# Signal AND System

Date: 31 - 12 - 2011

M.I Topics
1.) Laplace
2.) Z-transform
3.) fourier series
4.) Discrete time system
5.) RMS | Power of a signal.
6.) fourier transform.
7.) Basic System properties.

Books :- → B.P Lathi } theoretical
→ oppenhein
→ schaum's outline } Numerical
→ Simon hykins

Question Kanodia

Part I — 5 Subject
Part II — 6 - Subject

Syllabus :-

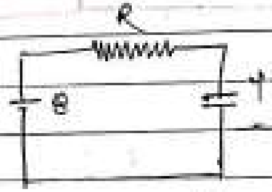
1.) Signal classification and different operation of signal.
2.) Basic System properties
→ Linear | Non-linear
→ Static | dynamic
→ Causal | Non-causal
→ Time variant | invariant.
→ Stable | unstable
3.) Linear time invariant System.
4.) fourier series

Fourier transform
Laplace transform.
Sampling theorem
Discrete time system
Z-transform

## Signal classification and different operating signal

Signal :-
A signal is a function representing physical quantity or variable, and contain information about nature behavior of problem.
mathmatical signal is represented as a function of independent variable.

eg:-




$V_c(t) = V_0(1 - e^{-t/RC})$
time is variable.

System:-
A system is interconnecting device or component that converts signal from one form into another.

Different operation on signal.
=1) shifting
2) Scaling.
3) Reversal

1) Shifting.
a:) time shifting (b) amplitude shifting.

a) time shifting :-

assume signal.


$x(t) = y(t) = x(t+k)$

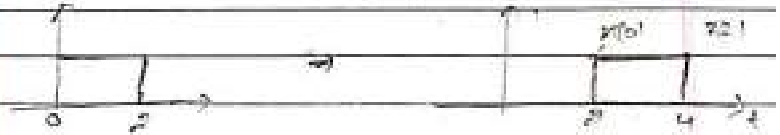
Case 1 $\underset{\sim}{k>0}$

$x(t) \longrightarrow y(t) = x(t+2)$


shifting will be left side (time advanced)

Case 2 :- $k<0$, $k=-2$

$x(t) = y(t) = x(t-2)$




shifting will be right side (time delay)

Amplitude shifting :-
$x(t) = y(t) = a + x(t)$
Case 1 :- $a>0$ let $a=2$.
$x(t) = y(t) = x(t)+2$

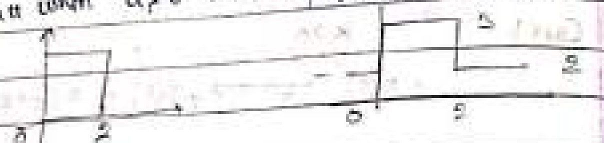
Mathematically

$$x(t) = \begin{cases} 0 & t \le 0 \\ 1 & 0 < t \le 2 \\ 0 & t > 2 \end{cases}$$

$y(t) = x(t) + 2$

$$= \begin{cases} 0 + 2 = 2 & t < 0 \\ 1 + 2 = 3 & 0 \le t \le 2 \\ 0 + 2 = 2 & t > 2 \end{cases}$$

Hence when $a > 0$ then signal is shifted upward



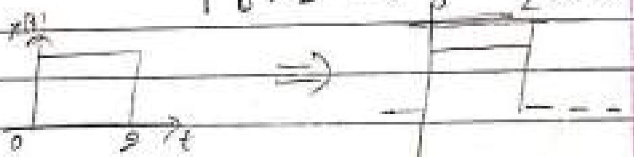
Case ② $a < 0$ let $a = -2$

$x(t) = x(t) - 2$

$$x(t) = \begin{cases} 0 & t < 0 \\ 1 & 1 \le t \le 2 \\ 0 & t < 2 \end{cases}$$

$y(t) = x(t) - 2$

$$\begin{cases} 0 - 2 = -2 & t < 0 \\ 1 - 2 = -1 & 1 \le t \le 2 \\ 0 - 2 = -2 & t > 2 \end{cases}$$



Hence when $a < 0$ then signal is shifted downward.

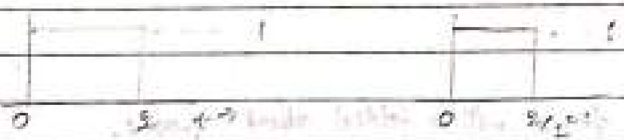
**Scaling :-**

(a) time scaling (b) Amplitude scaling.

(a) Time scaling :-

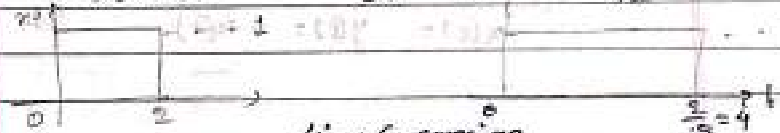
$x(t) \longrightarrow y(t) = x(kt)$

Case 1 :- $k > 1$ let $k = 2$



known as time compression.

Case 2 :- $0 < k < 1$ let $k = 0.5 = 1/2$

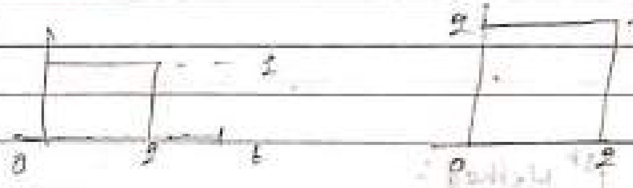


time expansion

(b) Amplitude Scaling :-

$x(t) = y(t) = a\,x(t)$

Case 1 :- $a > 1$ let $a > 2$



Amplification

Case 2 :- $0 < a < 1$ $a = 1/2$

Attenuation.

Reversal:

i) Time

ii) Amplitude.

(i) Time Reversal:

$x(t) \rightarrow y(t) = x(-t)$

Signal will be folded about y-axis.

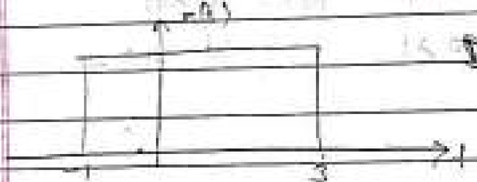
It is special case of time scaling.

(ii) Amplitude Reversal:

$x(t) = \quad y(t) = -x(t)$

Amplitude Reversal case of amplitude scaling

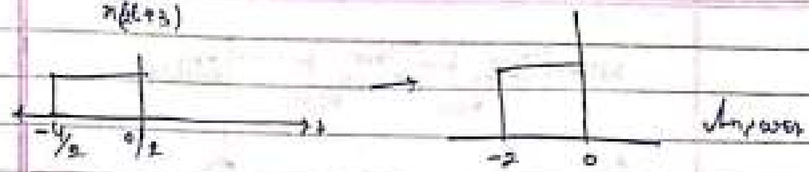
Que Draw $x(2t+3)$



Sol^n 1st Method:-

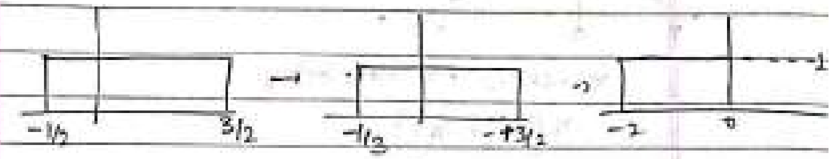
$x(t) \rightarrow \boxed{\text{time shifting}} \xrightarrow{x(t+3)} \boxed{\text{time scaling}} \rightarrow x(2t+3)$

Case of time advanced:

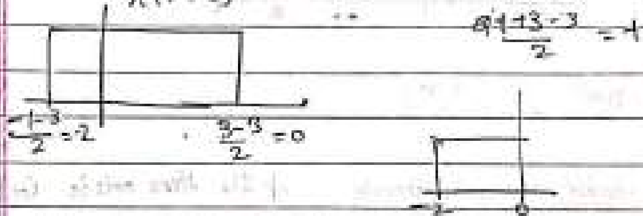


Method 2

$\rightarrow \boxed{\text{time scaling}} \xrightarrow{x(2t)} \boxed{\text{time shifting}} \rightarrow x(2t+3)$

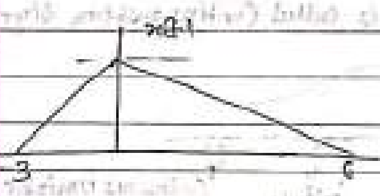


3rd Method:- (Trick)



$\frac{2t+3-3}{2} = t$

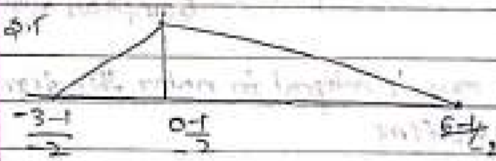
Que Draw $x(-2t+1)$

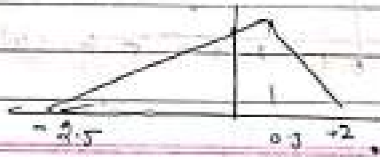


Sol^n

$\frac{-3-1}{-2} \quad \frac{0-1}{-2} \quad \frac{6-1}{-2}$

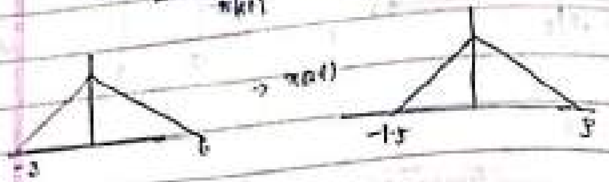
$= +2 \quad 0.5 \quad -2.5$

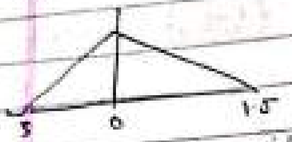




2nd method

$x(t) \rightarrow$ Time scaling $\xrightarrow{x(2t)}$ time reversal $\xrightarrow{x(-2t)}$ time shifting




$x(-2t+1) \rightarrow$ time delay

$x\left[-2\left[t - \frac{1}{2}\right]\right]$


Signal Classification:-

(1) Continuous and discrete: if the time axis is continuous in nature then signal is called continuous time signal.

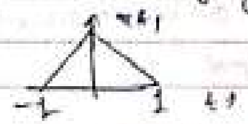
Eg:


(2) If time axis is integral in nature then signal is called discrete time.


(c) Even and odd signal:-

- For even signal $x(t) = x(-t)$
- even signal are mirror image of about Y-axis.

graphically




$x(t) = \cos t + t^2$

$x(-t) = \cos t + t^2$

- For odd signal $x(t) = -x(t)$

odd signal are anti symmetry about y axis.

Q1 $x(t) = t^3 + t^5 \cos t$

$x(-t) = -t^3 - t^5 \cos t$

$= -[t^3 + t^5 \cos t]$

$x(t) = -x(-t)$ odd signal.

Que $x(t) = 2$

Sol: even signal.

Const value are always even signal [D.C signal]

Some special point

⇒ EVEN * EVEN = EVEN

⇒ ODD * EVEN = ODD signal

⇒ ODD + ODD = EVEN

⇒ DC + EVEN = EVEN

⇒ DC + ODD = Niether even Nor ODD.

⇒ Derivative of even signal $\frac{d}{dt}$ (even) = odd.

$\frac{d(odd)}{dt}$ = even.

$\int \text{even}\, dt = \text{odd}$

$\int \text{odd}\, dt = \text{even}$

Any signal can be represented as a sum of even signal and odd signal i.e.

$x(t) = x_e(t) + x_o(t)$

$x(t) = 1 + 2t + 2t^2\cos t + t^7 \cdot \sin^7 t$

$x_e(t) = 1 + 2t^2\cos t$

$x_o(t) = 2t + t^7\sin^7 t + 2t^8 \sin t$

Periodic and Non periodic :-

If signal repeat itself after a certain time period then signal is called periodic

Mathematically $x(t) = x(t \pm nT)$

where $n$ = an integer = 1, 2, 3 - - -

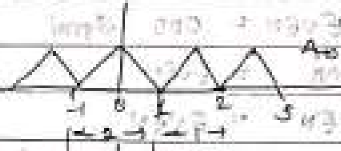
$T$ = fundamental time periodic = smallest +ve value of time for which signal is periodic.

$0 < T < \infty$

$T \neq 0$

$T \neq \infty$

Eg:- (1)



(2)

Given signal is non periodic

Ans $x(t) = A_0 \cos \omega_0 t$

$\omega_0$ = fundamental frequency (Rad/sec)

$= 2\pi f_0$

$T_0 = \dfrac{2\pi}{\omega_0}$

Ques Calculate $T$

(1) $x(t) = 2\sin^2 2t$

$= 1 - \cos 4t$

$\omega_0 = 4$

$T_0 = \dfrac{2\pi}{4} = \pi/2$

(2) $x(t) = 2\sin(2t + 30^\circ)$

$\omega_0 = 2$

$T_0 = \dfrac{2\pi}{\omega_0} = \pi$ Answer

(3) $x(t) = 2 + 2\sin(2t + 30^\circ)$

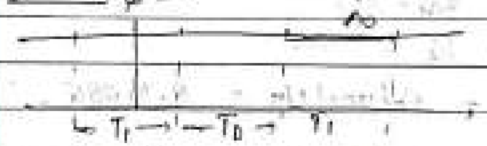
$\omega_0 = 2$

$T_0 = \dfrac{2\pi}{2} = \pi$ Answer

* fundamental time period or freq of a signal is un-affected by shift and phase shift.

* D.C ± periodic signal = Periodic signal

D.C Signal :- $x(t) = A_0$



→ DC signal is a periodic signal.

→ Time periods of D.C signal any +ve value b/w 0 to ∞

→ fundamental time periodic definition does not hold for DC signal.

$f = 1/T$

→ $x(t) = A_0 = \lim_{f\to 0} A_0 \cos(2\pi f t)$

⇒ frequency of DC signal is zero

$T = \frac{1}{f} = \frac{1}{0} = \infty$ ] ×

Equation holds only for

$f_0 = \frac{1}{T_0}$ { fundamental frequency & fundamental time periodic.

{ → fundamental frequency $(f_0)$ = fixed value b/w $(0, \infty)$ $\neq 0$ $\neq \infty$ }

* Sum or difference of two or more than two periodic signal will be periodic if Ratio of their time period or frequency are Rational No.

i.e $= \frac{T_1}{T_2} / \frac{f_1}{f_2} / \frac{\omega_1}{\omega_2}$ = Rational No.

$\boxed{\text{Rational No} = \frac{\text{int}}{\text{int}}}$

→ $\frac{4}{2} = 2$

→ $3/4 = 0.5$

→ $1/3 = 0.333$ - - -

$\frac{22}{7}$ = Rational No = 3.14285

$\pi$ = irrational No = 3.14

$\sqrt{3}$ = I Rational no.

$e^2$ = I.R No.

* $x(t) = x_1(t) + x_2(t) + x_3(t) + \dots$

Conjugate Symmetric antisymmetric.

• For Conjugate Symmetry.

$x(t) = x^*(-t)$

$x(t) = |x(t)| e^{-j\theta t}$

$|x(t)| = |x(-t)|$ even.

$\theta(t) = -\theta(-t)$ odd.

Que $x(t) = t^2 \cos t \; e^{j t^3 \cos^3 t}$

$E \times E = E$ $\quad O \times E = $ odd.

Conjugate Symmetry.

Que $x(t) = A(t) + jB(t)$

→ $A(t) = A(-t)$ ] even

→ $B(t) = -B(-t)$ ] odd

Que $x(t) = \underbrace{\sin^2 t}_{E} + j \underbrace{t^3}_{O} \underbrace{\cos^2 t}_{E}$ = odd.

Conjugate Symmetric.

Conjugate Antisymmetry:-

for Conjugate Antisymmetric

$\boxed{x(t) = -x^*(-t)}$ ?

$x(t) = A(t) + jB(t)$

→ $A(t) = -A(-t)$ even

→ $B(t) = B(-t)$ odd ?

Any Conjugate signal can be represented as a sum of two signal one is Conjugate Symmetrical and other is Conjugate antisymmetric

$$x(t) = x_{cs}(t) + x_{cas}(t)$$

$$x_{cs}(t) = \frac{x(t) + x^*(-t)}{2} \quad \text{v.v.I}$$

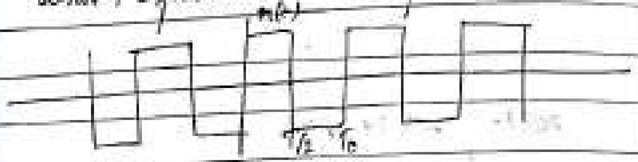
$$x_{cas}(t) = \frac{x(t) - x^*(-t)}{2} \quad \text{v.v.I}$$

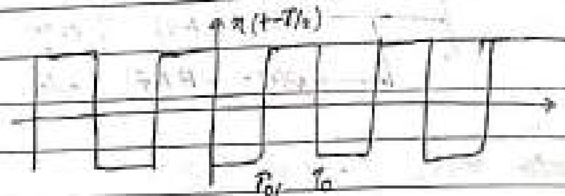
5) Half wave symmetry (HWS)

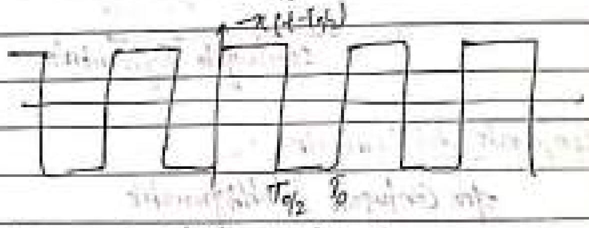
for HWS $\{x(t)\}$

$$\boxed{x(t) = -x\left(t \pm \frac{T}{2}\right)}$$

where $T$ = fundamental time period :




* Average value of signal $x(t)$

$$= \begin{cases} \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt, & \text{Non periodic} \\ \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt, & \text{periodic Signal} \end{cases}$$

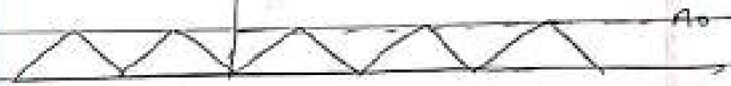
Area of $x(t) = \int_{-\infty}^{\infty} x(t)\,dt$

→ Avg. value of periodic signal, $x(t)$

$$= \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt$$

$$= \frac{\text{area of } x(t) \text{ over 'T'}}{T}$$

Q.




find avg. value.

$$\text{Avg. value} = \frac{1}{2}A_0T_0$$

$$= \frac{\frac{1}{2}A_0T_0}{T_0}$$

$$\boxed{x(t)\ \text{Avg value} = \frac{A_0}{2}}$$

*** To check Half wave Symmetricity of $x(t)$

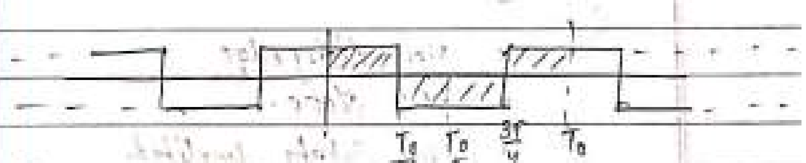
necessary

1) HWS Signal are periodic

2) Avg. value of HWS is '0'

| | Criteria | $t=0$ | | $t = \frac{T}{2}, -\frac{T}{2}$ |
|---|---|---|---|---|
| 3) (a) | Amplitude | $+A$ | → | $-A$ |
| | | $-A$ | → | $+A$ |
| (b) | Slope | $-K$ | → | $+K$ |
| | | $+K$ | → | $-K$ |
| (c) | Edge | Rising | → | falling |
| | | Rising | → | falling. |

Eg:

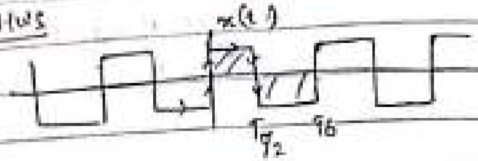



Check wether HWS or not condition

① ✓
② avg value = 0
③ 'o' is satisfied ✓

Here Hws

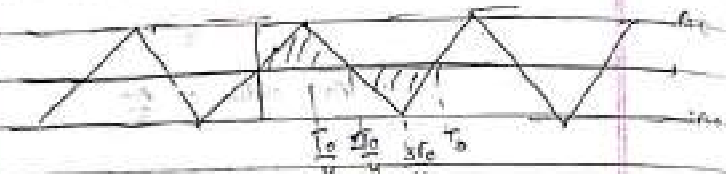
Ans




check Hws ?

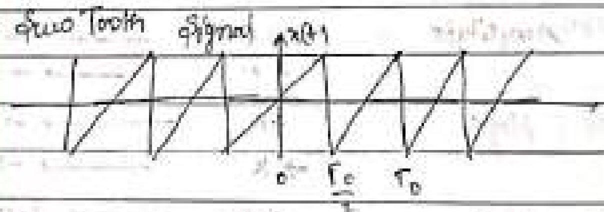
Sol (i) Periodic ✓
(ii) Avg. value ✓
(iii) o is satisfied & edge criterion.

Q:-




Sol ① Periodic ✓
② Avg value = 0 ✓
(3) Slope criteria ✓

Ans Saw Tooth Signal




Sol (i) ✓
(ii) ✓
(iii) ✗ Not Rising edge
✗ No slope.
✗ not satisfy Amplitud

6) Energy signal and Power signal :-

Energy signal.

For energy signal = finite, power = 0

$$\text{Energy of } x(t) = \begin{cases} \int_{-\infty}^{\infty} |x(t)|^2 dt, & x(t) \text{ is complex} \\ \int_{-\infty}^{\infty} x^2(t) dt, & x(t) \text{ is real} \end{cases}$$

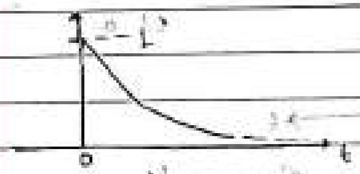
For periodic signal energy is infinite, so they are not energy signal.

An non-periodic signal will be an energy sig. if.

$$\int_{-\infty}^{\infty} |x(t)| dt < \infty$$

↳ signal should be absolutely integral. (magniti

Qus




$x(t) = e^{-at} u(t), a > 0$

$$E(t) = \int_{0}^{\infty} e^{-2at} u(t) dt.$$

$$= \left[ \frac{e^{-2at}}{-2a} \right]_0^{\infty} = \frac{1}{2a}$$

Power of x(t)

$$\Rightarrow \begin{cases} \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt, & R+NP \\ \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt, & C+NP \\ \frac{1}{N} \int_{-T/2}^{T/2} x^2(t) dt. & R+P. \end{cases}$$

$$P = \left\{ \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt, \; c+l \right.$$

$$\boxed{P = \lim_{T\to\infty} \frac{E}{T} = \frac{\text{finite}}{\infty} = 0}$$

Pulse Transform Graph

Area of $x(t) = \int_{-\infty}^{\infty} x(t)\,dt$

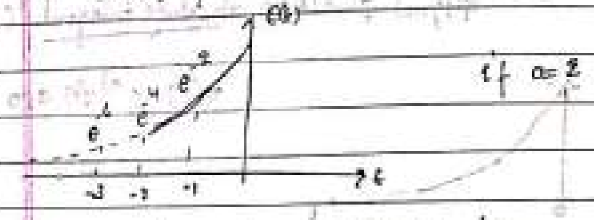
$$= \int_0^{\infty} e^{-at}\, dt = \left[\frac{e^{-at}}{-a}\right]_0^{\infty}$$

$$= \frac{1}{a}$$

2. $x(t) = e^{at}u(-t), \; a>0$

$E = ?$

Area $= ?$



if $a=2$

Area $= \frac{1}{a}$ Energy $= 1/2a$

Ques $x(t) = e^{-a|t|} \quad a>0$

$E = ?$, Area $= ?$

Soln

$$x(t) = \begin{cases} e^{-at} & t>0 \\ e^{at} & t<0 \end{cases}$$

$$E = E_{left} + E_{right}.$$

$$= 1/2a + 1/2a = 1/a$$

$$\text{Area} = \frac{1}{a} + \frac{1}{a} = 2/a$$

Power Signal :- for power signal.

Power = finite, energy = $\infty$

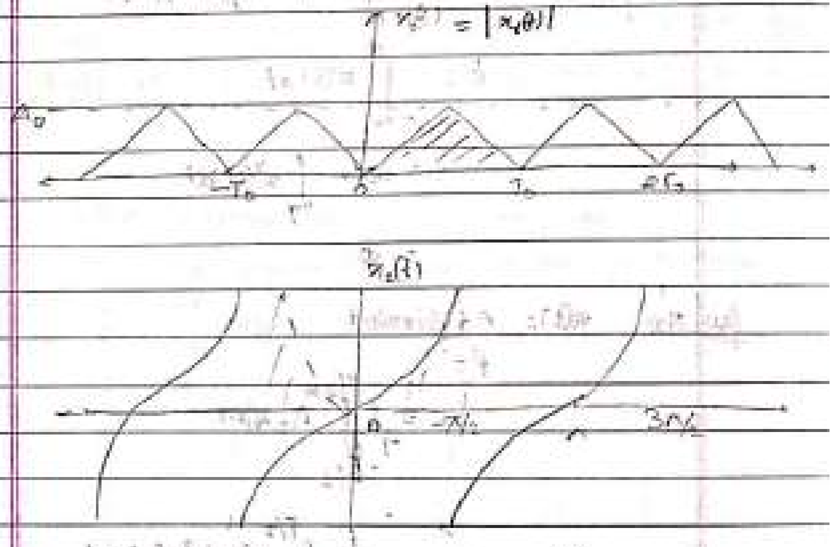
$$E = \lim_{T\to\infty} P \times T = \text{finite} \times \infty = \infty$$

A periodic signal will be a power signal if.

$$\frac{1}{T}\int_{-T/2}^{T/2} |x(t)|\, dt < \infty$$

Signal should be integrable over its time period

Exam



which is power signal ?

from $x_1(t)$

$$\text{area} = \int_{-T/2}^{T/2} x(t)\,dt < \infty$$

= area of $x_1$ over period 'T' finite.

from $x_2(t)$

$$\text{area} = \int_{-T/2}^{T/2} x_2(t)\,dt < \infty$$

= area of $x_2(t)$ over period T.

$= \infty$

Hence first signal is power signal. and second signal is not a power signal

☆ Second signal is periodic signal Hence it is not a energy signal.

first signal area of whole system is $\infty$.

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt.$$

$$= \infty \times \int_T x^2(t)\,dt.$$

$$= \infty.$$

Ques 8.) $x(t) = A_0 \sin\omega_0 t$

$P = ?$

$$= \frac{1}{T}\int_{-T/2}^{T/2} A_0 \sin t$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} A_0^2 \sin^2 \omega_0 t\, dt$$

$$= \frac{A_0^2 \times 2}{T} \int^{T/2} \frac{1-\cos 2\omega_0 t}{2}\,dt.$$

$$= \frac{A_0^2}{T}\left[t - \frac{\sin 2\omega_0 t}{2\omega_0}\right]_0^{T/2}$$

$$= \frac{A_0^2}{T}\left[(T/2 - 0) - \frac{\sin(2\omega_0 T/2)}{2\omega_0} - 0\right]$$

$$= \frac{A_0^2}{T} \times \frac{T}{2} = \frac{A_0^2}{2}$$

Power = $(RMS)^2$

RMS → Root mean square.

$$P = \left(\frac{A_0}{\sqrt{2}}\right)^2 = \frac{A^2}{2}$$

Ques $x(t) = A_0 \sin(\omega_0 t + \phi)$

$$P = \frac{A_0^2}{2}$$

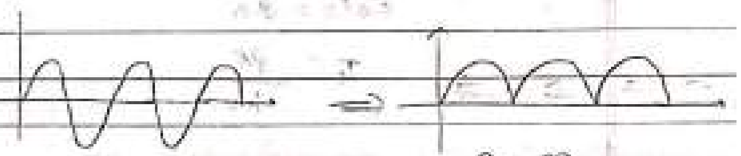
// Power and RMS of a power sig. is unaffected by time shift, phase shift, change in frequency or time period.

Ques $x(t) = A_0 \sin\omega_0 t\; u(t)$ is non period.

A nonperiodic signal will be a power signal if

(i) $\int_{-\infty}^{\infty} |x(t)|\,dt = \infty$

(ii) $x(t) \neq \infty$ at any 't'



$A = \infty$

$x(t) \neq \infty$ at any 't'

it is power signal. cause above 2 condition satisfied

$$\text{Power} = \left(\frac{A_0^2}{2}\right)/2 = \frac{A_0^2}{4}$$

* For Power signal :- P = finite, RMS = finite

* For Energy signal . P = 0 , RMS = 0.

* For Neither Energy Nor Power signal RMS = ∞

Ex: $x(t) = A_0 e^{j\omega_0 t}$ } Periodic signal.

P = ?

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} A_0^2\,dt = A_0^2$$

RMS = $A_0$

** → Complex exponential in Continuous time system are always periodic.

$$x(t) = x(t+T_0)$$

$$\Rightarrow e^{j\omega_0 t} = e^{+j\omega_0 (t+T_0)}$$

$$e^{j\omega_0 t} = e^{+j\omega_0 t} e^{j\omega_0 T_0}$$

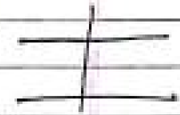
$$e^{j\omega_0 T_0} = 1 = e^{j2\pi} = \underbrace{\cos 2\pi}_{1} + j\underbrace{\sin 2\pi}_{0}$$

$$\omega_0 T_0 = 2\pi$$

$$T_0 = \frac{2\pi}{\omega_0}.$$

Ex: $x(t) = A_0$.

P = ? , RMS = ?



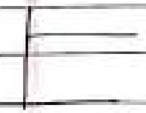
Sol: $P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$

$$= \frac{1}{T}\int_{-T/2}^{T/2} A_0^2\,dt$$

$$P = \frac{1}{T}\cdot A_0^2 T = A_0^2$$

RMS = $A_0$

Ex: $x(t) = A_0 u(t)$



$P = A_0^2/2$

RMS = $A_0/\sqrt{2}$

Root Mean Square Value (RMS) :-

$$= \begin{cases} \sqrt{\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt}, & NP+R \\ \sqrt{\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt}, & NP+C \end{cases}$$

$$= \begin{cases} \sqrt{\frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt}, & P+R \\ \sqrt{\frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt}, & P+C \end{cases}$$

orthogonal signal :-

Signal $x_1(t)$ & $x_2(t)$ are said to be orthogonal if

$\rightarrow \int_{-\infty}^{\infty} x_1(t) \cdot x_2(t) dt = 0$ } for non-periodic $x_1(t)$ & $x_2(t)$

$\rightarrow \int_{-T/2}^{T/2} x_1(t) \cdot x_2(t) dt = 0$ } for periodic $x_1(t)$ and $x_2(t)$

$T = LCM[T_1, T_2]$

* Effect of orthogonality on power and energy calculation of sum or difference of two or more signal.

if $x(t) = x_1(t) + x_2(t)$

and $x_1(t)$ and $x_2(t)$ are orthogonal then

* $E_{x(t)} = E_{x_1(t)} + E_{x_2(t)}$ } when $x_1$ and $x_2$ are energy sig

* $P_{x(t)} = P_{x_1(t)} + P_{x_2(t)}$ ] when $x_1(t)$ and $x_2(t)$ are power signal.

Some orthogonal pair :-

1) $\int_{T_0} \underbrace{\sin\omega_0 t \cdot \cos\omega_0 t}_{\text{orthogonal}} \, dt = 0$

$\int x_1(t) \cdot x_2(t) dt = 0$

Ex :- $x(t) = A_1 \cos\omega_0 t + A_2 \sin\omega_0 t$

$P_{x(t)} = ?$

Sol: $\cos\omega_0$ & $\sin\omega_0 t$ are orthogonal.

$P_{x(t)} = P_1 + P_2$

$= \frac{A_1^2}{2} + \frac{A_2^2}{2} = \frac{A_1^2 + A_2^2}{2}$

$RMS_{x(t)} = \sqrt{\frac{A_1^2}{2} + \frac{A_2^2}{2}}$

2) $\int \sin(\omega_1 t + \phi_1) \cdot \sin(\omega_2 t + \phi_2) \, dt = 0$

$\omega_1 \neq \omega_2$

Eg:- $x(t) = 2\sin(4\pi t + 36^\circ) + 4\sin(8\pi t + 37^\circ)$

$P_{x(t)} = ?$ orthogonal because of $\boxed{\omega_1 \neq \omega_2}$

$P_{x(t)} = P_1 + P_2 = \frac{2^2}{2} + \frac{4^2}{2}$

$= 10$

3) $\int_T \cos(\omega_1 t + \theta_1) \cos(\omega_2 t + \theta_2) \, dt = 0$

$\omega_1 \neq \omega_2$

Eg:- $x(t) = 8\cos(3\pi t + 77^\circ) + 6\cos(9\pi t + 97^\circ)$ orthogonal

$P_{x(t)} = ?$

$P_{x(t)} = P_1 + P_2 \Rightarrow \frac{8^2}{2} + \frac{36}{2} = 50.$

4) $\int (\sin(\omega_1 t + \phi) \cdot \cos(\omega_2 t + \phi)) \, dt = 0$

$\omega_1 \neq \omega_2$

* Two sinusoidal function of different frequency are always orthogonal irrespective of phase difference.

Eg:- $x(t) = 3\sin(3\pi t + 35^\circ) + 4\cos(9\pi t + 78^\circ)$

$P_{x(t)} = P_1 + P_2 \Rightarrow 9/2 + 16/2$

$= 25/2 = 12.5$

**Q. Sol**

$$\int_{T_1} A_0 \sin(\omega_1 t+\phi)\,dt = 0 \quad \text{sinusoidal signal.}$$

$$\int_{T_2} A_0 \cos(\omega_1 t+\phi_1)\,dt = 0$$

d.c and sinusoidal signal are always orthogonal.

**Qu**

$$x(t) = \begin{cases} 2+4\cos(8\pi t+60^\circ) \\ 2+4\sin(8\pi t+77^\circ) \end{cases}$$

$$P_{x(t)} = ? = P_1+P_2 = 2^2+\frac{4^2}{2} = 12$$

**Qu**

$$x(t) = A_1 \sin\omega_1 t + A_2 \sin(\omega_1 t+\phi_1)$$

$$A_1 \cos\omega_1 t + A_2 \cos(\omega_1 t+\phi_1)$$

$$\phi_1 \neq 0,\ \pi/2$$

**Sol**

$$= A_0 \cos[\omega_1 t + \theta_1]$$

$$P_{x(t)} = \frac{A_0^2}{2}$$

$$A_0 = \sqrt{A_1^2+A_2^2+2A_1A_2\cos\phi_1}$$

**Qu** find RMS of $x(t)$

$$x(t) = 2\sin 6t + 3\cos(6t+\pi/3)$$

$$A_0 = \sqrt{4+9+\frac{12}{2}} = \sqrt{19}$$

$$P_{x(t)} = \frac{19}{2} = 9.5$$

**Sol**

$$x(t) = 2\sin 6t + 3\sin\left[6t+\frac{\pi}{3}+\frac{\pi}{2}\right]$$

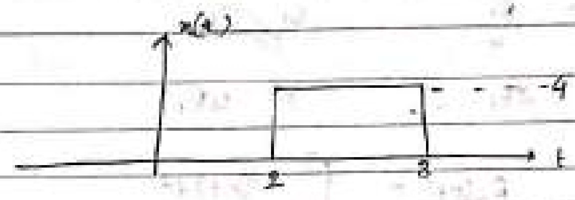
$$= 2\sin 6t + 3\sin\left(6t+\frac{5\pi}{6}\right)$$

$$A_0 = \sqrt{4+9+2\times2\times3\cos150^\circ} = \sqrt{13+12\left(\frac{-\sqrt3}{2}\right)}$$

$$RMS = \frac{A_0}{\sqrt2} = 1.14$$

$P_{x(t)} =$

**Qu**



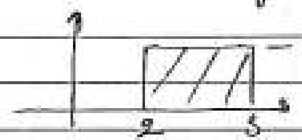

$E_x = ?$

**Sol**

$$E_{x(t)} = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_2^3 4^2\,dt = 16$$

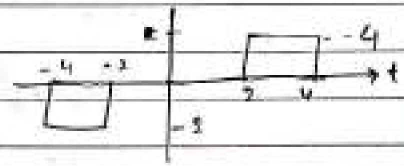
$$E_{x(t)} = \int_{-\infty}^{\infty} x^2(t)\,dt$$

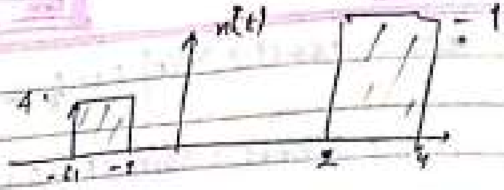
Area of $x^2(t)$




$= 16\times1 = 16$

**Qu**





$E_{x(t)} =$ Area of $x^2(t)$

$= 8 + 32 = 40$

Energy.

**Que** If

$x(t) \longrightarrow E_1 = \int_{-\infty}^{\infty} x^2(t)\, dt$

$x(2t) \longrightarrow ?$

(a) $\frac{E_1}{4}$ (b) $\frac{E_1}{2}$ ✓

(c) $2E_1$ (d) $4E_1$

$E_{x(2t)} = \int_{-\infty}^{\infty} x^2(2t)\, dt.$

let $2t = P \Rightarrow dt = \frac{dP}{2}$

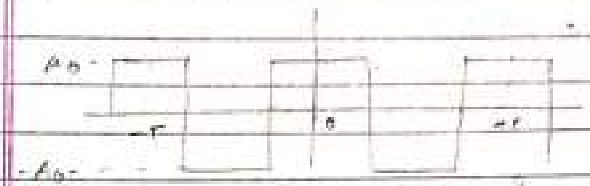
$E' = \int_{-\infty}^{\infty} x^2(P) \cdot \frac{dP}{2} = \frac{1}{2}\int_{-\infty}^{\infty} x^2(P)\, dP$

$= \frac{E_1}{2}$

$x(at),\ at \longrightarrow \frac{E_1}{|a|}$

Basic Signal :-

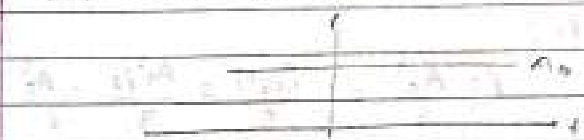
1) Symmetrical Square wave :-



Signal is even, Hence

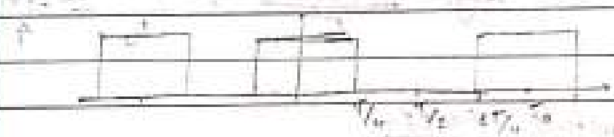
$P_{x(t)} = P_{y(t)} = A_0^2$

Signal having same modulus value contain same power.



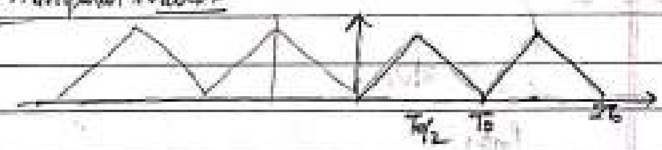
$\boxed{P_{y(t)} = A_0^2}$

**Ex**



$P_{x(t)} = \frac{P_{x(t)}}{2} = \frac{A_0^2}{2}$ half of $x(t)$ because the signal is half only.

2) Triangular Wave :-



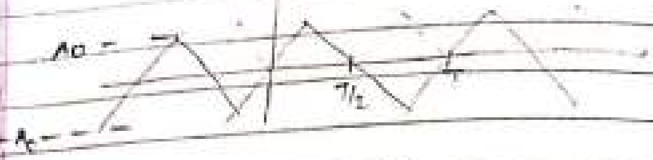
Avg Value $= \frac{\text{area over } T_0}{T_0} = \frac{\frac{1}{2} A_0 T_0}{T_0} = \frac{A_0}{2}$

$P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\, dt.$

$= \frac{2}{T}\int^{T/2} x^2(t)\, dt.$

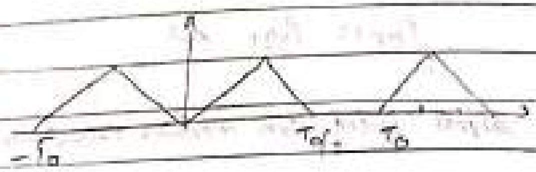
$$\frac{2}{T_0} \cdot 2 \int_0^{T_0/2} \left(\frac{2A_0}{T_0} t\right)^2 dt = \frac{A_0^2}{3}$$

Eg



$$P_{x(t)} = P_{x(t)} = \frac{A_0^2}{3}$$

Eg

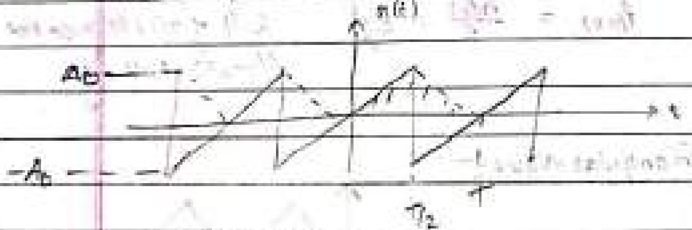


$P = ?$

$$P = \frac{A_0^2}{6} = \frac{P_{x(t)}}{2} = \frac{A_0^2/2}{3} = \frac{A_0^2}{6}$$

$$\text{Avg} = \frac{\text{area over } T_0}{T_0} = \frac{\frac{1}{2} A_0 \frac{T_0}{2}}{T_0} = \frac{A_0}{4}$$

Sawtooth wave :-



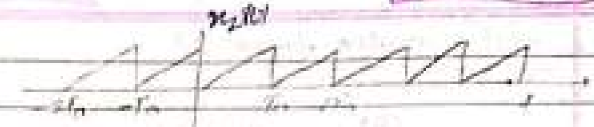
$x_0 \neq x(t)$

$P_{x(t)} = ?$

$$P_{x(t)} = \frac{A_0^2}{3}$$

Avg value of given signal = 0

Eg

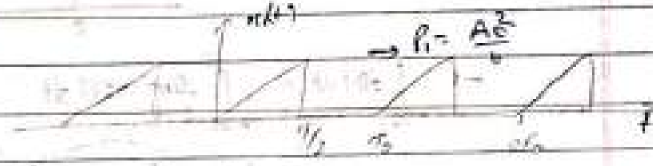


$$\text{Avg value} = \frac{\frac{1}{2} A_0 T_0}{T_0} = \frac{A_0}{2}$$

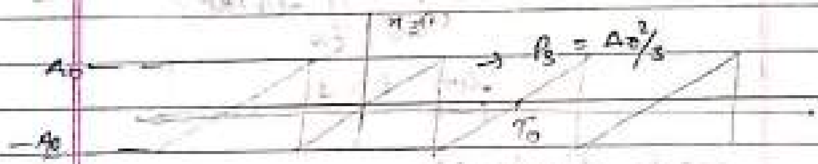
$$P_{x_2(t)} = \frac{1}{T_0}\int_0^{T_0} x^2(t)\,dt$$

$$= \frac{1}{T_0}\int_0^{T_0} \left(\frac{A_0}{T_0}\right)^2 t^2\,dt = \frac{A_0^2}{3}$$

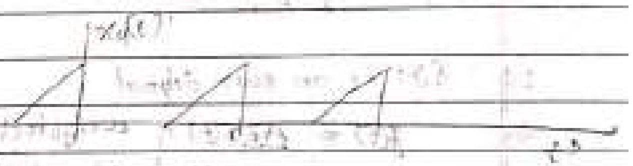
Eg :



$\rightarrow P_1 = \frac{A_0^2}{6}$



$\rightarrow P_3 = A_0^2/3$



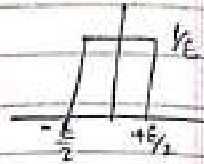
$$x(t) = x_1(t) + x_4(t)$$

$$P_{x_2} = P_{x_1} + P_{x_4}$$

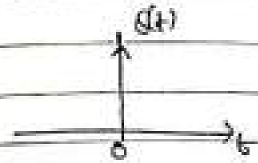
$$= \frac{A_0^2}{6} + \frac{A_0^2}{6} = \frac{A_0^2}{3}$$

## Unit impulse signal :-

$$\delta(t) = \lim_{\epsilon \to 0} x(t)$$




$$= \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$




$$\rightarrow \int_{-\infty}^{\infty} \delta(t)\,dt = \int_{-\infty}^{\infty} \lim_{\epsilon \to 0} x(t)\,dt$$

$$= \lim_{\epsilon \to 0} \int_{-\infty}^{\infty} x(t)\,dt$$

area of $x(t) = 1$

$$\lim_{\epsilon \to 0} 1 = 1$$

Properties of impulse :-

1) $\delta(t)$ is an even signal

2) $y(t) = A_0 \delta(t) =$ weighted impulse.
weight or strength of impulse.

$$\int_{-\infty}^{\infty} A_0 \delta(t)\,dt = A_0 \left[ \int_{-\infty}^{\infty} \delta(t)\,dt \right]$$

$$= A_0 \times 1 = A_0$$

Area under weighted impulse = weight of impulse.

$$y(t) = \begin{cases} A_0 \times \infty, & t = 0 \\ 0 & t \neq 0 \end{cases}$$


3) $$x(t-t_1) \cdot \underbrace{\delta(t-t_2)}_{t - t_2 = 0,\ t = t_2} = x(t_2 - t_1)\,\delta(t - t_2)$$

Eg.

$$\cos t \cdot \delta(t - \pi) = \ ?$$

$$= \cos \pi \,\delta(t-\pi) = -\delta(t-\pi)$$

4) $$\delta(at) = \frac{1}{|a|}\delta(t), \quad a \neq 0$$ Scaling property

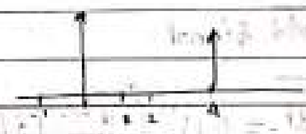
Eg :- $(1+t^2)\,\delta(2t-1) = \ ?$

$$= \left(1 + \frac{1}{4}\right) \cdot \frac{1}{2}\,\delta\left(t - \frac{1}{2}\right)$$

$$\frac{5}{8}\,\delta\left(t - \frac{1}{2}\right)$$

5) $$\int_{-\infty}^{\infty} x(t-t_1)\,\delta(t-t_2)\,dt = x(t_2 - t_1)$$ using property 3

Qu $$\int_{-1}^{2} \delta(t-4)\,dt = \ ? \quad 0$$




Qu $$\int_{-3}^{4} (2t^2 + 3t^3)\,\delta(2t-4)\,dt = \ ?$$

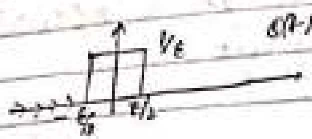
$$\int_{-3}^{4} (8 + 24)\,\delta(2(t-2))\,dt$$

$$= \frac{32}{2}\int_{-3}^{4} \delta(t-2)\,dt = 16$$

6th Doublet function

$1^{st}$ derivative of impulse is known as Doublet function

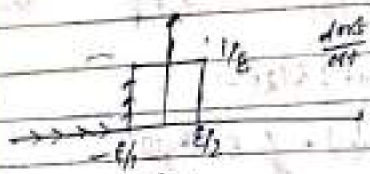
Doublet function $\frac{d}{dt}[\delta(t)]$



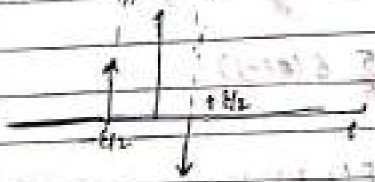
$\delta(t) = \lim_{\epsilon \to 0} x(t)$

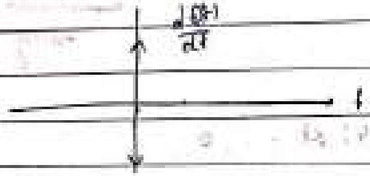
$= \frac{d}{dt}\left[\lim_{\epsilon \to 0} x(t)\right]$

$= \lim_{\epsilon \to 0}\left[\frac{d\,x(t)}{dt}\right]$



$\frac{dx(t)}{dt}$ = Slope of $x(t)$ wrt '$t$'





*** Doublet function is a odd signal

$$\boxed{\int_{-\infty}^{\infty} x(t)\cdot\delta^{n}(t-t_1)\,dt = (-1)^n \int_{-\infty}^{\infty} \frac{d^n x(t)}{dt^n}\bigg|_{t=t_1} \delta(t-t_1)\,dt}$$

$$\boxed{= (-1)^n \frac{d^n x(t)}{dt^n}\bigg|_{t=t_1}}$$

Que $E = \int_{-\infty}^{\infty} (t^2+2t+1)\,\frac{d^2}{dt^2}\delta(t-1)\,dt = ?$

$= (-1)^2 (2)\big|_{t=1}$

$= 2$

5th Unit step signal :→

$u(t) = \begin{cases} 1 & , t > 0 \\ 0 & , t < 0 \end{cases}$

→ $u(t)$ discontinuous at $t = 0$

→ Gibb's phenomenon :-

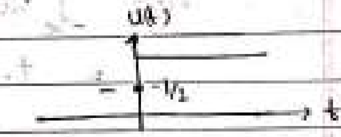
Signal value at the point of discontinuity is Given by the average of ~~the~~ signal value taking just before and after the point of discontinuity.

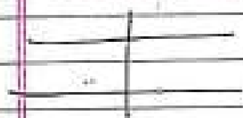
$$\boxed{u(0) = \frac{\overset{1}{u(0^+)} + \overset{0}{u(0^-)}}{2}} = 1/2$$

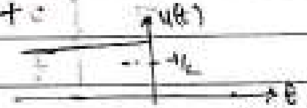
$u(t) = \begin{cases} 1 & ; t > 0 \\ 1/2 & ; t = 0 \\ 0 & ; t < 0 \end{cases}$

Properties of $u(t)$

1) $u(t) + u(-t) = 1$



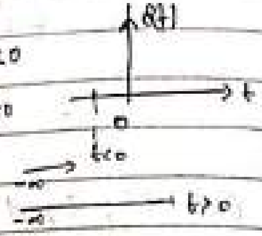




+

(3) $u(t)$ = is a power signal $P = 1/2$

(5) $$\int_0^t \delta(t)\, dt = u(t)$$
$$= \begin{cases} 0 ; & t<0 \\ 1 ; & t>0 \end{cases}$$
$$= u(t)$$

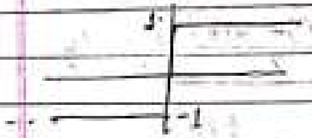


4) $$\frac{d\,u(t)}{dt} = \delta(t)$$

6) Signum function :-

$$sgn(t) = \begin{cases} 1, & t>0 \\ 0, & t=0 \\ -1, & t<0 \end{cases}$$

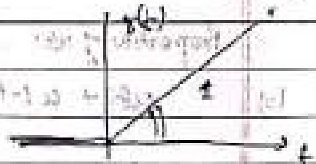
$$sgn(t) = \begin{cases} 1 & t>0 \\ -1 & t<0 \end{cases}$$



sgn(t) is discontinuous at t=0

7) Unit Ramp function :-

$$r(t) = \int_{-\infty}^{t} u(t)\, dt$$
$$= t \cdot u(t)$$



$$= \begin{cases} 0 & t<0 \\ t & t>0 \end{cases}$$

8) Parabolic function :->

$$p(t) = \int_{-\infty}^{t} r(t)\, dt = \frac{t^2}{2} \cdot u(t)$$

9) Sampling function :->

$$Sa(t) = \frac{\sin t}{t}$$

→ $Sa(0) = \lim_{t \to 0} \frac{\sin t}{t} = 1$

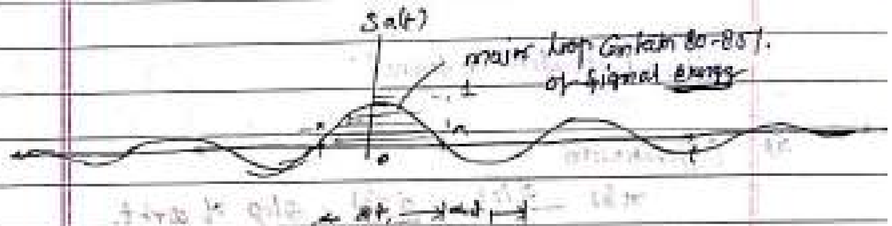
→ $Sa(\infty) \Rightarrow 0$

If $Sa(t) = 0$

$t = ?$

$\frac{\sin t}{t} = 0 \Rightarrow \sin t = 0$

$t = n\pi$ where $n = \pm 1, \pm 2, \ldots$




⇒ fourier Transform

$$E_{Sa(t)} = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_{-\infty}^{\infty} \left(\frac{\sin t}{t}\right)^2 dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \left(\frac{1-\cos 2t}{t^2}\right) dt = \frac{1}{2}\int_{-\infty}^{\infty} \left(\frac{1}{t^2} - \frac{\cos 2t}{t^2}\right) dt$$

$$= \pi \quad \text{Ans}$$

10 → Sinc function :-

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$= Sa(\pi t)$$ $\boxed{F.T.}$

$\text{sinc}(0) = 1$

$\text{sinc}(\infty) = 0$

if $\text{sinc}(t) = 0$ $t = ?$

$$\frac{\sin \pi t}{\pi t} = 0$$

$\sin \pi t = 0$

$\pi t = n\pi$

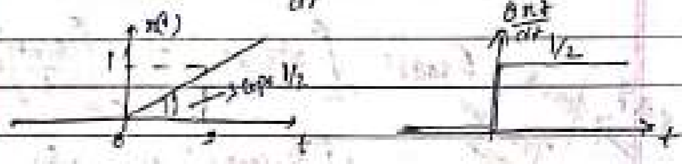
$\boxed{t = n}$ $n = \pm 1, \pm 2, ---$

Different operation in signal

a) Differentiation

$$x(t) \xrightarrow{d/dt} \frac{d\,x(t)}{dt} = \text{slop of curve } t.$$

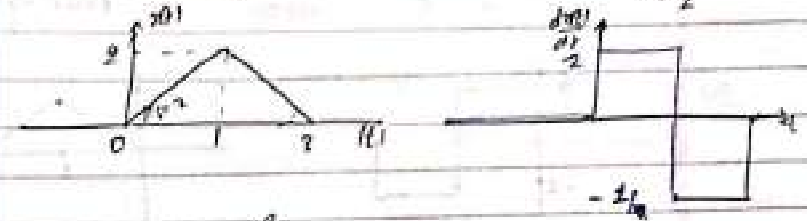
Ex)



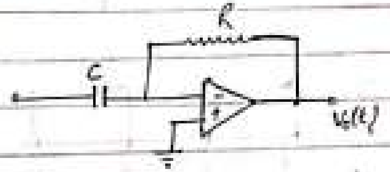
Ex)

Ex)

Ex)



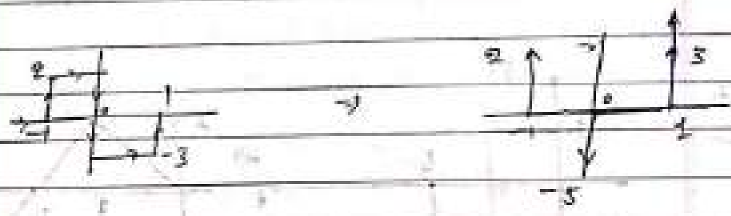
Ex)



$\boxed{RC = 1}$

$$V_o(t) = -RC\frac{dV_i(t)}{dt} = \frac{-d\,x(t)}{dt}$$

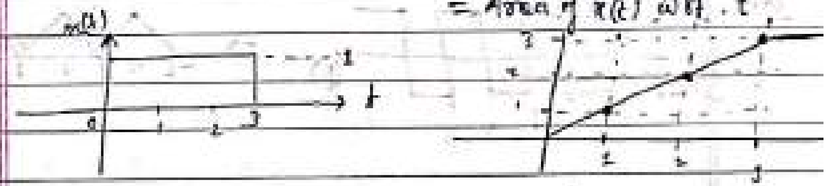
Ex)



Integration :-

$$x(t) \xrightarrow{\int_{-\infty}^{t}} y(t) = \int_{-\infty}^{t} x(t)\,dt$$

= Area of $x(t)$ w.r.t. 't'.



$y(t) = \int^{t} x(t)\,dt.$

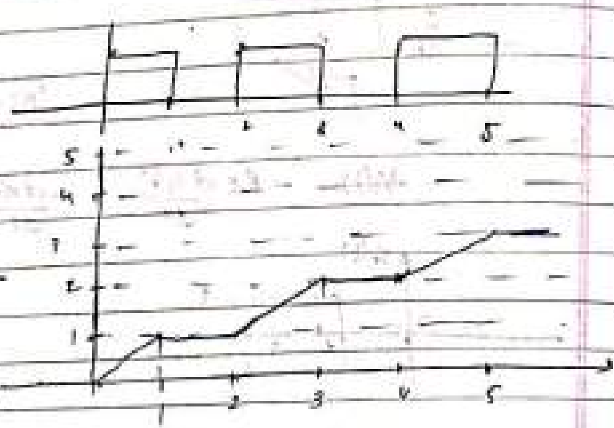
beyond zero $-\infty$

$\begin{cases} t=1 & \text{area} = 1 \\ t=2 & \longrightarrow -2 \\ t=3 & \longrightarrow 3 \\ t=4 & \end{cases}$

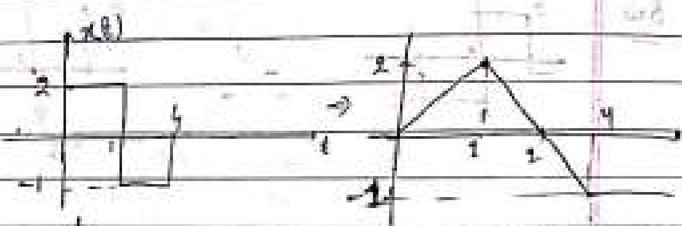
$t=5$ area $= 3$ cause after $t=3$ signal

Qu

Q.

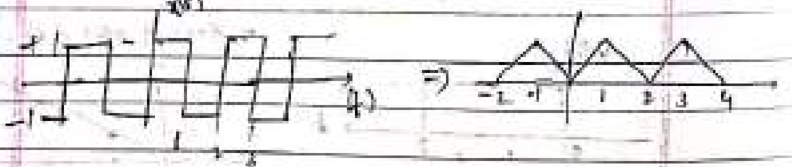


Qu

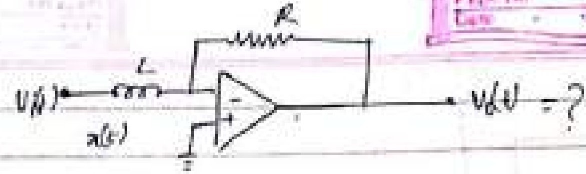


$$\int_{-\infty}^{t} x(t)\,dt$$

Qu



Qu



$V_o(t) = ?$

$\frac{L}{R} = 1$

$$V_o(t) = -\frac{R}{L}\int_0^t V_i(t)\,dt$$

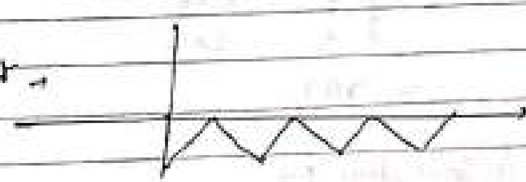
$V_i(t) = x(t)$ from prev

$$= -\int_0^t V_i(t)\,dt = -\int_0^t x(t)\,dt$$

wrong bcz of given opamp's inverting

$\int x(t)dt \rightarrow$

$-\int x(t)dt \rightarrow$



6 ⇒ Convolution

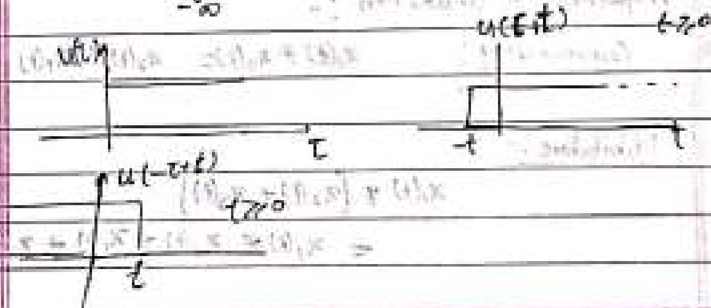
$y(t) = x_1(t) * x_2(t)$

Convolution operator

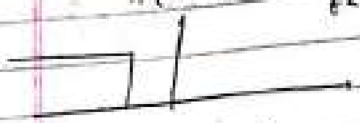
$$= \int_{-\infty}^{\infty} x_1(\tau)\cdot x_2(t-\tau)\,d\tau$$

Qu $y(t) = u(t) * u(t)$

$$= \int_{-\infty}^{\infty} u(\tau)\cdot u(t-\tau)\,d\tau$$



$u(-\tau+t)$ $t \geq 0$



$$y(t) = \int_{-\infty}^{\infty} u(\tau) \cdot x(t-\tau)\, d\tau$$

$$= \begin{cases} 0 & t \leq 0 \\ \int_0^t 1 \cdot 1\, dt & t > 0 \end{cases}$$

$$= \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}$$

$$= r(t)$$

Steps in Convolution :-

$$y(t) = x_1(t) * x_2(t)$$

$$= \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau)\, d\tau$$

$x_1(t)$ . $x_2(t)$

$x_2(\tau+t)$ ⟶ shifting

⟶ $\times$ $x_2(-\tau+t)$ ⟶ folding

↓ multiplication

$\int_{-\infty}^{\infty}$ integration

Properties of Convolution :-

1) Commutative :- $x_1(t) * x_2(t) = x_2(t) * x_1(t)$

2) Distributive :-

$$x_1(t) * [x_2(t) + x_3(t)]$$

$$= x_1(t) * x_2(t) + x_1(t) * x_3(t)$$

3) Associative :-

$$[x_1(t) * x_2(t)] * x_3(t) = x_1(t) * [x_2(t) * x_3(t)]$$

4) Impulse Response property :-

$$x(t-t_1) * \delta(t-t_2) \Rightarrow x[(t-t_2)-t_1]$$

Q. ① $x(t) * \delta(t) = ?$

$= x(t)$

② $x(t) * \delta(t-t_1) = x(t-t_1)$

5) Step Response property of Convolution

$$x(t) * u(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

6) Differentiation

If $y(t) = x_1(t) * x_2(t)$

$$\frac{dy(t)}{dt} = \frac{dx_1(t)}{dt} * x_2(t)$$

$$= x_1(t) * \frac{dx_2(t)}{dt}$$

7) Duration property

If $y(t) = x_1(t) * x_2(t)$

$x_1(t)$ ⟶ $t_1 \leq t \leq t_2$

$x_2(t)$ ⟶ $t_3 \leq t \leq t_4$

$y(t)$ = $(t_1+t_3) \leq t \leq (t_2+t_4)$

8) Scaling Property :-

Let $x_1(t) * x_2(t) = y(t)$

then $x_1(at) * x_2(at) = \frac{1}{|a|} y(at)$, $a \neq 0$

Que $y(t) = u(t) * u(t)$

Sol^n 2^nd Method

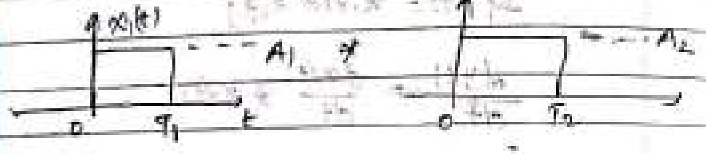
$$\frac{dy(t)}{dt} = \underbrace{\frac{du(t)}{dt}}_{\to \delta(t)} * u(t)$$

$$y(t) = \int_{-\infty}^{\infty} \frac{dy(t)}{dt}\, dt = \int_{-\infty}^{t} \delta(t) * u(t)$$

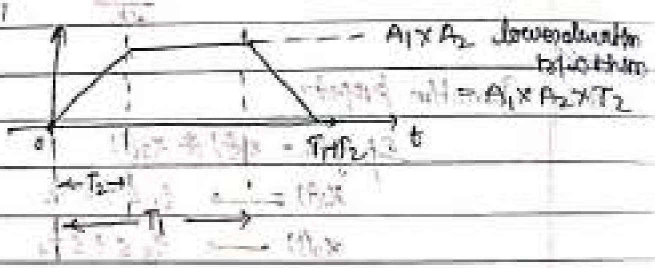
$$= \int_{-\infty}^{t} u(t)\, dt = r(t)$$

V.V.I: Convolution of two Rectangular pulses of ~~two~~ unequal duration will be a trapezoid

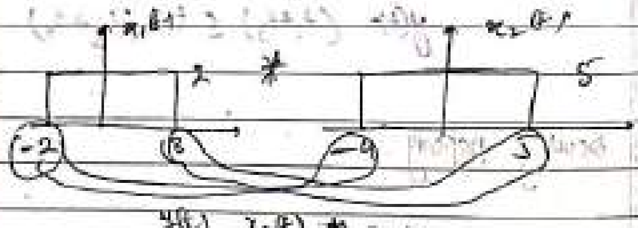
Ex




Assume $T_2 < T_1$



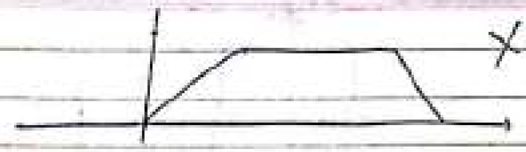

Que

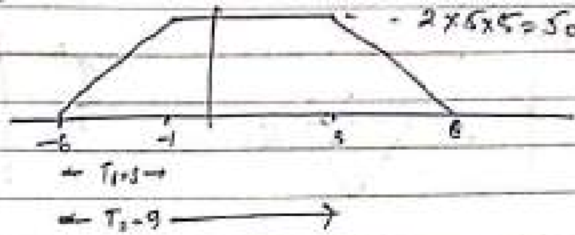



$y(t) = x_1(t) * x_2(t)$

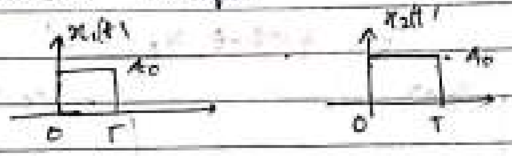
Sol^n

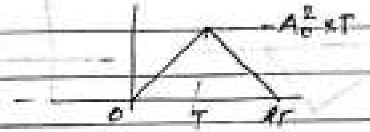
$\boxed{T_1 = 5, \; T_2 = 9}$



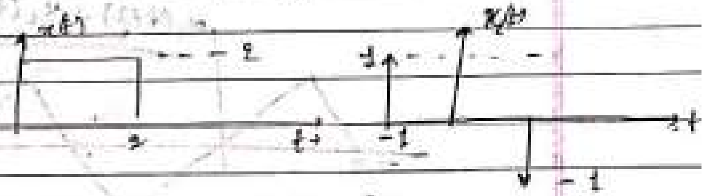



Imp. Convolution of two Rectangular pulses of equal duration will be a Triangle.




Que




$y(t) = x_1(t) * x_2(t)$

$x_2(t) = \delta(t+1) + \delta(t-1)$

$x_1(t) * x_2(t) = x_1(t+1) + x_1(t-1)$