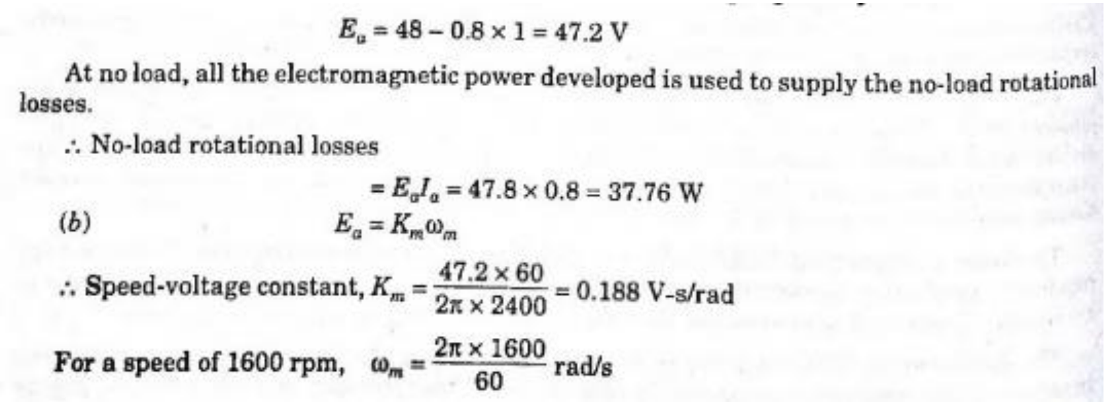
$$E_1 = V - I_{a_1} R_a = 220 - 100 \times 0.1 = 210 \text{ V}$$

$$E_2 = V - I_{a_2} R_a = 220 - 70.7 \times 0.1 = 212.93 \text{ V}$$

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\Phi_1}{\Phi_2} = \frac{E_2}{E_1} \times \frac{I_{a_1}}{I_{a_2}}$$

$$\frac{N_2}{800} = \frac{212.93}{210} \times \frac{100}{70.7}$$

$$N_2 = 1147.3 \text{ r.p.m.}$$



$$E_a = 48 - 0.8 \times 1 = 47.2 \text{ V}$$

At no load, all the electromagnetic power developed is used to supply the no-load rotational losses.

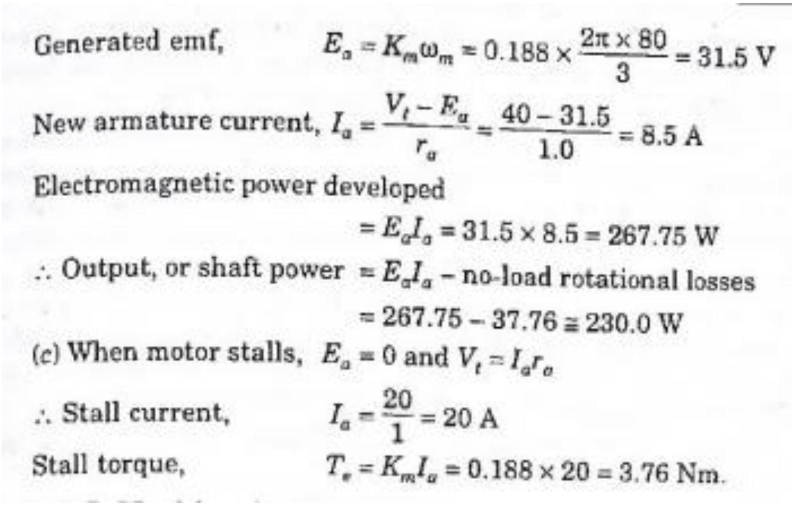
$\therefore$ No-load rotational losses

$$= E_a I_a = 47.8 \times 0.8 = 37.76 \text{ W}$$

(*b*) $$E_a = K_m \omega_m$$

$\therefore$ **Speed-voltage constant,** $K_m = \dfrac{47.2 \times 60}{2\pi \times 2400} = 0.188$ **V-s/rad**

For a speed of 1600 rpm, $\omega_m = \dfrac{2\pi \times 1600}{60}$ **rad/s**



Generated emf, $E_a = K_m \omega_m = 0.188 \times \dfrac{2\pi \times 80}{3} = 31.5 \text{ V}$

New armature current, $I_a = \dfrac{V_t - E_a}{r_a} = \dfrac{40 - 31.5}{1.0} = 8.5 \text{ A}$

Electromagnetic power developed

$$= E_a I_a = 31.5 \times 8.5 = 267.75 \text{ W}$$

$\therefore$ Output, or shaft power $= E_a I_a -$ no-load rotational losses

$$= 267.75 - 37.76 \cong 230.0 \text{ W}$$

(*c*) When motor stalls, $E_a = 0$ and $V_t = I_a r_a$

$\therefore$ Stall current, $I_a = \dfrac{20}{1} = 20 \text{ A}$

Stall torque, $T_e = K_m I_a = 0.188 \times 20 = 3.76 \text{ Nm.}$

SUB-SYNCHRONOUS MOTOR

Manual rotor rotation at synchronous speed

↓

Input supply to stator

↓

Alternating field in airgap

↓

Stator and rotor poles interlocking

↓

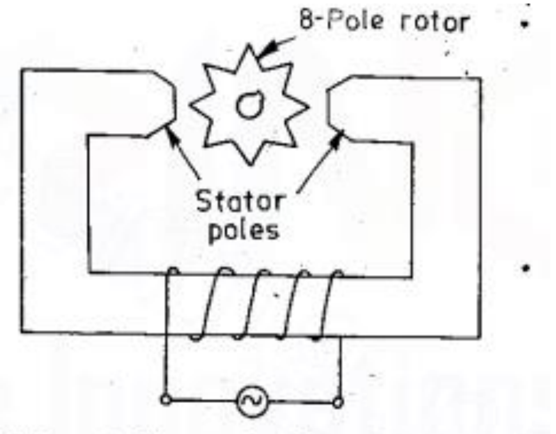
Rotor poles go ahead due to inertia

↓

Magnetic flux goes to zero & increases in Reverse direction

↓

Stator poles attract rotor poles again



Schematic diagram of a sub-synchronous motor.